LECTURES

ON

SELECT SUBJECTS

IN

MECHANICS,
HYDROSTATICS,
HYDRAULICS,
PNEUMATICS,

OPTICS,
GEOGRAPHY,
ASTRONOMY, AND
DIALLING.

BY JAMES FERGUSON, F. R. S.

WITH

NOTES, AND AN ADDITIONAL VOLUME,

CONTAINING

THE MOST RECENT DISCOVERIES IN THE ARTS AND SCIENCES.

BY DAVID BREWSTER, LL. D.

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OF MOUNT ANNAN,

FELLOW OF THE ROYAL SOCIETIES OF LONDON & EDINBURGH,

THIS ENLARGED EDITION

OF

FERGUSON'S LECTURES

IS INSCRIBED,

IN TESTIMONY OF THE RESPECT AND ESTEEM

OF

THE EDITOR.

CONTENTS OF THE SECOND VOLUME.

CHAPTER I.

On Mechanical Agents, or the First Movers of Machinery *page* 1

Sect. I. On the Strength of Men and Animals . . . 2

Sect. II. On the Force of Water, and the mode of applying it to drive machinery. 11

On Undershot Wheels . . . . *ib.*

On Horizontal Mills 38

On Double Corn-Mills 42

On Besant's Undershot Wheel 44

On Breast-Wheels 45

On Overshot Wheels 47

On a Double Overshot Wheel, with a Chain of Buckets 57

On the Performance of Overshot and Undershot Wheels 59

On Wheels driven by the reaction of Water, or Barker's Mill 60

Method of keeping off the back-water from Water-Wheels 63

Sect. III. On the Force of Wind, and the mode of applying it to drive machinery 64

On Vertical Windmills 64

On the Form and Position of Windmill Sails . . . 69

On the Effect of Windmill Sails 81

Sect. IV. On the Force of Steam, and the method of applying it to drive machinery 89

Description of Mr. Watt's Steam-engine 91

Account of Mr. Woolf's Steam-engine 104

On High Pressure Steam-engines 107

On the Power of Steam-engines 109

On Steam-boats 112

On Steam-carriages 118

CHAPTER II.

On the Formation of the Teeth of Wheels, the Leaves of Pinions, and the Wipers of Stampers 119

On the Teeth of Rack-work 124

On the Form of Spindles for Lanterns 125

On the Wipers of Stampers 127

On Bevelled or Conical Wheels 135

On Interior and Exterior Epicycloids, and the Disposition of the Teeth of Wheels 138

Tables of the Pitches of Wheels 146

CHAPTER III.

On the Nature of Friction, and the method of diminishing its effects in machinery . . . . page 148
On the Friction of Surfaces 154
On the Friction of Axles 163
On the Friction of Ropes 166
On the Friction and Form of Pivots 169
On the Method of diminishing Friction in machinery 173

CHAPTER IV.

On the Elements of Machinery, and the Contrivances used in the Composition of Machines 176

CHAPTER V.

On the Nature and Operation of Fly-wheels, and other Contrivances for regulating the motion of machinery . . 194
On Prony's Condenser of Forces 199
On Samuel's Regulator of the Action of Horses . . 201

CHAPTER VI.

On the Maximum Effect of Machines 204

CHAPTER VII.

On Wheel-carriages 210
On the Size and Form of Carriage Wheels . . . 210
On the Position of Carriage Wheels 221
On the Line of Traction, and the Length of Traces 222
On the Position of the Centre of Gravity, &c. . . . 226

CHAPTER VIII.

On the Forces of Elasticity and Torsion 229
Gravesende's Apparatus for measuring Elasticity . 229
Mechanical Teinometer 231
Chromatic Teinometer 232
Coulomb's Torsion Balance 235
Coulomb's Apparatus for measuring the flexion of Plates 242

CHAPTER IX.

On the Strength of Materials 244
1. When pulled in the direction of their Length . . 244
2. When compressed in the direction of their Length 246
3. When exposed to a Lateral Strain 250
4. When twisted 259

CHAPTER X.

Description of different Mechanical Engines . . . *page* 261
1. Description of a simple and powerful Capstane . . *ib.*
2. On the Principle and Construction of Lever Presses 264
3. Description of the Thrashing Machine 267

CHAPTER XI.

Account of the Apparatus for making and burning Gas . . 271

CHAPTER XII.

Account of the Safety Lamp for lighting Mines 275

CHAPTER XIII.

On the Principles and Construction of Balloons 278

CHAPTER XIV.

Description of the Diving Bell 281

CHAPTER XV.

Description of some Hydrostatic and Hydraulic Engines . 285
1. Description of the Water Blowing Machine . . . 285
2. Description of Whitehurst's Engine for raising Water 287
3. Description of Montgolfier's Hydraulic Ram . . 288

CHAPTER XVI.

On the Principles and Construction of Optical Instruments . 293
1. On the Single Microscope 294
2. On the Compound Microscope 296
3. On the Refracting Telescope 298
4. On the Gregorian and Cassegrainian Telescope . . 300
5. On the Newtonian Telescope 303
6. On a new Reflecting Telescope 307
7. On Achromatic Object-glasses 309
8. On Achromatic Eye-pieces, &c. 323
9. On the Camera Obscura 330
10. On the Phantasmagoria 331
11. On the Dichroscope 333
12. On the Kaleidoscope 336

CHAPTER XVII.

On the Method of Forming the Lenses and Specula of Refracting and Reflecting Telescopes 338
1. On the Method of grinding and polishing Lenses . 338
2. On the Method of casting, grinding, and polishing the Specula of Reflecting Telescopes 341

CHAPTER XVIII.

On the Phenomena of Double Refraction . . . . *page* 345
1. On Crystals with one axis of Double Refraction . . 346
2. On Crystals with two axes of Double Refraction . 348

CHAPTER XIX.

On the Polarisation of Light 349
1. On the Polarisation of Light by Reflexion . . . 350
2. On the Polarisation of Light by Refraction . . . 355
3. On the Colours produced by transmitting Polarised Light through thin crystallized plates 358
4. On the System of Rings produced by Crystals with one axis of Double Refraction 361
5. On the Double System of Rings produced by Crystals with two axes of Double Refraction 363
6. On Circular Polarisation in Rock Crystal, Amethyst, and certain Fluids 364
7. On Singly Polarising Crystals 367
8. On the Dichroism of Crystals, and the Absorption of Polarised Light . . . . *ib.*
9. On the Polarisation of Light by the Transmission of Heat through Glass, &c. 369
10. On the Communication of the Polarising Structure by Compression and Dilatation 372
11. On the Polarisation of Light by Metals, and the Second Surfaces of Transparent Bodies 373
12. On the Influence of the Surfaces of Crystals on the Light which they reflect 376
13. On the Structure of Tesselated Apophyllites . . 378
14. On the Multiplication of Images, and the Colours which accompany them in Calcareous Spar, &c. . 379

CHAPTER XX.

Description of New Dials 381
1. Description of an Analemmatic Dial, which sets itself
2. Description of Lambert's Dial 386

CHAPTER XXI.

On the Cause of the Tides on the side of the Earth opposite to the Moon 388

Table of Specific Gravities 390
Index 397

APPENDIX

TO

FERGUSON'S LECTURES

ON

MECHANICS, &c.

THE object of the following Appendix to *Ferguson's Lectures* is to supply in some measure their defects and omissians, and at the same time to give a full account of the discoveries and improvements in Practical Mechanics, and other branches of Natural Philosophy, in a manner suited to the popular and unpretending character of the original work. In order to do this in a somewhat systematic manner, we shall follow as much as possible the order of the Lectures, and give such a form to the Supplementary Chapters, that they may be read with advantage without any particular reference to the corresponding subjects in the first volume.

CHAPTER I.

ON MECHANICAL AGENTS, OR THE FIRST MOVERS OF MACHINERY.

As every kind of machine must be set in motion, and kept in a state of action by the continued exertion of some mechanical agent, either animate or inanimate, we are led first to inquire into the nature, and mode of application, of the different powers which have been employed as the first movers of machinery.

These powers are—

1. The force of men and animals;
2. The force of water;
3. The force of wind; and
4. The elastic force of steam and heated air.

To these might be added the force of springs, and the force of gravity; but as springs require the force of man to renew their state of tension, and as weights require to be elevated by the same means, they ought rather to be considered as intermediate or secondary agents for producing small mechanical effects during the intermissions of the first mover.

Sect. I.—*On the Strength of Men and Animals.*

The force of man was no doubt the earliest mechanical agent that was employed to produce any useful effect, and he formed not only the first mover, but the machine itself. The singular adaptation of the human frame to the purposes of active life renders the force of man one of the most valuable, and, at the same time, one of the most universally applicable first movers of machinery. It is therefore a matter of great importance to ascertain the manner in which his exertions will yield the greatest quantity of useful work with the least quantity of bodily fatigue, or with a degree of fatigue compatible with the preservation of his bodily health.

Excepting a few insulated, though useful observations, by Bernoulli, Smeaton, Desaguliers, Emerson, and other authors, the subject of the strength of men has been indebted almost wholly for its illustration to the labours of M. Coulomb.

1. The first object of this able philosopher was to ascertain the quantity of action which was exerted when ascending a stair either loaded or unloaded.

He found that a man could ascend the stairs of a house at the rate of 16 yards in a minute, provided he did not ascend more than 20 or 30 yards. Hence he concluded, that if the average weight of a man is taken at 150 lbs. avoirdupois, the quantity of action which he thus furnishes will be

2560 lbs. avoirdupois, raised 1 yard in 1 minute;

and if he is supposed capable of continuing this action 4 hours a-day, his daily action will be

614,400 lbs. avoirdupois, raised 1 yard in 1 hour.

M. Coulomb likewise proved that a man unloaded ascended to

the height of 164 yards by steps cut in the rock in 20 minutes, but none of the workmen would engage to continue this for 6 hours, or do it 18 times a-day, for the ordinary wages. By examining the rate at which M. Borda, with a party of sailors, ascended the Peak of Teneriffe, Coulomb concludes that a man in ascending with ease a convenient stair, will give daily a quantity of action equal to

480 lbs. avoirdupois, raised 1000 yards;

or, since Coulomb considers this as decidedly too low, we may safely take the easily recollected expression of 500 lbs. avoirdupois, raised 1000 yards. In order to compare this quantity of action with that which a man can furnish loaded, Coulomb found that a man loaded with 150 lbs. of firewood could raise daily about

250 lbs. avoirdupois 1000 yards,

and that a strong man could by great exertion raise about

300 lbs. avoirdupois 1000 yards.

Hence, taking the first result, which is an average one, the quantity of action of a man when unloaded and loaded, is in the ratio of 500 to 250, or two to one nearly.

When a man ascends a stair loaded, he raises his own weight along with the load, but the only *useful effect* which he produces is the elevation of the load. If the load were gradually increased, the total quantity of his daily action would diminish, and if it amounted to about 330 lbs. avoirdupois, he would scarcely be able to move it. If, on the contrary, he ascended without a load, the useful effect would be nothing, although his actual quantity of action was then a maximum. There must therefore be between these limits a certain load which will give a maximum effect. By following Coulomb's method, we shall find that the effect will be a maximum when the load is about 3-4ths, or 0.756 of the weight of the man, or about $113\frac{1}{2}$ lbs. avoirdupois; and that the useful effect of a man ascending a stair and properly loaded, is

137 lbs. avoirdupois raised 1000 yards.

Consequently this mode of employing a man consumes nearly 3-4ths, or 0.756 of his real action; and he will cost five times more than a man who after having ascended the stair unloaded, raises a weight by allowing himself to descend by gravity through the height to which he ascended.

If we suppose the man is so loaded as to perform no work,

then the formulæ would give for the action 260 lbs. avoirdupois, which is not far from the weight which an ordinary man can lift or just carry.[1]

The next object of Coulomb's consideration is the quantity of action which a man can furnish when *he walks on a horizontal road*, either loaded or unloaded.

When a man travels for several successive days unloaded, he can easily walk about 54,680 yards, or about 31 miles a-day, which gives for his quantity of action

7700 lbs. avoirdupois, carried 1094 yards.

We likewise find that the average quantity of action furnished by porters carrying furniture or loads of any kind amounting to 130 lbs. is

4400 lbs. avoirdupois, carried 1094 yards.

Hence the quantity of action lost is

3300 lbs. avoirdupois, carried 1094 yards.

And if we suppose the losses of action to be proportional to the load, we shall have

$$130 : L = 33{,}000 : \frac{33{,}000}{130} = 25.37\,L.$$

Hence the real quantity of action will be $7700 - 25.37\,L$; and making this equal to nothing, we obtain $L = 304$ lbs. as the greatest load which a man can carry, a result coinciding very nearly with that given in the preceding note.[2]

[1] The following very simple formulæ I have deduced from those given by Coulomb, by supposing that the number of kilogrammes which a man can raise through one kilometre daily, when ascending a stair unloaded, is *thrice* his own weight, and that the quantity of action lost by being loaded is $1\frac{1}{2}$ times the load. Putting L for the load, h the height to which the man ascends with that load daily, and W the man's weight, then $L\,h = \frac{(3\,W - 1\frac{1}{2}\,L)\,L}{W + L}$ and $h = \frac{3\,W - 1\frac{1}{2}\,L}{W + L}$ which becomes a maximum when $L = W(\sqrt{3} - 1) = 0.732\,W$. When $L = 2\,W$ we have $3\,W - 1\frac{1}{2}\,L = 0$ and $h = 0$, which shews that when the load is equal to twice the weight of the man, or about 300 lbs. he loses the power of ascending.

[2] By considering that the weight which a man can carry daily through the distance of 1 kilometre, or 1094 yards, is 50 times his own weight, supposing what is very nearly the case, that the loss of action when he is loaded with 130 lbs. is 25 times the load raised through the height above mentioned, I have deduced the following simple formula from those of Coulomb's, d being the distance through which he carries the load L.

$$L\,d = \frac{(50\,W - 25\,L)\,L}{L + W} \text{ and } d = \frac{50\,W - 25\,L}{L + W}$$

which becomes a maximum when $L = W(\sqrt{3} - 1) = 0.732\,W$. When

The next case considered by Coulomb is that in which porters return unloaded to carry away a new load.

His general result is, that a maximum effect will be produced when the porter carries a load equal to nine tenths of his own weight, and that his daily quantity of action will be

1533 lbs. carried 1094 yards.

The load usually carried by porters under the circumstances of this case, is very nearly 135 lbs. or 0.9 W.[3]

From the above results it follows, that the quantity of action furnished by a man walking unloaded, and a man walking under the circumstances of the present case, is as 505 to 100, or nearly as 5 to 1.

Although the quantities of action of a man ascending a stair is not of the same kind with that of a man walking freely on a horizontal road, yet it is interesting to compare them together, as Coulomb has done.

The quantities of action in these two cases are as 205 to 3500, or as 1 to 17. Now, if we take the height of a step at 6 inches, we shall have $17 \times 6 = 102$ inches, the length of horizontal road which a man can travel with the same degree of fatigue which he experiences in ascending a step of 6 inches, and if the pace of a man is 30 inches, it follows, that a man experiences the same degree of fatigue in ascending one step of 6 inches as in advancing about three paces and a half.

$L = W$, then $50\,W - 25\,P = 0$, and $d = 0$, which shews that when the load is equal to *twice* the weight of the man, or 300 lbs. he can no longer carry it.

It is a singular circumstance that the ratio between L and W, when the effect is a maximum, should be so exactly the same in the two cases of a man carrying a burden on a level road and up a pair of stairs, and that this should take place by such a slight change on the numbers of Coulomb. His results are—

In ascending a height, - - -	$L = 0.754\ W$
In walking along a horizontal plain, -	$L = 0.72\ W$
Mean, - - -	$L = 0.737\ W$,

a mean which is nearly the same as the result $L = 0.732\ W$, which we have just found for both cases.

[3] I have also simplified Coulomb's formulæ for this case by considering that $W D = 50\ W$; that D, the distance a man can travel unloaded, $= 50$ kilometres, and substituting 25. in place of 25.37, as before. Coulomb's formula is $L\,x = \frac{L\,(W D^2 - b\,D\,L)}{2\,W D + L\,(D - b)}$, the portion of action which is equal to the useful effect which a man can furnish in a day. By the above substitutions we have $L\,x = \frac{L\,(2500\ W - 25\,D^2\,L)}{\frac{1}{2}\,D\,(100\ W + L)}$, which will be a maximum when $L = 280\,(\sqrt{1\tfrac{1}{2}} - 1)$ or L 0.8989 W, nearly the same as Coulomb's result.

Having thus followed Coulomb through the most general and most useful part of his investigation, we shall present the remainder of his results, along with those which we have already given, in a tabular form, the weight of a man being supposed 70 kilogrammes.[4]

Table, shewing the Quantity of Action which a Man can furnish in a day, when his strength is exerted in various ways.

	Kilogrammes.	Kilometre.
1. When he walks on level ground unloaded	2500	carried 1
2. When he walks loaded with 58 kilogrammes, and returns unloaded for another burden, the *useful effect* is	692.4	1
3. When he walks always loaded -	912	1
4. When a man carries a load on a wheel-barrow, the *useful effect* is -	1022.7	1
5. When a man ascends a stair unloaded, the *mechanical effect* is - -	205	1
6. When he raises a weight for driving piles, the *useful effect* is - -	75	1
7. When he draws water from a well by a double bucket, the *useful effect* is -	72	1
8. A man coining money by raising a weight[5] - - -	40.	1
9. When he ascends a stair loaded with 53 kilogrammes, the *useful effect* is	56	1
10. A mean of these three results -	62	1
11. When he exerts himself in turning a winch - - -	116	1
12. When he digs with a spade, the total effect is* - - -	100.4	1
This effect is composed of three separate ones.		
1. The raising of the earth and the spade is	43	1
2. The action of sinking the spade into the ground, is - - - - -	53.6	1
3. The labour of breaking the clods, and smoothing the surface, which is about $\frac{1}{23}$ of the whole work is - - -	3.8	1

[4] A kilogramme is 2 lbs. 3 oz. 5 dr. avoirdupois, and a kilometre is nearly 1094 yards.

[5] This result is so much less than the preceding, because at the mint the men wrought 15 months in succession; whereas, those who drove piles went to another kind of work when they were fatigued.

[6] In fine weather a man dug 181 square metres. The spade went about 25

Such are the leading results of Coulomb's experiments, which establish many points of singular importance. We would recommend them to practical men as deserving their particular study, and as likely to be of great advantage to those who may judiciously carry them into effect on a large scale.

The following results, given on the same scale as those of Coulomb, have been collected by M. Hachette. The number expressing the kilogrammes, both in this and the preceding table, are called by the French writers *dynamical units*, a dynamical unit being the weight of a cubic metre of water, raised to the height of a metre. Hence, the number of kilogrammes that are raised one kilometre are dynamical units.

	Kilogram.	Kilom.
1. A man weighing 70 kilogrammes, who walks $7\frac{1}{2}$ hours a day, on a declivity of 14 centimetres to 1 metre, with a load of 7 or 8 kilogrammes - -	225	raised 1
2. A man marching in a mountainous country with a load - - -	140	1
3. A porter carrying wood up a stair (the weight of the porter included) -	109	1
N. B. This is the case, No. 6. of Coulomb's Table, where we did not include the weight of the porter.		
4. A porter carrying coals up a stair, the weight of the porter included	112 to 120	1
5. A man raising a weight by a pulley[6]	40	1
6. A man raising a weight by a pulley for driving piles - - -	48	1
N. B. This result was obtained by M. Lamande, from the work of 38 men, who raised a weight of 587 kilogrammes for driving piles.		
7. A man drawing a boat *a la bricole* -	110	1

In the following examples, the dynamical unit is a cubic metre of water carried a metre upon a horizontal road.

8. A soldier loaded with from 20 to 25 kilogrammes, and travelling 20 kilometres a-day - - -	1800 to 1900	carried 1

centimetres deep. Each spadeful weighed 6 kilogrammes, and was raised to the height of about 4 decimetres. A cubic metre of the soil weighed 1898 kilogrammes.

[6] Some results are so high as 75.

	Kilogram.	Kilom.
9. A Roman soldier on forced marches of 40 kilometres a-day -	4400 to 4800 carried	1
10. Hawkers with crotchets (weight of man not included) - -	792 to 880	1
11. Porters drawing a small waggon on 4 wheels over ground pretty unequal, and often clayey. Guinevau *Essais sur les Machines*, p. 271 - -	626	1
12. Porters drawing a small waggon on 4 wheels, *running upon horizontal planks*,	900 to 1000	1
13. Porters running on the ground, where there are inequalities of surface -	600	1
14. A man drawing a boat on a canal (50,000 kilogrammes transported 11 kilometres)	550,000	1

Before we conclude this part of the subject, we shall subjoin various other estimates of the force of men and horses, which may be of use in particular cases, though the results have been generally obtained from experiments not very judiciously conducted.

Table of the Strength of Men, according to different Authors.

No. of pounds raised.	Height to which the weight is raised.	Time in which it is raised.	Duration of the Work.	Names of the Authors.
1000	180 feet	60 minutes		Euler
60	1	1 second	8 hours	Bernouilli
25 French	220 French	145 seconds		Amontons
170	1	1 second	half an hour	Coulomb
1000	330	60 minutes		Desaguliers
1000	225	60 minutes		Smeaton
30	$3\frac{1}{2}$	1 second	10 hours	Emerson
30	2.43 feet	1 second		Schulze

According to M. Amontons, a man weighing 133 French pounds, ascended 62 French feet by steps, but was completely exhausted.

A sawyer, according to the same author, made 200 strokes of 18 French inches each, with a force of 25 pounds in 145 seconds.

An ordinary man, according to Desaguliers, can turn a winch with the force of 30 pounds for 10 hours, with a velocity of $2\frac{1}{2}$ feet per second.

Two men, according to Desaguliers, working at a windlass with handles at right angles to each other, can raise 70 pounds more easily than one man can raise 30, an additional effect of five pounds being produced on the work of each man, in consequence of the uniform action arising from the handles being at right angles to each other.

A man may, according to the same author, exert a force of 80 pounds with a fly, when the motion is pretty quick.

A man may also, with a good pump, raise a hogshead of water 10 feet high in a minute, and continue the work for a whole day.

According to Dr. Robison, a feeble old man raised 7 cubic feet of water = $437\frac{1}{2}$ pounds avoirdupois, $11\frac{1}{2}$ feet high in a minute, for 8 or 10 hours a-day, by walking backwards and forwards on a lever.

A young man, weighing 135 pounds, and carrying 30 pounds, raised $9\frac{1}{4}$ cubic feet of water, = $578\frac{1}{10}$ pounds avoirdupois, $11\frac{1}{2}$ feet high for 10 hours a-day without being fatigued.

According to Mr. Buchanan, the forces exerted by a man acting at a winch, pumping, ringing, and rowing, are as the numbers 1742, 2856, 3883, and 4095. Now, if we take Coulomb's measure of action for turning a winch, which is 116 kilogrammes through 1 kilometre, or 256 pounds avoirdupois through 3281, we shall have the

Nature of the Action.	Mechanical Effect. Kilog. Kilom.	lbs. avoird.	Feet.
Mechanical effect of turning a winch	116 through 1	256	3281
Mechanical effect of pumping	190 through 1	419	3281
Mechanical effect of ringing .	259 through 1	572	3281
Mechanical effect of rowing .	273 through 1	608	3281

On the Force of Horses.

The following table contains the relative strength of horses, asses, and men.

1 horse is equal to 5 men,	{ Desaguliers. Smeaton.
1 . . . . . . 7 men,	Bossut, &c.
1 . . . . . . 14 men,	Schulze.
1 ass . . . . 2 men,	Bossut.

2 horses, according to Amontons, exert a force of 150 pounds when yoked in a plough.

1 horse, according to Desaguliers, can draw with a force of 200 lbs. $2\frac{1}{2}$ miles an hour, and continue this action for 8 hours every day. When he exerts a force of 240 pounds, he can continue it only 6 hours every day.

1 horse, by means of pumps, can, according to Mr. Smeaton, raise 250 hogsheads of water 10 feet high in an hour.

1 horse walking on a good road, and loaded with about 2 hundred weight, can travel 25 miles in 7 or 8 hours.

1 horse raising coals by means of a wheel and axle, and moving at the rate of about 2 miles an hour, can, according to Mr. Fenwick, raise a load of 1000 pounds avoirdupois with a velocity of 13 feet per minute, and continue this for 12 hours.

1 horse, according to the same author, can exert a force of 75 pounds, moving at the rate of 13 feet per minute, and continue it for $9\frac{1}{2}$ hours.

According to Regnier, the mean draught of 4 horses was 36 myriogrammes in 794 hours.

1 horse can draw more up a steep hill than three men can carry; that is, from 450 to 750 pounds. Desaguliers.

1 strong horse can draw 2000 pounds in a cart, up a steep hill which is but short. Desaguliers.

1 horse has sometimes carried 650 or 700 pounds for 7 or 8 miles without resting, as its ordinary work. Desaguliers.

1 horse at Stourbridge carried 11 hundred weight of iron, or 1232 pounds for 8 miles. Desaguliers.

1 mule works in the West Indies, 2 hours out of about 18, with a force of about 150 pounds, walking 3 feet in one second. Cazand.

From Desaguliers's measure of the force of a horse already given, it follows that its force is,

	Pounds.		Foot.	
According to Desaguliers,	44,000	raised	1	in 1 min.
According to Smeaton,	22,916	. . .	1	in 1
According to Hachette,	28,000	. . .	1	in 1
According to Mr. Watt,	33,000	. . .	1	in 1

Mr. Watt's steam-engines are, however, calculated to work equal to 44,000 pounds, raised 1 foot per minute, as he considers the difference, or $44{,}000 - 33{,}000 = 11{,}000$ pounds raised 1 foot per minute, to be lost in the friction, &c. of the engine itself.

The power of one horse is supposed capable of driving 100 spindles with preparation cotton water twist, 1000 spindles with preparation cotton mule yarn, and 75 spindles with preparation flax yarn. The following are the results respecting horses given by M. Hachette.

	Kilog.	Kilom.	Daily action, measured by the draught and the road travelled.	Useful effect in Dynamical Units.
1. A cart horse	140 ×	40	5600	
2. A post horse	90 ×	38	3420	
3. A horse moving in a circle, and working a pump, ...				585
4. Id. working 12 hours, and raising plaster,			1684	842
5. Mean of 3 horses working at a pump,			1185	595
6. Mean of 8 horses raising water by a pump, ...			2948	675
7. Two horses raising coals, ...				1560
8. A horse drawing a load of 150,000 kilogrammes through 8 kilometres,			800	Doubtful.
9. The force of a horse acting 24 hours is equal to			5974	dynamical units. 9

Sect. II.—*On the Force of Water, and the mode of applying it to drive Machinery.*

In employing water as the first mover of machinery, it is applied to the circumference of wheels, from the axis of which the power is immediately conveyed to the other parts of the machine.

Water-wheels derive their names from the different ways in which the power of the water is applied to them, and are divided into,

1. Undershot wheels.
2. Overshot wheels.
3. Breast wheels; and
4. Wheels driven by the re-action of water; which we shall describe in their order.

1. *On Undershot wheels.*

An undershot water-wheel is a wheel having a number of plane surfaces called float-boards projecting from its circumference, for the purpose of receiving the impulse of the water, which is delivered by a sloping canal, and with great velocity, upon the under part of the wheel.

[9] This result is deduced from the fact, that a horse can draw 140 pounds with the velocity of 200 feet in a minute. For farther information on this subject, see the *Edinburgh Encyclopœdia*, Art. *Mechanics*, vol. xiii, p. 560.

This kind of wheel is represented in Plate I, Fig. 1, where *W w* is the wheel with 24 float-boards, and *n o* one of the float-boards receiving the impulse of water which has acquired great velocity by descending the inclined mill-course *A B D F K M*. In the erection of undershot wheels, the principal points to be attended to relate to the construction of the mill-course, the size of the wheel, the number, form, and position of the float-boards, and the relation between the velocity of the water and that of the wheel, in order to produce a maximum effect; but as these wheels are commonly applied to mills for grinding corn, we shall introduce under this section a description of corn-mills.

On the Construction of the Mill-Course.

On the mill-course.

As it is of importance to have the height of the fall as great as possible, the bottom of the canal or dam, which conducts the water from the river, should have a very small declivity; for the height of the water-fall will diminish in proportion as the declivity of the canal is increased.

Plate I. Fig. 1.

On this account, it will be sufficient to make *A B* slope about one inch in 200 yards, taking care to make the declivity about half an inch for the first 48 yards, in order that the water may have a velocity sufficient to pre vent it from flowing back into the river. The inclination of the fall, represented by the angle *G C R*, should be 25° 50′; or *C R*, the radius, should be to *G R* the tangent of this angle, as 100 to 48, or as 25 to 12; and since the surface of the water *S b* is bent from *a b* into *a c*, before it is precipitated down the fall, it will be necessary to incurvate the upper part *B C D* of the course into *B D*, that the water at the bottom may move parallel to the water at the top of the stream. For this purpose, take the points *B*, *D*, about 12 inches distant from *C*, and raise the perpendiculars *B E*, *D E*; the point of intersection *E* will be the centre from which the arch *B D* is to be described; the radius being about $10\frac{1}{6}$ inches. Now, in order that the water may act more advantageously upon the float-boards of the wheel *W W*, it must assume a horizontal direction *H K*, with the same velocity which it would have acquired when it came to the point *G*: but, in falling from *C* to *G*, the water will dash upon the horizontal part *H G*, and thus lose a a great part of its velocity; it will be proper, therefore, to make it move along *F H*, an arch of a circle to which *D F* and

K H are tangents, in the points *F* and *H*. For this purpose, make *G F* and *G H* each equal to three feet, and raise the perpendiculars *H I*, *F I*, which will intersect one another in the point *I*, distant about 4 feet 9 inches and 4-10ths from the points *F* and *H*, and the centre of the arch *F H* will be determined. The distance *H K*, through which the water runs before it acts upon the wheel, should not be less than two or three feet, in order that the different portions of the fluid may have obtained a horizontal direction: and if *H K* be much larger, the velocity of the stream would be diminished by its friction on the bottom of the course. That no water may escape between the bottom of the course *K H* and the extremities of the float-boards, *K L* should be about 3 inches, and the extremity *o* of the float-board *n o* should be beneath the line *H K X*, sufficient room being left between *o* and *M* for the play of the wheel, or *K L M* may be formed into the arch of a circle *K M* concentric with the wheel. The line *L M V*, called by M. Fabre the *course of impulsion (le coursier d'impulsion)*, should be prolonged, so as to support the water as long as it can act upon the float-boards, and should be about 9 inches distant from *O P*, a horizontal line passing through *O* the lowest point of the fall; for if *O L* were much less than 9 inches, the water having spent the greater part of its force in impelling the float-boards, would accumulate below the wheel and retard its motion. For the same reason, another *course*, which is called by M Fabre, the *course of discharge (le coursier de decharge)*, should be connected with *L M V*, by the curve *V N*, to preserve the remaining velocity of the water, which would otherwise be destroyed by falling perpendicularly from *V* to *N*. The course of discharge is represented by *V Z*, sloping from the point *O*. It should be about 16 yards long, having an inch of declivity in every two yards. The canal which reconducts the water from the course of discharge to the river, should slope about 4 inches in the first 200 yards, 3 inches in the second 200 yards, decreasing gradually till it terminates in the river. But if the river to which the water is conveyed should, when swoln by the rains, force the water back upon the wheel, the canal must have a greater declivity, in order to prevent this from taking place. Hence it will be evident, that very accurate levelling is necessary for the proper formation of the mill-course.

In order to find the breadth of the course of discharge, multiply the quantity of water expended in a second,[1] measured in cubic feet, by 756, for a first number. Multiply the square root of $d\,K$ ($d\,K$ being found by subtracting $O\,K$, or $P\,R$, each equal to a foot, from $d\,O$ or $b\,R$, the height of the fall) by $O\,L$, or $\frac{3}{4}$ of a foot, and also by 1000, and the product will be a second number. Divide the first number by the second, and the quotient will be nearly the least breadth of the course of discharge. If the breadth of the course, thus found, should be too great or too small, the point L has been placed too far from O, or too near it. Increase, therefore, or diminish $O\,L$; and having subtracted from $d\,O$ or $b\,P$, the quantity by which $O\,K$ is greater or less than a foot, repeat the operation with this new value of $d\,K$, and a more convenient answer will be found. The preceding rule will give too large a breadth to the course, when the expense of water is great, and the height of the fall inconsiderable. But the course of discharge ought always to have a very considerable breadth, which should be greater than that of the course of impulsion, that the water having room to spread, may have less depth; and that a greater height may be procured to the fall, by making $O\,L$, and consequently $O\,K$, as small as possible; for the breadth of the course is inversely as $O\,L$, that is, it increases as $O\,L$ diminishes, and diminishes as it increases. The reader may suppose that this rule still leaves us to guess at the breadth of the course of discharge; but, from the purposes for which it is used, it is easy to know when it is excessively large or small; and it is only when this is the case, that we have any occasion to seek for another breadth, by taking a new value of $O\,L$.

The section of the fluid at K should be rectangular, the breadth of the stream having a determinate relation to its depth. If there is very much water, the breadth should be triple the

[1] The quantity of water expended in a second may be found pretty accurately by measuring the depth of the water at a ($A\,B$, the bottom of the canal, being nearly horizontal, and its sides perpendicular), and the breadth of the canal at the same place. Take the cube of the depth of the water in feet, and extract the square root of it. Multiply this root by the breadth of the canal, and also by 507. Divide the product by 100, and the quotient will be the expense of water in a second, measured in cubic feet. This rule is founded on the formula, $x = 5.07\ b \times d^{\frac{3}{2}}$; where x is the quantity of water expended in a second, b the breadth of the canal, and d its depth.

depth; if there is a moderate quantity, the breadth should be double the depth; and, if there is very little water, the breadth and depth should be equal. That this relation may be preserved, the course at the point *K* must have a certain breadth, which may be thus found:—Divide the square root of *d K* (found as before) by the quantity of water expended in a second, and extract the square root of the quotient. Multiply this root by .623, if the breadth is to be triple the depth; by .515, if it is to be double; and by .364, if they are to be equal, and the product will be the breadth of the course at *K*. The depth of the water at *K* is therefore known, being either one third, or one half of the breadth of the course, or equal to it, according to the quantity of water furnished by the stream.

Plate I. Fig. 1.

In Fig. 1, *b P* is called the *absolute fall*, which is found by levelling. Draw the horizontal lines *b d*, *P O*; *d O* will thus be equal to *b P*, and will likewise be the absolute fall. The *relative fall* is the distance of the point *d* from the surface of the water at *K*, when the depth of the water is considerably less than *d K*, but is reckoned from the middle of the water at *K*, when *d K* is very small.[2] The relative fall, therefore, may be determined by subtracting *O K*, which is generally a foot, from the absolute fall *d O*, and by subtracting also either the whole or one half of the natural depth of the water at *K*, according as *d K* is great or small in proportion to this depth.

Breadth of the course at the top of the fall.

The next thing to be determined is the breadth of the course at the top of the fall *B*, and the breadth of the canal at the same place. To find this, multiply the quantity of water expended in a second by 100, for a first number; take such a quantity as you would wish, for the depth of the water, and, having cubed it, extract its square root, and multiply this root by 507, for a second number; divide the first number by the second, and the quotient will be the breadth required. The breadth, thus found, may be too great or too small in relation to the depth. If this be the case, take one half of the breadth, thus found, and add to it the number taken for the depth of the water; the sum will

[2] The depth of the water, here alluded to, is its natural depth, or that which it would have if it did not meet the float-boards. The effective depth is generally two and a half times the natural depth, and is occasioned by the impulse of the water on the float-boards, which forces it to swell, and increases its action upon the wheel.

be the true depth, with which the operation is to be repeated, and the new result will be better proportioned than the first.

Quantity of water furnished in a second.

The mill-course being thus constructed, we may now find more exactly the quantity of water furnished in a second. For this purpose, subtract one half the depth of the water at *K* from *d K*, and having multiplied the remainder by .5719, extract the square root of the product. Multiply this root by the breadth of the course at *K* multiplied into the depth of the water there,[3] and the result will be the true expense of the source in cubic feet.

In order to know whether the water will have sufficient force to move the least millstone which should be employed, namely, a millstone weighing, along with its axis and trundle, 1550 pounds avoirdupois, take the relative fall, increased by one half the natural depth of the water at *K*, viz. *d K* (Fig. 1), and multiply it by the expense of the source in cubic feet; if the product is 32.95, or above it, the machine will move without interruption. If the product be less than this number, the weight of the millstone ought to be less than 1550 pounds, and the meal will not be ground sufficiently fine; for the resistance of the grain will bear up the millstone, and allow the meal to escape before it is completely ground.

As it is of great consequence that none of the water should escape, either below the float-boards, or at their sides, without contributing to turn the wheel, the course of impulsion, *K V*, should be wider than the course at *K*, as represented in Fig. 2, where *C D*, the course of impulsion, corresponds with *L V* in Fig. 1, *A B* corresponds with *H K*, and *B C* with *K L*. The breadth of the float-boards, therefore, should be wider than *m n*, and their extremities should reach a little below *B*, like *n o* in Fig. 1. When this precaution is taken, no water can escape, without exerting its force upon the float-boards.[4]

Plate I. Fig. 1 and 2.

On the size of the water-wheel, and on the number, magnitude, and position, of its float-boards.

Size of the water-wheel.

The diameter of the wheel should be as great as possible, unless some particular circumstances in the

[3] That is, by the area of the rectangular section of the stream at *K*.

[4] See Du Buat's *Traité D'Hydraulique*, and Fabre *Sur les Machines Hydrauliques*.

construction prevent it; but ought never to be less than seven times the natural depth of the stream at K, the bottom of the course.[5] It has been much disputed among mechanical writers, whether the wheel should be furnished with a small or a great number of float-boards. M. Pitot has shewn, that when the float-boards have different degrees of obliquity, the force of impulsion upon the different surfaces will be reciprocally as their breadth: thus, in Fig. 3, the force upon $h\,c$ will be to the force upon $D\,O$ as $D\,O$ to $h\,e$.[6] He therefore concludes, that the distance between the float-boards should be equal to one half of the arch plunged in the stream, or that, when one is at the bottom of the wheel, and perpendicular to the current, as $D\,E$, the preceding float-board $B\,C$ should be leaving the stream, and the succeeding one $F\,G$ just entering into it.[7] For, when the three float-boards $F\,G$, $D\,E$, $B\,C$, have the same position as in the figure, the whole force of the current $N\,M$ will act upon $D\,E$, having the most advantageous position for receiving it: whereas, if another float-board $d\,e$ were inserted between $F\,G$ and $D\,E$, the part $i\,g$ would cover $D\,O$, and, by thus substituting an oblique for a perpendicular surface, the effect would be diminished in the proportion of $D\,O$ to $i\,g$. Upon this principle it is evident, that the depth of the float-board $D\,E$ should always be equal to the versed sine of the arch between any two floating-boards, $D\,E$ being the versed sine of $E\,G$. For the use of those who may wish to follow M. Pitot, though we are of opinion that he recommends too small a number of float-boards, we have calculated the following table upon the above principles. It exhibits the diameter of water-wheels, the number of float-boards they should contain, and the size of the float-boards, when any two of these quantities are given. According to M. Pitot, the proper relation between these is of so great importance, that if a water-wheel, 16 feet diameter, with its float-

Plate I. Fig. 3.

Number of float-boards, according to Pitot.

5 The diameter here meant is double the *mean radius*, or the distance between the centre of the wheel and the middle of the natural stream, which impels it, or what is called the centre of impulsion. By adding or subtracting the half of the stream's natural depth, to or from the mean radius, we have the *exterior* and *interior* radius of the wheel.

6 See *Traite d'Hydrodynamique*, § 771.

7 *Mem. Acad. Par.* 1729, 8vo, p. 359.

boards three feet deep, should have nine instead of seven, one twelfth of the whole force of impulsion would be lost.[8]

Table of the number of float-boards in undershot wheels.

Diameter of the wheel in feet.	Depth of the float-boards in feet.						
	1	1.5	2	2.5	3	3.5	4
10	10	8	7	6	5	5	5
11	10	8	7	6	5	5	5
12	11	9	8	7	6	6	5
13	11	9	8	7	6	6	5
14	12	9	8	7	7	6	6
15	12	9	8	7	7	6	6
16	12	10	9	8	7	7	6
17	12	10	9	8	7	7	6
18	13	11	9	8	8	7	6
19	13	11	10	9	8	7	7
20	14	11	10	9	8	7	7
21	14	12	10	9	8	7	7
22	15	12	10	9	8	8	7
23	15	12	10	9	8	8	7
24	15	12	11	10	9	8	8
25	16	13	11	10	9	8	8
26	16	13	11	10	9	8	8
27	16	13	11	10	9	8	8
28	17	13	12	10	9	9	8
29	17	14	12	11	10	9	8
30	17	14	12	11	10	9	9
32	18	14	12	11	10	9	9

In order to find from the preceding table the number of float-boards for a wheel 20 feet in diameter (the diameter of the wheel being reckoned from the extremity of the float-boards), their depth being two feet ;—enter the left hand column with the number 20, and the top of the table with the number 2, and in a line with these numbers will be found 10, the number of float-boards which such a wheel would require.

As the numbers representing the depths of the float-boards, and the diameter of the wheel, increase more rapidly than the numbers in the other columns, the preceding table will not shew us with accuracy the diameter of the wheel when the number and depth of the float-boards are given ; ten float-boards, for example, two feet deep, answering to a wheel either 19, 20, 21, 22, or 23 feet diameter. This defect, however, may be supplied by the following method.—Divide 360 degrees by the number of float-boards, and the quotient will be

[8] Desaguliers has adopted the rule given by Pitot. See his *Experimental Philosophy*, vol. ii, p. 424.

the arch between each. Find the natural versed sine of this arch, and say, as 1000 is to this versed sine, so is the wheel's radius to the depth of the float-boards; and to find the diameter of the wheel, say, as the above versed sine is to 1000, so is the depth of the float-boards to the wheel's radius.

The rule of Pitot inaccurate.

Plate I. Fig. 3.

We have already said, that the number of float-boards found by the preceding table is too small. Let us attend to this point, as it is of considerable importance. It is evident from Fig. 3, that when one of the floats, as $D E$, is perpendicular to the stream, it receives the whole impulse of the water in the most advantageous manner; but when it arrives at the position $d e$, and the succeeding one $F G$ into the position $f g$, so that the angle $e A g$ may be bisected by the perpendicular $A E$, they will have the most disadvantageous situation; for a great part of the water will escape below the extremities g and e, of the float-boards, without having any effect upon the wheel; and the part $i g$ of the float-board, which is really impelled, is less than $D E$, and oblique to the current. The wheel, therefore, must move irregularly, sometimes quick, and sometimes slow, according to the position of the floats with respect to the stream; and this inequality will increase with the arch plunged in the water. M. Pitot proceeds upon the supposition, that if another float $f g$, were placed between $F G$ and $D E$, it would destroy the force of the water that impels it, and cover the corresponding part $D O$ of the preceding float-board. But this is not the case. The water, after acting upon $f g$, still retains a part of its motion, and bending round the extremity g, strikes $D E$ with its remaining force. Considerable advantage, therefore, must be gained by using more float-boards than M. Pitot recommends.[9]

Number of float-boards according to Bossut.

M. Bossut[1] has shewn, that when the wheel has an uniform velocity, the most advantageous number of floats is determined. Having fixed upon the radius and velocity of the wheel, and on the portion of

[9] In Mr. Smeaton's experiments, the water-wheel, which was 25 inches in diameter, had 24 floats; and he observes, "That, when the number was reduced to 12, it caused a diminution of the effect, on account of a greater quantity of water escaping between the floats and the floor; but a circular sweep being adapted thereto, of such a length, that one float entered the course before the preceding one quitted it, the effect came so near to the former, as not to give hopes of advancing it by increasing the number of floats beyond 24 in this particular wheel." *Smeaton's Experimental Enquiry*, p. 24; or, *Phil. Trans.* 1759, v. 51.

[1] *Traite d'Hydrodynamique*, notes on chap. x; also § 778.

its circumference that ought to be plunged in the stream, he imagines the wheel to have different numbers of float-boards, and then computes the momentum of the water against all the parts of those that are immersed. The number of float-boards which gives the greatest momentum should be adopted as the most advantageous. When the velocity of the stream was thrice that of the wheel, and when 72 degrees of the circumference were immersed, Bossut found that the number of float-boards should be 36. When a greater arch is plunged in the stream, the velocity continuing the same, the number should be increased, and *vice versa*.

The float-boards should be as numerous as possible.

This rule, however, is too difficult to be of use to the practical mechanic. From what has been said, it is evident, that in order to remove any inequality of motion in the wheel, and prevent the water from escaping beneath the tips of the float-boards, the wheel should be furnished with the greatest number of float-boards possible, without loading it, or weakening the rim on which they are placed.[2] This rule was first given by Dupetit Vandin,[3] and afterwards by M. Fabre,[4] and it is not difficult to see, that if the millwright should err in furnishing the wheel with too many float-boards, the error will be perfectly trifling, and that he would lose much more by erring on the other side. The float-boards should not be rectangular, like $a\ b\ n\ c$ in Fig. 3, but should be bevelled like $a\ b\ m\ c$. For if they were rectangular, the extremity $b\ n$ would interrupt a portion of the water, which would otherwise fall on the corresponding part of the preceding float-board. The angle $a\ b\ m$ may be found thus :— Subtract from 180° the number of degrees contained in the immersed arch $C\ E\ G$, and the half of the remainder will be the angle required. It has been already observed, that the effective depth of the water at K (Fig. 1) is generally two and a half times greater than the natural depth. The height $D\ E$, therefore, of the float-boards should be two and a half times the natural depth of the current at K (Fig. 2). The breadth of the float-boards should always be a little greater than the breadth of the course at K, the method of finding which has been already pointed out.

[2] Brisson (*Traite Elementaire de Physique*) observes, that there should be 48 floats, instead of 40, as generally used in a wheel 20 feet in diameter.

[3] *Mem. des Savans Etrangers*, tom. i.

[4] *Sur les Machines Hydrauliques*, p. 55, No. 103.

Inclination of the float-boards.

M. Pitot has shewn,[5] that the float-boards should be perpendicular to the rim, or, in other words, a continuation of the radius. This, indeed, is true in theory, but it appears from the most unquestionable experiments, that they should be inclined to the radius. This was discovered by Deparcieux, in 1753 (not in 1759, as Fabre asserts), who shews, that the water will thus heap up on the float-boards, and act, not only by its impulse, but also by its weight.[6] This discovery has been confirmed also by the Abbe Bossut,[7] who found, that when the velocity of the water is about $\frac{300}{27}$ of a foot, or 11 feet per second, the inclination of the float-board to the radius should be between 15 and 30 degrees. M. Fabre, however, is of opinion, that when the velocity of the stream is 11 feet per second, or above this, the inclination should never be less than 30 degrees; that when this velocity diminishes, the inclination should diminish in proportion; and that when it is four feet, or under, the inclination should be nothing. In order to find the inclination for wheels of different radii, let $A\,H$ (Fig. 3) be the radius, bisect $P\,H$, the height of the float-board, in i, and having drawn $P\,K$ perpendicular to $P\,A$, set off $P\,K$ equal to $P\,i$, and join $H\,K$; $H\,K$ will be the position of the float-board inclined to the radius $A\,H$ by the angle $K\,H\,P$. This construction supposes the greatest value of the angle $K\,H\,P$ to be 26° 34′.[8]

On the formation of the spur-wheel and trundle.

Size of the spur-wheel.

The radius of the spur-wheel is found by multiplying the mean radius of the water-wheel by that

[5] *Mem. Acad. Par.* 1729, 8vo, p. 350.

[6] *Mem. Acad.* 1754, 4to, p. 614, 8vo, p. 944.

[7] *Traite d'Hydrodynamique*, § 814 and § 817.

[8] Boecklerus, a German writer on mechanics, makes the diameter of the great wheel 48 feet, and the number of float-boards 86, when the force of the water is great; the diameter of the second wheel 18 feet, with 180 teeth, and the number of staves in the third wheel 60. Casatus (*Mechan.* lib. 5, p. 560) observes, that the diameter of the water-wheels in mills on the Po was commonly 10 cubits, the diameter of the second wheel $5\frac{1}{2}$ cubits, with 108 teeth; the number of spindles in the trundle 9; the thickness of the millstone 6 or 7 inches, and its diameter $2\frac{1}{2}$ cubits. Florinus makes the diameter of the great wheel 18 feet, when the fall of water is 4 or 5 feet; the number of float-boards 30 or 36; the breadth of one of the float-boards 12 or 14 digits, and its height one foot; the teeth of the second wheel 72; and the number of staves in the trundle 6, 8, or 9. Wolfius (*Opera Mathemat.* tom. 1, p. 690) observes, that the diameter of the large wheels, in most of the double overshot mills near

of the lantern, which may be of any size, and also by the number of turns, which the spindle or axis of the lantern performs in a second,[9] and then by the number 2.151. This product being divided by the square-root of the relative fall, the quotient will be the radius required. The number of teeth in the wheel should be to the number of staves in the trundle as their respective radii. In order to find the exact number, take the proper diameter of the teeth and the staves, which ought to be two and a half inches each in common machines, and determine also how much is to be allowed for the play of the teeth, which should be about two and a half tenths of an inch; add these three numbers, and divide by this sum the mean circumference of the spur-wheel,[1] the quotient will be nearly the number of teeth in the wheel. Let us call this quotient x, to avoid circumlocution. Multiply x by the mean radius of the trundle, and divide the product by the radius of the spur-wheel. If the quotient is a whole number, it will be the exact number of staves in the trundle, and x, if it were an integer, will be the exact number of teeth in the wheel. But should the quotient be a mixed number, diminish the integer, which may still be called x, by the numbers 1, 2, 3, &c. successively, and at every diminution, multiply x, thus diminished, by the radius of the trundle, and divide the product by the radius of the wheel. If any of these operations give a quotient without a remainder, this quotient will be the number of staves in the trundle, and x, diminished by one or more units, will be the number of teeth in the wheel. Thus let the radius of the trundle be one foot, that of the wheel four feet, the thickness of the teeth and the staves two and a half inches, or $\frac{30}{144}$ of a foot, and the space for the play of the teeth two and a half tenths of an inch, or $\frac{3}{144}$; the sum of the three quantities will be $\frac{63}{144}$ or $\frac{7}{16}$ of a foot; and 25 feet, or $\frac{176}{7}$ of a foot, the circumference of the wheel, divided by $\frac{7}{16}$ will give $\frac{2816}{49}$, or $57\frac{23}{49}$ feet. Multiply the integer x or 57 by

Number of teeth in the wheel and trundle.

Hall did not exceed 16 feet. In South Wales there is an overshot water-wheel above fifty feet in diameter. According to Belidor, the diameter of the millstones should be from 5 to 7 feet, and their thickness 12, 15, or 18 inches.

[9] The method of determining the velocity of the spindle, or the millstone, will be afterwards pointed out. The axis of the lantern should, in general, make about 90 turns in a minute.

[1] The mean radius is reckoned from the centre of the wheel to the centre of the teeth.

1, the radius of the lantern; but as the product 57 will not divide by 4, the radius of the wheel, let us diminish x, or 57, by unity, and the remainder 56 being multiplied by 1, the radius of the trundle, and divided by four, the radius of the wheel, gives 14 without a remainder, which, therefore, will be the number of staves, while 56, or x diminished by unity, is the number of teeth in the spur-wheel.

Had it been possible to make the number of teeth equal to $57\frac{23}{49}$, $2\frac{1}{2}$ inches would be the proper thickness for the teeth and the staves; but as the number must be diminished to 56, there will be an interval left, which must be distributed among the teeth and staves, so that a small addition must be made to each. To do this, divide the circumference of the wheel $\frac{176}{7}$ of a foot by the number of teeth 56, and, from the quotient $\frac{450}{1000}$ subtract the interval for the play of the teeth $\frac{3}{144}$ or $\frac{20}{1000}$ the remainder $\frac{430}{1000}$ being halved, will give $\frac{215}{1000}$ of a foot, or 2 inches and 5.8 tenths, for the thickness of every tooth and stave, $\frac{8}{100}$ of an inch being added to each tooth and stave to fill up the interval.

It may sometimes happen, however, that, in diminishing x successively by unity, a quotient will never be found without a remainder. When this is the case, seek out the mixed number which approaches nearest an integer, and take the integer to which it approximates for the number of staves in the lantern. Thus, when the radius of the wheel is $4\frac{1}{13}$ feet, the different quotients obtained, after diminishing x by one, two, three, four, will be $14\frac{226}{1000}$, $13\frac{981}{1000}$, $13\frac{735}{1000}$, $13\frac{490}{1000}$, and $13\frac{245}{1000}$. The nearest of these to an integer is $13\frac{981}{1000}$, being only $\frac{19}{1000}$ less than 14, which will therefore be the number of staves in the trundle.[2]

Form of the teeth.

In a succeeding article on the teeth of wheels, we have shewn what form must be given them in order to produce an uniformity of action. The following method, however, will be pretty accurate for very common works. In Plate I, Fig. 4, take EB, equal to the radius of the trundle,[3] and describe the acting part BA, and with the same radius describe CD. When the teeth of the wheel are perpendicular to its plane, as in the spur-wheels of corn-mills, we must bisect BD in n, and drawing mn perpendicular to BD, make the plane $BACD$ move round upon mn as an axis; the figure

[2] See Fabre, *Sur les Machines Hydrauliques*, p. 304, § 546.

[3] The staves of the trundle should be as short as possible.

thus generated like $a\ b\ c\ d$, Fig. 5, will be the proper shape for the teeth.

Size of the gudgeons.

The pivots, or gudgeons, on which vertical axes move, should be conical; and those which are attached to horizontal arbors, should be cylindrical, and as small and short as possible. A gudgeon two inches in diameter will support a weight of 3239 pounds avoirdupois, though we often meet with gudgeons three or four inches in diameter, when the weight to be supported is considerably less. By attending to this, the friction of the gudgeons will be much diminished, and the machine greatly improved. Particular care, too, should be taken, that the axis of the gudgeons be exactly in a line with the axis of the arbor which they support, otherwise the action or motion of the wheels which they carry will be affected with periodical inequalities.

On the formation, size, and velocity of the millstone, &c.

On the surfaces of the millstones.

In the fourth lecture,[4] Mr. Ferguson has given several useful directions for the formation of the grinding surfaces of the millstones, to which we have only to add, that when the furrows are worn shallow, and consequently new dressed with the chisel, the same quantity of stone must be taken from every part of the grinding surface, that it may have the same convexity or concavity as before. As the upper millstone should always have the same weight when its velocity remains unchanged, it will be necessary to add to it as much weight as it lost in the dressing. This will be most conveniently done by covering its top with a layer of plaster, of the same diameter as the layer of stone taken from its grinding surface, and as much thicker than the layer of stone, as the specific gravity of the stone exceeds the specific gravity of the plaster.[5] That the reader may have some idea of the manner in which the furrows, or channels, are arranged, we have represented, in Plate I, Fig. 6, the grinding surface of the under millstone A, as fitted up in an iron frame, and with adjusting screws, according to the method used by Mr. Austen. $B\ B$ is the bed of cast iron, $C\ C\ C$ the adjusting screws, on

[4] The diameter of the gudgeon must be proportional to the square root of the weight which it supports.

[5] The relative weights of the stone and plaster may be determined from the table of specific gravities in this volume.

the points of which the bed-stone rests, *D D D D* the screws for adjusting it laterally. The iron frame in Fig. 6 is adapted for laying on an old floor.

In Fig. 7 we have given a section of the millstones, spindle, &c. The under millstone *A*, which never moves, may be of any thickness. Its grinding surface should be a portion of a cone, whose height is not above half an inch. The upper millstone *O*, which is fixed to the spindle *P*, and is carried round with it, should be so hollowed that the angle formed by the grinding surfaces may be of such a size that their greatest distance may be equal to the thickness of two grains of corn. The diameter of the mill-eye, or hole in the upper stone, should be between 8 and 14 inches; and the weight of the upper millstone, joined to the weight of the spindle and the trundle (the sum of which three numbers is called the *equipage* of the turning millstone), should never be less than 1550 pounds avoirdupois, otherwise the resistance of the grain would bear up the millstone, and the meal be ground too coarse. In Fig. 7, *E E* are the two beams on which the floor and cast iron beds rest, *H H* a circle of wood to cover the adjusting screws, *K K* the case which incloses the stones, *L* the hopper, *M* the shoe, and *N* the damsel.[6]

Weight of the equipage.

In order to find the weight of the equipage, divide the third of the radius of the gudgeon by the radius of the water-wheel which it supports, and having taken the quotient from 2.25, multiply the remainder by the expense of the source, by the relative fall, and by the number 19911, and you will have a first quantity, which may be regarded as pounds. Multiply the square root of the relative fall by the weight of the arbor of the water-wheel, by the radius of its gudgeon, and by the number 1617, and a second quantity will be had, which will also represent pounds. Divide the third part of the radius of the gudgeon by the radius of the water-wheel, and having augmented the quotient by unity, multiply the sum by 1005, and a third quantity will be obtained. Subtract the second quantity from the first, divide the remainder by the third, and the quotient will express the number of pounds in the equipage of the millstone.

The weight of the equipage being thus found, extract its

[6] See the *Transactions of the Society of Arts*, 1820, vol. xxxviii, p. 69.

square root, expressed in pounds, and multiply it by 0.39, and the product will be the radius of the millstone in feet.[7]

Size of the millstones. In order to find the weight and thickness of the upper millstone, the following rules must be observed:—

1. To find the weight of a quantity of stone equal to the mill-eye:—Take any quantity which seems most proper for the weight of the spindle and the lantern, and subtract this quantity from the weight of the millstone's equipage, for a first quantity. Find the area of the mill-eye, and multiply it by the weight of a cubic foot of stone of the same kind as the millstone (found from the table of specific gravities), and a second quantity will be had. Multiply the area of the millstone by the weight of a cubic foot of the same stone, for a third quantity. Multiply the first quantity by the second, and divide the product by the third, and the quotient will be the weight required.

2. To find the number of cubic feet in the turning millstone, supposing it to have no eye:—From the weight of the spindle and lantern subtract the quantity found by the preceding rule, for the first number. Subtract this first number from the weight of the equipage, and a second number will be obtained. Divide this second quantity by the weight of a cubic foot of stone of the same quality as the millstone, and the quotient will be the number of cubic feet in the upper millstone, the eye being supposed to be filled up.

3. To find the thickness of the millstone at its centre and circumference:—Divide the solid content of the millstone, as found by the preceding rule, by its area, and you will have a first quantity. Add the height of the conical surface of *A*, which is generally about half an inch or an inch, to twice the diameter of a grain of corn, for a second quantity. Add the first quantity to one third of the second, and the sum will be the thickness of the millstone at the circumference. Subtract the third of the second quantity from the first quantity, and the remainder will be its thickness at the centre.[8]

[7] This rule supposes, that when the diameter of the millstone is 5 feet, the weight of the equipage should be 4307 avoirdupois pounds.

[8] These rules are founded upon formulæ, which may be seen in *Fabre sur les Machines Hydrauliques*, pp. 172, 239.

Velocity of the millstone.

The size of the millstone being thus found, its velocity is next to be determined. M. Fabre, observes, that the flour is the best possible when a millstone 5 feet in diameter makes from 48 to 61 revolutions in a minute. Mr. Ferguson allows 60 turns to a millstone 6 feet in diameter, and Mr. Imison 120 to a millstone $4\frac{1}{2}$ feet in diameter. In mills upon Mr. Imison's construction, the great heat that must be generated by such a rapid motion of the millstone, must render the meal of a very inferior quality: much time, on the contrary, will be lost, when such a slow motion is employed as is recommended by M. Fabre and Mr. Ferguson. In the best corn-mills in this country, a millstone 5 feet in diameter revolves, at an average, 90 times in a minute.[9] The number of revolutions in a minute, therefore, which must be assigned to millstones of a different size, may be found by dividing 450 by the diameter of the millstone in feet.

Spindle.

The spindle *P*, which is commonly 6 feet long, may be made either of iron or wood. When it is of iron, and the weight of the millstone 7558 pounds avoirdupois, it is generally three inches in diameter; and when made of wood it is 10 or 11 inches in diameter. For millstones of a different weight, the thickness of the spindle may be found by proportioning it to the square root of the millstone's weight, or, which is nearly the same thing, to the weight of the millstone's equipage.

Pivots.

The greatest diameter of the pivot *D*, upon which the millstone rests, should be proportional to the square root of the equipage, a pivot half an inch diameter being able to support an equipage of 5398 pounds. In most machines, the diameter of the pivots is by far too large, being capable of supporting a much greater weight than they are required to bear. The friction is therefore increased, and the performance of the machine diminished.

Bridge-tree.

The bridge-tree *S T* (Plate III. Fig. 2, Vol. I.) is generally from 8 to 10 feet long, and should not be elastic as is recommended by Belidor and others. When its length is 9 feet, and the weight of the equipage 5182 pounds, it should be 6 inches square; and when the length remains un-

[9] Mr. Fenwick of Newcastle, an excellent practical mechanic, observes, that, in the best corn-mills in England, millstones from $4\frac{1}{2}$ to 5 feet in diameter revolve from 90 to 100 times in a minute.

changed, and the equipage varies, the thickness of the bridge-tree should be proportional to the square root of the equipage.

On the Performance of Undershot Mills.

Performance of undershot mills.

The performance of any machine may be properly represented by the number of pounds which it will elevate, in a given time, by means of a rope wound upon a spindle and passing over a pulley.[1] In order to find the weight which a given machine will raise:—Divide the third part of the radius of the gudgeon of the water-wheel, by the mean radius of the wheel itself, and having subtracted the quotient from 2.25, multiply the remainder by the expense of water in a second in cubic feet, by the height of the relative fall, and by the number 19911, for a first quantity. Multiply the weight of the arbor of the water-wheel, and its appendages (viz. the water-wheel itself and the spur-wheel), by the radius of the gudgeon in decimals of a foot, by the square of the relative fall, and the number 1617, and divide the product by the mean radius of the water-wheel, and a second quantity will be had. Divide the third part of the gudgeon's radius by the mean radius of the water-wheel, augment the quotient by unity, and multiply the sum by the radius of the spindle for a third quantity. Subtract the second quantity from the first, and divide the remainder by the third quantity, the quotient will be the number of pounds which the machine will raise. Multiply the diameter of the spindle by 31,416, and you will have a quantity equal to the height which W will raise by one turn of the spindle; this quantity, therefore, being multiplied by the number of turns which the spindle performs in a minute, will give the height through which the weight W will rise in the space of a minute.

According to Mr. Fenwick.

Mr. Fenwick[2] found, by a variety of accurate experiments made upon good corn-mills, whose upper millstone, being from $4\frac{1}{2}$ to 5 feet in diameter, revolved from 90 to 100 times in a minute, that a mill, or any power capable of raising 300 pounds avoirdupois with a velocity of 210 feet per minute, will grind *one* boll of good corn in

[1] It was in this way that Smeaton measured the performance of his models.

[2] *Four Essays on Practical Mechanics*, 2d edit. 1802, p. 60.

an hour; and that two, three, four, or five bolls will be ground in an hour, when a weight of 300 pounds is raised with a velocity of 350, 506, 677, or 865 feet respectively in a minute.[3]

Or, to arrange the numbers more properly:

Number of bolls ground in an hour.........	1	2	3	4	5	6
Number of feet through which 300lb. is raised in a minute..........................	210	350	506	677	865	1069

Supposing it, therefore, to be found, by the preceding rules, that a mill would raise 600 pounds through 253 feet in a minute of time, we have 300 : 600 = 253 : 506; that is, the same power that can raise 600 pounds through 253 feet, will raise 300 pounds through 506 feet, consequently such a mill will be able to grind three bolls of corn in an hour.[4]

According to M. Fabre, the quantity of meal ground in an hour may be determined by multiplying 62.4 Paris pounds by the square of the radius of the millstone, and the product will be the number of pounds of meal. But, as this rule is founded upon an erroneous supposition, that the quality of the flour is best when a millstone, 5 feet in diameter, performs 48 revolutions in a minute, we have made the calculation anew, upon the supposition, that the velocity of a millstone, 5 feet in diameter, should be 90 revolutions in a minute, and have found, that, when mills are constructed upon this principle, the quantity of flour ground in an hour, in pounds avoirdupois, will be equal to the product of the square of the millstone's radius, and the number 125.

According to M. Fabre.

The following measures of the performance of undershot corn-mills, are given by M. Hachette.

M. Aitken at Senonges has constructed corn-mills which can grind in one hour seven measures of corn of 120 kilogram-

[3] As the differences of these numbers increase nearly by 16, they may be continued by always augmenting the difference between the two last numbers by 16, and adding the difference thus augmented to the last number, for the number required. Thus, by adding 16 to 188, the difference between 677 and 865, we have 204, which being added to 865, gives 1069 for the number of feet, nearly, through which the power must be able to raise a weight of 300 pounds in a minute, in order to grind six bolls of corn in an hour.

[4] The proper result of Mr. Fenwick's experiment was, that a power requisite to raise a weight of 300 pounds avoirdupois, with a velocity of 190 *feet per minute*, would grind one boll of good corn in an hour; but, in order to make the above numbers accurate in practice, he increased the velocity $\frac{1}{10}$, and made it 210 feet per minute.

mes, with a spring which discharges 1500 cubic feet in a minute, and a fall of 10 feet. Taking for the small dynamical unit a cubic foot of water raised one foot, the spring would yield in an hour for the grinding of a measure of corn of 120 kilogrammes (which is called a *setier* or bushel), 60 times $\frac{15000}{7}$ cubic feet of water, falling 10 feet, or 128,571 small dynamical units, which is equivalent to 1431 large dynamical units. (See p. 7.)

A mill on the Beuvronne, ground, in 24 hours, 25 bushels of flour, with a spring of 900 times 560[5] cubic feet, and a fall of 10 feet. Hence a bushel consumes 201,600 dynamical units.

The mill of Claye near Paris, ground 26 bushels in 24 hours, with a spring of 978 times 560 cubic feet, and a fall of 11 feet. Hence each bushel consumes 229,815 dynamical units.

The mills of Thoulouse, which give 500 cubic feet in a minute, and have a fall of 8 feet, grind about two bushels in an hour. Hence each bushel consumes 120,000 dynamical units.

A mill at Malmaison gave a bushel in four hours, with a fall of 20 feet and a spring of 20 cubic feet a minute. Hence each bushel consumed 96,000 dynamical units.

A steam engine at Paris, which turned six millstones, consumed 162,000 dynamical units for each bushel.

The following important maxims have been deduced from Mr. Smeaton's accurate experiments on undershot mills, and merit the attention of the practical mechanic.

Explanation of Smeaton's maxims.

Maxim 1. That the virtual or effective head of water being the same,[6] the effect will be nearly as the quantity of water expended. That is, if a mill, driven by a fall of water whose virtual head is 10 feet, and which discharges 30 cubic feet of water in a second, grinds four bolls in an hour; another mill having the same virtual head,

[5] 560 cubic feet is called by the French engineers *un pouce de Fontanier*, or a hydraulic inch.

[6] The *virtual*, or *effective head* of water moving with a certain velocity, is equal to the height from which a heavy body must fall in order to acquire the same velocity The height of the virtual head, therefore, may be easily determined from the water's velocity, for the heights are as the squares of the velocities, and the velocities, consequently, as the square roots of the heights. Mr. Smeaton observes, that in the large openings of mills and sluices, where great quantities of water are discharged from moderate heads, the real head of water, and the virtual head, as determined from the velocity, will nearly agree. See his ***Experiments on Mills***, p. 23.

but which discharges 60 cubic feet of water, will grind eight bolls of corn in an hour.

Maxim 2. That the expense of water being the same, the effect will be nearly as the height of the virtual or effective head.

Maxim 3. That the quantity of water expended being the same, the effect is nearly as the square of its velocity. That is, if a mill, driven by a certain quantity of water, moving with the velocity of four feet per second, grinds three bolls of corn in an hour; another mill, driven by the same quantity of water, moving with the velocity of five feet per second, will grind nearly $4\frac{7}{10}$ bolls of corn in an hour, because $3 : 4\frac{7}{10} = 4^2 : 5^2$ nearly; that is, as 16 to 25, the squares of the respective velocities of the water.

Maxim 4. The aperture being the same, the effect will be nearly as the cube of the velocity of the water. That is, if a mill driven by water, moving through a certain aperture, with the velocity of four feet per second, grind three bolls of corn in an hour; another mill driven with water, moving through the same aperture with the velocity of five feet per second, will grind $5\frac{43}{50}$ bolls nearly in an hour, for $3 : 5\frac{43}{50} = 4^3 : 5^3$ nearly; that is, as 64 to 125, the cubes of the water's respective velocities.

On the method of constructing Mill-wrights' Tables on correct principles.

Constructing of a new millwrights' table.

Although a mill-wright's table has been constructed by Mr. Ferguson,[7] and afterwards altered a little by Mr. Imison, so far as concerns the velocity of the millstone; yet, as we shall now shew, the principles upon which it is computed are far from being correct. It is evident that the great wheel must always move with less velocity than the water, even when there is no work to be performed; for a part of the impelling power is necessarily spent in overcoming the inertia of the wheel itself; and if the wheel has little or no velocity, it is equally manifest that it will produce a very small effect. There is consequently a certain proportion between the velocity of the water and the wheel, when the effect is a maximum. Parent and Pitot found this proportion to be as 1 to 3;

Relative velocity of the water and the wheel.

[7] See vol. i, p. 64.

and Desaguliers,[8] Maclaurin,[9] and Ferguson have adopted their determination.[1] But Mr Smeaton has shewn, that instead of the wheel moving with $\frac{1}{3}$ of the velocity of the water, when the effect is a maximum, as Parent imagined, the greatest effect is produced when the velocity of the wheel is between $\frac{1}{3}$ and $\frac{1}{2}$, the maximum being much nearer to $\frac{1}{2}$ than $\frac{1}{3}$. He observes also, that $\frac{1}{2}$ would be the true maximum "if nothing were lost by the resistance of the air, the scattering of the water carried up by the wheel, and thrown off by the centrifugal force, &c. all which tend to diminish the effect more at what would be the maximum if these did not take place, than they do when the motion is a little slower."[2] But in making this alteration, we are warranted not merely by the results of Mr Smeaton's experiments, but also by the deductions of theory. In the investigations from which Parent and Pitot concluded that the velocity of the wheel should be $\frac{1}{3}$ of the velocity of the water in order to produce a maximum effect, they considered the impulse of the stream upon one float-board only, and therefore made the force of impulsion proportional to the square of the difference between the velocities of the stream and the float-board. The action of the current, however, is not confined to one float-board, but is exerted on several at the same time, so that the float-boards which are acurately fitted to the mill-course, abstract from the water its excess of velocity, and the force of impulsion becomes proportional only to the difference between the velocities of the stream and the float-boards. From this circumstance, the Chevalier de Borda has shewn in his *Memoire sur les Roues Hydrauliques*,[3] that in theory the velocity of the wheel is $\frac{1}{2}$ that of the current, and that in practice it is never more than $\frac{3}{8}$ of the stream's velocity, when the effect is a maximum.[4]

The constant number, too, which is used by Mr. Ferguson

[8] Desagulier's *Experimental Philosophy*, vol. ii, p. 424, Lect. 12.

[9] Maclaurin's *Fluxions*, Art. 907, p. 728.

[1] M. Lambert has also adopted the determination of Parent, in his *Memoir on Undershot Mills* in the *Nouv. Mem. de l'Acad. de Berlin*, 1775, p. 63.

[2] *Smeaton on Mills*, p. 77. M. Bossut and M. Fabre, along with Smeaton, make the velocity of the wheel $\frac{2}{5}$ of the velocity of the water. See *Traite d'Hydrodynamique, par Bossut*, § 808.9, Fabre, § 66. The great hydraulic machine at Marly was found to produce a maximum effect when the velocity of the wheel was $\frac{2}{5}$ that of the current.

[3] *Memoires de l'Acad. Par.* 1767, 4to, p. 285.

[4] Borda's investigation will be found in the *Edinburgh Encyclopædia*, Art. *Hydrodynamics*, vol. xi, p. 557.

for finding the velocity of the water from the height of the fall, viz. 64.2882 is not correct. From the recent experiments of Mr. Whitehurst on pendulums, it appears, that a heavy body falls 16.087 feet in a second of time; consequently the constant number should be 64.348.

In Mr. Ferguson's table, the velocity of the millstone is too small; and Mr. Imison, in correcting this mistake, has made the velocity too great. From this circumstance, the mill-wrights' table will admit of some improvement. Proceeding, therefore, upon the practical deductions of Smeaton, as confirmed by theory, and employing a more correct constant number, and a more suitable velocity for the millstone, we may construct a new mill-wrights' table by the following rules.

Method of constructing the table.

1. Find the perpendicular height of the fall of water in feet above the bottom of the mill-course at *K* (Fig. 1, Plate I); and having diminished this number by one half of the natural depth of the water at *K*, call that the height of the fall.[5]

2. Since bodies acquire a velocity of 32.174 feet in a second, by falling through 16.087 feet, and since the velocities of falling bodies are as the square roots of the heights through which they fall, the square root of 16.087 will be to the square root of the height of the fall as 32.174 to a fourth number, which will be the velocity of the water. Therefore the velocity of the water may be always found by multiplying 32.174, by the square root of the height of the fall, and dividing that product by the square root of 16.087. Or it may be found more easily by multiplying the height of the fall by the constant number 64.348, and extracting the square root of the product, which, abstracting the effects of friction, will be the velocity of the water required.[6]

[5] The height of the fall here meant is the relative or virtual height, and it is supposed that the mill-course is so accurately constructed, that the water will have the same velocity at *K* as it would have at *R* by falling perpendicularly through *C R*. This will be nearly the case when the mill-course is formed according to the directions formerly given; though in general a few inches should be taken from the fall, in order to obtain accurately its relative or virtual height.

[6] That the velocity of the water is equal to the square root of the product of the height of the fall, and the constant number 64.348, may be shewn in the following manner. Let x be the velocity of the water, m the height of the fall, $a = 16.087$, and consequently $2\ a = 32.174$. Then by the first part of the second rule $\sqrt{a} : \sqrt{m} = 2\,a : x$ therefore $x = \frac{2\,a\sqrt{m}}{\sqrt{a}}$; multiplying by $\sqrt{a}$ we have

3. Take *one half* of the velocity of the water, and it will be the velocity which must be given to the float-boards, or the number of feet they must move through in a second, in order that the greatest effect may be produced.

4. Divide the circumference of the wheel by the velocity of its float-boards per second, and the quotient will be the number of seconds in which the wheel revolves.

5. Divide 60 by this last number, and the quotient will be the number of revolutions which the wheel performs in a minute. Or the number of revolutions performed by the wheel in a minute, may be found by multiplying the velocity of the float-boards by 60, and dividing the product by the circumference of the wheel, which in the present case is 47.12.

6. Divide 90 (the number of revolutions which a millstone 5 feet diameter should perform in a minute) by the number of revolutions made by the wheel in a minute, and the quotient will be the number of turns which the millstone ought to make for one revolution of the wheel.

7. Then, as the number of revolutions of the wheel in a minute is to the number of revolutions of the millstone in a minute, so must the number of staves in the trundle be to the number of teeth in the wheel, in the nearest whole numbers that can be found.[7]

8. Multiply the number of revolutions performed by the wheel in a minute, by the number of revolutions made by the millstone for one of the wheel, and the product will be the number of revolutions performed by the millstone in a minute.

In this manner the following table has been calculated for a water-wheel fifteen feet in diameter, which is a good medium size, the millstone being five feet in diameter, and revolving 90 times in a minute.

$x \surd = 2\, a \surd\, m$; putting all the quantities under the radical sign there comes out $\surd\, x^2\, a = 4\, a^2\, m$; extracting the square root of both sides, we have $x^2\, a = 4\, a^2\, m$, dividing by a gives $x^2 = 4\, a\, m$ or $x = \surd\, 4\, a\, m$. But since the constant number 64.348 is double of 32.174, it will be equal to 4 a; then by the latter part of rule second we have $x = \surd\, 4\, a\, m$, which is the same value of x, as was found from the first part of the rule

[7] We have filled up the *sixth* column of the tables in the common way; but, for the proper method of finding the relation between the radius of the spur wheel and trundle, and the exact number of teeth in the one, and staves in the other, we must refer the reader to p. 22 of this volume.

TABLE I.—A New Mill-Wright's Table, in which the velocity of the wheel is one-half the velocity of the stream, the effects of friction not being considered.

Height of the effective fall of water.	Velocity of the water *per* second, friction not being considered.	Velocity of the wheel *per* second, being one half that of the water.	Revolutions of the wheel *per* minute, its diameter being 15 feet.	Revolutions of the mill-stone for one of the wheel.	Teeth in the wheel and staves in the trundle.		Revolutions of the mill-stone *per* minute by these staves and teeth.
Feet.	Ft. 100ths.	Ft. 100ths.	Rev. 100ths.	Rev. 100ths.	Teeth.	Staves.	Rev. 100ths.
1	8.02	4.01	5.10	17.65	106	6	90.01
2	11.34	5.67	7.22	12.47	87	7	90.03
3	13.89	6.95	8.85	10.17	81	8	90.00
4	16.04	8.02	10.20	8.82	79	9	89.96
5	17.94	8.97	11.43	7.87	71	9	89.95
6	19.65	9.82	12.50	7.20	65	9	90.00
7	21.22	10.61	13.51	6.66	60	9	89.98
8	22.69	11.34	14.45	6.23	56	9	90.02
9	24.06	12.03	15.31	5.88	53	9	90.02
10	25.37	12.69	16.17	5.57	56	10	90.06
11	26.60	13.30	16.95	5.31	53	10	90.00
12	27.79	13.90	17.70	5.08	51	10	89.91
13	28.92	14.46	18.41	4.89	49	10	90.02
14	30.01	15.01	19.11	4.71	47	10	90.00
15	31.07	15.53	19.80	4.55	48	11	90.09
16	32.09	16.04	20.40	4.45	44	10	89.96
17	33.07	16.54	21. 5	4.28	47	11	90.09
18	34.03	17. 2	21.66	4.16	50	12	90.10
19	34.97	17.48	22.26	4.04	44	11	89.93
20	35.97	17.99	22.86	3.94	48	12	90.07
1	2	3	4	5	6		7

TABLE II.—A New Mill-Wright's Table, in which the velocity of the wheel is three-sevenths of the velocity of the water, and the effects of friction on the velocity of the stream reduced to computation.

Height of the fall of water.	Velocity of the water *per* second, friction being considered.	Velocity of the wheel *per* second, being 3-7ths that of the water.	Revolutions of the wheel *per* minute, its diameter being 15 feet.	Revolutions of millstone for one of the wheel.	Teeth in the wheel and staves in the trundle.		Revolutions of the mill-stone *per* minute, by these staves and teeth.
Feet.	Ft. 100ths.	Ft. 100ths.	Rev. 100ths.	Rev. 100ths.	Teeth.	Staves.	Rev. 100ths.
1	7.62	3.27	4.16	21.63	130	6	89.98
2	10.77	4.62	5.88	15.31	92	6	90.02
3	13.20	5.66	7.20	12.50	100	8	90.00
4	15.24	6.53	8.32	10.81	97	9	89.94
5	17.04	7.30	9.28	9.70	97	10	90.02
6	18.67	8.00	10.19	8.83	97	11	89.98
7	20.15	8 64	10.99	8.19	90	11	90.01
8	21.56	9.24	11.76	7.65	84	11	89.96
9	22.86	9.80	12.47	7.22	72	10	90.03
10	24.10	10.33	13.15	6.84	82	12	89.95
11	25.27	10.83	13.79	6.53	85	13	90.05
12	26.40	11.31	14.40	6.25	72	12	90.00
13	27.47	11.77	14.99	6.00	72	12	89.94
14	28.51	12.22	15.56	5.78	75	13	89.94
15	29.52	12.65	16.13	5.58	67	12	90.01
16	30.48	13.06	16.63	5.41	65	12	89.97
17	31.42	13.46	17.14	5.25	63	12	89.99
18	32.33	13.86	17.65	5.10	61	12	90.01
19	33.22	14.24	18.13	4.96	64	13	89.92
20	34.17	14.64	18.64	4.83	58	12	89.84
1	2	3	4	5	6		7

Explanation and Use of the Mill-wrights' Tables.

It has already been observed, that, according to theory, an undershot wheel will produce the greatest possible effect, when the velocity of the stream is double the velocity of the wheel; and, upon this principle, the first of the preceding tables has been computed. When we consider, however, that, after every precaution is observed, a small quantity of water will escape between the mill-course and the extremities of the float-boards; and that the effect is diminished by the resistance of the air, and the dispersion of the water carried up by the wheel, the propriety of making the wheel move with $\frac{3}{7}$ of the velocity of the water will readily appear. The Chevalier de Borda supposes it never to exceed $\frac{3}{8}$, and Mr Smeaton found it to be much nearer $\frac{1}{2}$ than $\frac{2}{5}$. With $\frac{3}{7}$, therefore, as a proper medium, the numbers in Table II have been computed for this new velocity of the wheel. In Table I, the water is supposed to move with the same velocity as falling bodies. Owing to its friction on the mill-course, &c. this is not exactly the case; but the error, arising from the neglect of friction, might be in a great measure removed, by diminishing the height of the fall a few inches, in order to have the effective height, with which the other numbers are to be taken out of the table. As this mode of estimating the effects of friction is rather uncertain, we have deduced the velocity of the water from the following formula: $V = \sqrt{\frac{172}{3} \times Rb - \frac{1}{2} Hh}$, in which V is the velocity of the water, Rb the absolute heights of the fall, and Hh the depth of the water at the bottom of the course. This formula is founded on the experiments of Bossut, from which it appears, that if a canal be inclined $\frac{1}{10}$ part of its length, this additional declivity will restore that velocity to the water which was destroyed by friction.

We would not advise the mechanic, however, to trust to the second column of Table II for the true velocity of the stream, or to any theoretical results, even when deduced from formulæ that are most agreeable to experience. Bossut, with great justice, remarks, "It would not be exact, in practice, to compute the velocity of a current from its declivity. This velocity ought to be determined by immediate experiment in every particular case."—*Traité*, § 645 Let the velocity of the water, where it strikes the wheel, be determined by the method

which we shall now explain. With this velocity, as an argument, enter column second of either of the tables, according to the velocity which is required for the wheel, and take out the other numbers from the table.

Method of measuring the Velocity of Water.

Different methods of ascertaining the velocity of running water.

A variety of methods have been proposed, by different philosophers, for measuring the velocity of running water. The method by floating bodies, employed by Mariotte; the bent tube (*tube recourbe*) of Pitot;[8] the regulator of Guglielmini;[9] the quadrant,[10] the little wheel,[1] and the method proposed by the Abbé Mann,[2] have each their advantages and disadvantages. The little wheel was employed by Bossut. It is the most convenient mode of determining the superficial velocity of the water; and when constructed, in the following manner, will be very accurate. The small wheel *W W* should be formed of the lightest materials. It should be about 10 or 12 inches in diameter, and furnished with 14 or 16 float-boards. This wheel moves upon a delicate screw *a B* passing through its axle *B b*; and when impelled by the stream, it will gradually approach towards *D*, each revolution of the wheel corresponding with a thread of the screw. The number of revolutions performed, in a given time, are determined upon the scale *m a*, by means of the index *O h*, fixed at *O*, and moveable with the wheel, each division of the scale being equal to the breadth of a thread of the screw, and the extremity *h* of the index *O h* coinciding with the beginning of the scale, when the shoulder *b* of the wheel is screwed close to the scale *a*. The parts of a revolution are indicated by the bent index *m n* pointing to the periphery of the wheel, which is divided into 100 parts. When this instrument is to be used, take it by the handles *C*, *D*, screw the shoulder *b* of the wheel close to *a*, so that the indices may both point to 0, the commencement of the scales; then by means of a stop-watch, or a pendulum, find how many revolutions of the wheel are performed in a given time. Multiply the mean circumference of the wheel, or the circumference deduced from the mean radius,

Instrument for measuring the velocity of water.

Plate I. Fig. 8.

[8] Mem. Acad. Par. 1732.

[9] *Aquarum fluentium Mensura*, lib. iv.

[10] Bossut, *Traite d'Hydrodynamique*, § 654.

[1] *Id. Id.* § 655.

[2] *Phil. Trans.* v. lxix.

which is equal to the distance of the centre of impulsion from the axis $b\,B$, by the number of revolutions, and the product will be the number of feet which the water moves through in the given time. On account of the friction of the screw, the resistance of the air, and the weight of the wheel, its circumference will move with a velocity a little less than that of the stream; but the diminution of velocity, arising from these causes, may be estimated with sufficient precision for all the purposes of the practical mechanic.

A very convenient and useful measure of the velocity of water may be made by attaching to one end of the axis of a wheel like *W W*, one of the *Odometers*, as improved by Mr. Hunter of Thurston.[3] This instrument will register the number of revolutions of the wheel either in an hour or in a day, and will thus give a more correct mean velocity than could be obtained in a shorter interval.

The velocity of water might be also determined from its dissolving power, by measuring the time in which it dissolves a piece of alum, or any other very soluble substance, and taking at the same time the temperature of the current.

On Horizontal Mills.

Horizontal mills.

Although horizontal water-wheels are very common on the continent, and are strongly recommended to our notice by the simplicity of their construction, yet they have almost never been erected in this country, and are therefore not described in any of our treatises on practical mechanics. In order to supply this defect, and recommend them to the attention of the mill-wright, we shall give a brief account of their construction. In Fig. 10, we have a representation of one of these mills. *A B* is the large water-wheel,

Plate I. Fig. 10.

[3] This curious and useful instrument, called an *Odometer* or *Waywiser*, was originally intended to be applied to the axletrees of carriages, in order to register the number of revolutions made by one of the wheels. It is represented in Fig. 9, Plate I, where *A B* is a frame, made hollow, to receive an endless screw *H*, the teeth of which are visible on each side of the index *F*. It is kept in its place by a nut at *A*. The teeth of the endless screw work in the teeth of two concentric wheels *G*, *H*, the first of which has 100 teeth, and the second 101. When *G* has made a complete revolution of 100 teeth, 100 teeth of *H* will also have passed forward; but as *H* has 101 teeth, the index of *G* will have separated from the index of *H* one division, and will consequently point to 1 on the scale of *H*. After a second revolution of *G*, its index will point to 2 on the scale of *H*. Hence, when the index of *G* points to 101 on the scale of *H*, the endless screw *C D* will have performed 10,100 revolutions. This method of registering revolutions may be obviously extended without limit by additional wheels. The *Odometer* is manufactured by Messrs. Howdens, South Bridge, Edinburgh.

which moves horizontally upon its arbor *C D*. This arbor passes through the immoveable millstone; and being fixed to the upper one, carries it once round, for every revolution of the great wheel. The water is commonly conducted along the inclined canal *M N*, and strikes the float-boards *F*, *F′*, &c. placed obliquely to the plane of the wheel.

Mill-course.

When the float-boards are placed perpendicular to the rim, and the water strikes them horizontally, the mill-course is constructed in the same manner for horizontal as for vertical wheels, with this difference only, that the part *m B n C*, Fig. 2, of which *K L*, in Fig. 1, is a section, instead of being rectilineal like *m n*, must be circular like *m P*, and concentric with the rim of the wheel, sufficient room being left between it and the tips of the float-boards, for the play of the wheel.

The equipage[4] of the millstone of a horizontal mill may be found by multiplying the product of the 100th part of the expense of the water in cubic feet, and the relative fall, by 5078, and the product will be the weight of the equipage in pounds avoirdupois.

The mean radius of the wheel *A B* is to be determined by multiplying the product of the relative fall, and the square root of the expense of water in a second by 0.062.

What has been said respecting the number, position, and form of the float-boards of vertical wheels, may be applied also to horizontal ones. In the latter, however, the float-boards must be inclined, not only to the radius, but also to the plane of the wheel, with the same angle as they are inclined to the radius, so that the lowest and the outermost sides of the float-boards may be farthest up the stream.

Velocity of the millstone.

Since the millstone of horizontal mills performs the same number of revolutions as the water-wheel; and since a millstone five feet in diameter should never make less than 48 turns in a minute, the wheel must perform the same number of revolutions in the same time; and in order that the effect may be a *maximum*, or the greatest possible, the velocity of the current must be *double* that of the wheel. Suppose the millstone, for example, to be five feet diameter, and the water-wheel six feet, it is evident that the millstone and wheel must at least revolve 48 times in a minute; and since the circumference of the wheel is 18.8 feet, the float-boards will move

[4] The equipage comprehends the millstone, the water-wheel, and its arbor.

through that space in the 48th part of a minute, that is nearly at the rate of 15 feet per second, which being doubled makes the velocity of the water 30 feet, answering, as appears from the preceding table, to a fall of 14 feet. But if the given fall of water be less than 14 feet, we may procure the same velocity to the millstone by diminishing the diameter of the wheel. If the wheel, for instance, is only five feet diameter, its circumference will be 15.7 feet, and its floats will move at the rate of 12.56 feet in a second, the double of which is 25.12 feet per second, which answers to a head of water less than ten feet high. As the diameter of the water-wheel, however, should never be less than seven times the breadth of the mill-course at *K* (Fig. 1), there will be a certain height of the fall beneath which we cannot employ horizontal wheels,[5] without making the millstone revolve too slowly. This height will be found by the following table.

Method of finding whether horizontal or vertical mills should be erected.

			Ft. Dec.
When the natural depth of the water at the bottom of the fall is to the breadth of the mill-course at the same place, as	3 to 1	The relative fall beneath which we cannot employ horizontal mills will be	7.314
	2 to 1		8.602
	1 to 1		11.350
	$\frac{1}{2}$ to 1		14.976
	$\frac{1}{3}$ to 1		17.613

Thus, if the natural depth of the water at *K* (Fig. 1) is three times the width of the mill-course at the same place, the relative fall beneath which we cannot employ a horizontal wheel will be 7.314 feet. Since the depth of the water is so great in this case, a great quantity of it will be discharged in a second, and therefore it requires a less velocity, or a less height of the fall, to impel the wheel, whereas if the depth of the water had been only one third of the width of the mill-course, such a small quantity would be discharged in a second that we must make up for the want of the water by giving a great velocity to what we have, or by making the height of the fall 17.613 feet.

In order to find the radius of the millstone in horizontal mills, multiply the expense of water in cubic feet in a second, by the relative fall; extract the square root of the product, and multiply this root by 0.267; the product will be the radius of the millstone in feet.

[5] This applies only to mills for grinding corn, where the millstone is fixed on the arbor of the water-wheel, and must move with a determinate velocity. For any other purpose horizontal wheels may be used, however small be the fall of water.

The quantity of meal ground in an hour may be found by the rules already given for vertical mills, or by multiplying the product of the expense of water, and the relative fall, by 456lbs. and the result will be the quantity required.

Performance of horizontal mills.

The thickness of the millstone at the centre and circumference, the thickness of the arbor and pivots, may be determined by the rules already laid down for vertical mills.

In horizontal wheels, the mill-course is sometimes differently constructed. Instead of the water assuming a horizontal direction before it strikes the wheel, as in the case of undershot-mills, the float-board is so inclined as to receive the impulse perpendicularly, and in the direction of the declivity of the waterfall. When this construction is adopted, the greatest effect will be produced when the velocity of the float-boards is not less than $\frac{5.67 \sqrt{H}}{2 \text{ Sin. } A}$, in which H represents the height of the waterfall, and A the angle which the direction of the fall makes with a vertical line. But since this quantity increases as the sine of A decreases, it follows, that without taking from the effect of these wheels, we may diminish the angle A, and thus augment considerably the velocity of the float-boards, according to the nature of the machinery employed; whereas, in vertical wheels, there is only one determinate velocity, which produces a maximum effect.[6]

Horizontal mills with inclined float-boards.

In the southern provinces of France, where horizontal wheels are very generally employed, the float-boards are made of a curvilineal form, so as to be concave towards the stream, as represented in Plate I, Fig. 11. The Chevalier de Borda observes, that in theory a double effect is produced when the float-boards are concave, but that this effect is diminished in practice, from the difficulty of making the fluid enter and leave the curve in a proper direction. Notwithstanding this difficulty, however, and other defects which might be pointed out, horizontal wheels with concave float-boards are always superior to those in which the float-boards are plain, and even to vertical wheels, when there is a sufficient head of water. When the float-boards are plain, the wheel is driven merely by the impulse of the stream; but when they are concave, a part of the water acts by its weight, and increases the velocity of

With curvilineal float-boards.

[6] See *Mem. Acad. Par.* 1767, p. 285.

the wheel. If the fall of water be five or six feet, a horizontal wheel with concave float-boards may be erected, whose maximum effect will be to that of ordinary vertical wheels as 3 to 2.[7]

Conical horizontal wheel with spiral float-boards.

Plate I. Fig. 12.

In the provinces of Guyenne and Languedoc, another species of horizontal wheels is employed for turning machinery. They consist of an inverted cone, *A B*, with spiral float-boards of a curvilineal form winding round its surface. The wheel moves on a vertical axis in the building *D D*, and is driven chiefly by the impulse of the water conveyed by the canal *C* to the oblique float-boards. When the water has spent its impulsive force, it descends along the spirals, and continues to act by its weight till it reaches the bottom, where it is carried off by the canal *M*.

On Double Corn-Mills.

Double corn-mills.

It frequently happens that one water-wheel drives two millstones, in which case the mill is said to be double; and when there is a copious discharge of water from a high fall, the same water-wheel may give sufficient velocity to three, four, or five millstones. Mr. Ferguson has given a brief description of a double mill in Vol. I, p. 66, and a drawing of one in Plate III, Fig. 4, but has laid down no maxim of construction for the use of the practical mechanic. In supplying this defect, and following M. Fabre, let us first attend to double horizontal mills, in which the axis *C D* (Fig. 7) is furnished with a wheel which gives motion to two trundles, the arbors of which carry the millstones.

In order to find the weight of the equipage for each millstone, multiply the product of the expense of water, and the relative fall, by 48116 lbs, and divide the product by 2000, if there are two millstones, 3000 if there are three, and so on; the quotient will be the weight of the equipage of each millstone.

Size of the wheel that drives the trundles.

To determine the radius of the wheel that drives the trundles, find first the radius of the millstones by the rules already given, and having added it to

[7] A new horizontal water-wheel has been recently described by Mr. Adamson. It consists of a horizontal wheel, with a number of vertical float-boards, descending below the general level of the wheel. The water is introduced into a cylindrical reservoir, which surrounds the wheel, and issues from a number of cuts at the bottom of the reservoir, in the direction of tangents to the wheel's circumference. Hence the water acts against all the float-boards at the same time. The power of this wheel is said to be double that of an undershot wheel. A full account of it will be found in the *Journal of the Royal Institution*, vol. iv, p. 46.

half the distance between the two neighbouring mill-stones,[8] subtract from this *sum* the radius of the lantern, which may be taken at pleasure, and the remainder will be the radius required when there are two millstones. But if there are three millstones, or four, or five, or six, before subtracting the radius of the lantern, divide the sum by 0.864, 0.705, 0.587, 0.5, respectively.

Size of the water-wheel.

The mean radius of the water-wheel may be found by multiplying the square root of the relative fall by the radius of the millstone, by the radius of the wheel that drives the trundles, and by 231, and then dividing the product by the radius of the lantern multiplied by 1000, the quotient will be the wheel's radius. It may happen, however, that the diameter of the wheel found in this way is too great. When this is the case, we may be certain that the radius of the lantern has been taken too small. In order then to get a less value for the wheel's radius, increase a little the radius of the lantern, and find new numbers both for the water-wheel, and that which drives the trundles, by the preceding rule. It may happen also, that in giving an arbitrary value to the radius of the lantern, the diameter of the wheel found by the rule may be too small, that is, less than seven times the breadth of the mill-course at the bottom of the fall. When this takes place, make the diameter of the water-wheel seven times the width of the mill-course, and you may find the radius of the other wheel and lanterns by the following rules.

Size of the wheel that drives the trundles.

1. To find the radius of the wheel that impels the trundles; add the radius of the millstone to half the distance between any two adjoining millstones for a first quantity. Multiply the square root of the relative fall by the radius of the millstone and by .231; and having divided the product by the radius of the water-wheel, add unity to the quotient, and multiply the sum by 1 if there are two millstones, by .864 if there are three, by .705 if there are four, by .587 if there are five, and by .5 if there are six, and the result will be a second quantity. Divide the first by the second quantity, and the quotient will be the radius of the wheel that drives the trundles.

Size of the lantern.

2. To find the radius of the lantern, multiply the radius of the wheel as found by the preceding rule, by the square root of the relative fall, and by .231, and divide

[8] This quantity may be taken at pleasure, and should never be less than 2 feet, however great be the number of the millstones.

the product by the radius of the water-wheel; the quotient will be the lantern's radius.

By the rules formerly given, find the quantity of meal ground by one millstone, and having multiplied this by the number of millstones, the result will be the quantity ground by the compound mill.

If the equipage of the millstone of a vertical mill, as found in page 25, should be too great, that is, if it should require too large a millstone, then we must employ a double mill, like that which is represented in Plate III, Fig. 4, Vol. I, or one which has more than two millstones.

In order to know the equipage of each millstone, find it by the rule for a single mill, and having multiplied the quantity by .947, divide the product by the number of millstones, and the quotient will be the equipage of each millstone.

The radius of the wheel *D* (Plate III, Fig. 4, Vol. I) will be found by the same rule which was given for horizontal mills; but it must be attended to, that the lantern whose radius is there employed is not *B B*, but *F G* or *E H*.

Size of the spur-wheel.

To determine the mean radius of the large spur-wheel *A A*, which is fixed upon the arbor of the water-wheel, multiply the square of the radius of the lanterns *F G* or *E H*, by the radius of the water-wheel, and also by 4302, and a first quantity will be had. Multiply the square root of the relative fall by the radius of one of the millstones, and by the radius of the wheel *D*, and by 1000, and a second quantity will be obtained. Divide the first quantity by the second, and the quotient will be the mean radius of the wheel *A A*.

The quantity of meal ground by a compound mill of this kind, is found by the same rule that was employed for compound mills driven by a horizontal water-wheel.

Besant's Undershot Wheel.

Besant's undershot wheel.

The water-wheel invented by Mr. Besant of Brompton is constructed in the form of a hollow drum, so as to resist the admission of water. The float-boards are fixed obliquely in pairs on the periphery of the wheel, each pair forming an acute angle, open at its vertex. This is represented in Plate I, Fig. 13, where *A B* is the wheel, *C D* its axle, and *m n*, *o p*, the position of the float-boards. In common undershot wheels, their motion is greatly retarded by the resistance opposed by the tail water to the ascending

float-boards; and their velocity is still farther diminished by the resistance of the air. But when the preceding construction is adopted, the resistance of the air and the tail water is greatly diminished by the oblique position of the float-boards.

Undershot Wheel moving at Right Angles to the Stream.

Undershot wheels have sometimes been constructed like windmills, having large inclined float-boards, and being driven in a plane perpendicular to the direction of the current. Albert Euler, who has examined theoretically this species of water-wheel, concludes that the effect will be twice as great as in common undershot wheels, and that in order to produce this effect, the velocity of the wheel, computed from the centre of impression, should be to the velocity of the water as radius is to thrice the sine of the inclination of the float-boards to the plane of the wheel. When the inclination is 60°, the ratio will be as 5 to 13 nearly, and when it is 30°, it will be nearly as 2 to 3. In this kind of wheel, a considerable advantage may also be gained by inclining the float-boards to the radius. In this case, the area of the float-boards ought to be much greater than the section of the current, and before one float-board leaves the current, the other ought to have fairly entered it. This construction may be employed with advantage in deep rivers that have but a small velocity.

On the Construction of Breast Wheels.

Breast wheels.

A breast water-wheel is a wheel in which the water is delivered at a point intermediate between the upper and under point of a wheel with float-boards. It is generally delivered at a point below the level of the axis, as in Plate I, Fig. 14, but sometimes at a point higher than the level of the axis, as in Fig. 15. On breast wheels, buckets are never employed, but the float-boards are fitted accurately, with as little play as possible, to the mill-course, so that the water, after acting upon the float-boards by its impulse, is retained between the float-boards and the mill-course, and acts by its weight till it reaches the lowest part of the wheel.

A breast wheel, as constructed by Mr. Smeaton, is represented in Fig. 14, where *A B* is a portion of the wheel, *N M* the canal which conveys the water to the wheel, *M O P* the curvilineal mill-course accurately fitted to the extremity of the float-boards, and *c d* the shuttle moved by a pinion *a*, for the purpose of regulating the admission of water upon the wheel.

An improved breast wheel is shewn in Fig. 15. The water is delivered on the wheel through an iron grating *a b*, and its admission is regulated by two shuttles *c*, *d*, the lowermost of which, *d*, is adjusted till a sufficient quantity of water passes over it; while the other *c*, which is generally moved by machinery, is made to descend upon *d*, so as to stop the wheel.

According to Mr. Smeaton, "the effect of a breast-wheel is to the effect of an undershot wheel, whose head of water is equal to the difference of level between the surface of water in the reservoir, and the part where it strikes the wheel, added to that of an overshot whose height is equal to the difference of level between the part where it strikes the wheel, and the level of the tail water."

M. Lambert observes, that when the fall of water is between 4 and 10 feet, a breast water-wheel should be erected, provided there is enough of water; that an undershot wheel should be used when the fall is below 4 feet, and an overshot wheel when the fall exceeds 10 feet. He recommends also that when the fall exceeds 10 feet, it should be divided into two, and two breast wheels erected upon it. The following Table, which may be of utility to the practical mechanic, is calculated from the formulæ of Lambert,[9] and exhibits at one view the result of his investigations.

Table for Breast Mills.

Height of the fall in feet.	Breadth of the float-boards.	Depth of the float-boards.	Radius of water wheel reckoned from extremity of float-boards.	Velocity of the wheel per second.	Time in which the wheel performs one revolution.	Turns of the mill-stone for one of the wheel.	Force of the water upon the float-boards.	Water required per second to turn the wheel.
	Ft. Dec.	Ft. Dec.	Ft. Dec.	Ft. Dec.	Secs. Dec.		lbs. Avoir.	Cub. Ft.
1	0.17	198. 6	0.75	2.18	1.92	4.80	1536	74.30
2	0.34	35. 1	1.50	3.09	2.72	6.80	1084	37.15
3	0.51	12. 7	2.26	3.78	3.33	8.32	886	24.77
4	0.69	6. 2	3.01	4.36	3.84	9.60	762	18.57
5	0.86	3.57	3.76	4.88	4.28	10.70	686	14.86
6	1.03	2.25	4.51	5.35	4.70	11.76	626	12.38
7	1.20	1.53	5.26	5.77	5.08	12.70	581	10.61
8	1.37	1.10	6.02	6.17	5.43	13.58	543	9.29
9	1.54	0.81	6.77	6.55	5.76	14.40	512	8.26
10	1.71	0.77	7.52	6.90	6.07	15.18	486	7.43
1	2	3	4	5	6	7	8	9

[9] *Nouv. Mem. de l'Acad. de Berlin*, 1775, p. 71.

It is evident from the preceding table, that, when the height of the fall is less than 8 feet, the depth of the float-boards is so great, and their breadth so small, that the breast-wheel cannot well be employed; and, on the contrary, when the height of the fall approaches to 10 feet, the depth of the float-boards is too small in proportion to their breadth. These two extremes, therefore, must be avoided in practice. The ninth column contains the quantity of water necessary for impelling the wheel, but the total expense of water should always exceed this by the quantity, at least, which escapes between the mill-course and the sides and extremities of the float-boards.

On the Construction of Overshot Wheels.

Overshot wheels.

An overshot wheel of the common kind is represented in Plate II, Fig. 1, where $ABCD$ is the rim of the wheel, having a number of buckets a, b, c, d, arranged round its circumference. When the wheel is in a state of rest upon its axis O, and water is introduced into the bucket c from the horizontal mill-course or canal EF, the weight of the water in the bucket, acting at the end of a lever equal to $m\,O$, puts the wheel in motion in the direction $c\,d$. When the subsequent bucket b comes into the position c, it is also filled with water, and so on with all the rest. When the bucket c reaches the situation of d, its mechanical effect to turn the wheel is increased, being now equal to the weight of water acting at the end of a lever $n\,O$, equal to the distance of its centre of gravity d, from a vertical line passing through the axis O, so that the mechanical effect of the water in the bucket increases all the way to B, and of course diminishes while the buckets are moving from B to C.

The buckets, however, between B and C, have not the same power upon the wheel as those between A and B; for the water begins to fall out of the buckets before they approach to B, and are almost completely empty when they reach the point H. The construction of the buckets, therefore, as shewn in the figure, is very improper, as it not only allows the water to escape before it has reached the point B, where its mechanical effect is a maximum, but also to escape completely, long before they have reached the lowest point C of the wheel. The power, therefore, of an overshot wheel must depend principally upon the form which is given to the buckets, which

should always be fullest when they are at the point B, and should retain the water as long as possible. If the buckets were to consist of a single partition in the direction of the radii of the wheel, all the water would escape from the buckets before they passed the point B on a level with the axis O.

The form of a bucket, which has been regarded as the best, is shewn in Fig. 2, by the line $D\,C\,B\,A\,G\,I\,K\,L$, where it is represented as composed of three partitions, viz. $A\,B$ and $G\,I$, called the *start* or *shoulder*, which lies in the direction of the radius; $B\,C$ and $I\,K$, called the *arm*, and inclined at an obtuse angle to the radius; and $C\,D$, $K\,L$, called the *wrist*, and inclined at an angle less than 180° to the arm $B\,C$ or $I\,K$. The depth $A\,G$ of each bucket is about $1\frac{1}{5}$ of $G\,H$; $A\,B$ is $\frac{1}{2}$ of $A\,M$; and the angle $A\,B\,C$ is such, that $B\,C$ and $G\,I$ prolonged would pass through the same point H. It ends, however, in C: so that $F\,C$ is $\frac{5}{6}$ths of $G\,H$; and $C\,D$ is placed so, that $H\,D$ is nearly $\frac{1}{5}$th of $H\,M$. Hence it follows, that the area $F\,A\,B\,C$ is nearly equal to $D\,A\,B\,C$; so that the quantity of water $F\,A\,B\,C$ will still continue in the bucket when $A\,D$ is a horizontal line, which happens when $A\,B$ forms an angle of about 35° with a vertical line. The preceding construction of the buckets is obviously too complicated, and very little additional power is gained by the angle $B\,C\,D$. Hence the general practice is to continue $B\,C$ to H, and $A\,B$ is generally only $\frac{1}{3}$d of $G\,H$.

New form of the buckets.

Such is the general view of the construction of buckets, which is given by Dr. Robison; but we cannot agree with him in thinking that this form is the best. It must be obvious, upon the slightest consideration, that the power of the wheel would be a maximum, if the whole of its semi-circumference were loaded with water. This effect would be produced, if the buckets had the shape shewn in Fig. 3, where $A\,B\,C$ is the form of the bucket, $A\,B$ being in the direction of the radius, and $B\,C$ part of the circumference of the wheel, and nearly equal to $A\,D$. This construction is, however, impracticable, as the aperture $E\,C$ is not large enough either for the admission or the escape of the water, and when the last portion of the water flows out along $B\,C$, it would strike against the bottom $D\,E$ of the bucket immediately above it. We must therefore consider what modification this form should receive, in order to give a free passage to the

water at *E C*. This may be effected, by making *B C* (Fig. 4) a little larger than *B E*, and diminishing *A B*, so as to make the angle *A B C* a little greater than 90°. In this way an aperture *d E* will be obtained, of sufficient magnitude both for the introduction and the discharge of the fluid ; and the last portion of water will no longer strike against the bottom *D d* of the upper bucket. The angle *A B E* may be brought still nearer to a right angle, as in Fig. 3, by rounding the angles at *B* and *d*. This will allow *E* to be brought nearer *d*. When the water is properly introduced by the methods afterwards to be described, this construction will be found to give great additional power to the wheel. Hence we see the reason why the inclination of *D C*, in Fig. 2, is advantageous, as it is an approximation to the preceding construction.

Double buckets. Fig. 5.

The late Mr. Robert Burns of Cartside in Renfrewshire, a most ingenious millwright and mechanic, proposed what appeared to be a very great improvement upon the form of the buckets in overshot wheels. It consisted in using a double bucket, as shewn in Fig. 5, where *L M* is a partition almost concentric with the rim, and placed so as to make the inner and outer portions of the bucket hold equal quantities of water. When these buckets are filled $\frac{1}{3}$d, they retain the whole water at 18° from the bottom of the arch, and they retain $\frac{1}{2}$ of the water at 11°. Another great advantage of this construction is, that when there is little water to drive the wheel, it may be directed, by a slight adjustment of the spout, into the outer bucket, so as to make up, by the additional length of lever, for the small quantity of water which is in use. These advantages, however, are found in practice to be counterbalanced by disadvantages which cannot be got the better of. The water is found never to fill the inner buckets, and on this account we believe Mr. Burns did not put the construction in practice.

It has in general been assumed by writers on water wheels, that the diameter of overshot wheels should always be less than the height of the fall of water by which it is to be put in motion, and various ratios have been assigned between the height of the fall and the diameter of the wheel. The Chevalier de Borda has shewn, that overshot wheels will produce a maximum effect when their diameter is equal to the greatest

height of the fall, but that a slight diminution of the wheel's diameter produces only a very small diminution of the maximum effect. If the height of the fall, for example, is 12 feet, and if the diameter of the wheel is made only 11 feet, the effect si diminished only $\frac{1}{12}$. This theoretical result has been confirmed by the admirable experiments of Mr. Smeaton, who found, "*that the higher the wheel is in proportion to the whole descent, the greater will be the effect;*" because, as he remarks, "it depends less upon the impulse of the head, and more upon the gravity of the water in the buckets; and if we consider how obliquely the water issuing from the head must strike the buckets, we shall not be at a loss to account for the little advantage that arises from the impulse thereof, and shall immediately see of how little consequence this impulse is to the effect of an overshot wheel."

If the diameter of the wheel were equal to the whole height of the fall, the water would be laid in the buckets without having acquired any velocity; so that a portion of the power of the wheel would be spent in dragging this inert mass into motion, and also by the impulse of the buckets against the water, which will dash a part of it over the wheel. Hence it is necessary that the difference between the head of water and the diameter of the wheel should be such, that the water may acquire in its descent through that space a velocity a little greater than that of the circumference of the wheel. In this view of the subject, the water should fall through a height of $2\frac{1}{2}$ or 3 inches per second, in order to acquire the velocity of the wheel; and therefore the diameter of the wheel should be only 3 inches less than the height of the fall.

The determination of the diameter of an overshot wheel, as given by Borda, Smeaton, Robison, and other authors, is founded upon the assumption, that it never should exceed the height of the fall. Let us suppose that we have a fall of 12 feet, and that the wheel should have a diameter of 11 feet according to Borda, then it appears to us, that a great advantage will be derived from making the wheel 15 feet. Now it is obvious, that the advantage of using the 15 feet wheel is, that we apply the water where it will act most perpendicularly to the line *O B* (Fig 1), or the radius of the wheel, whereas the disadvantage of such a wheel is, that it begins to lose its water

much sooner than the small one. We differ in opinion from Dr. Robison when he says, that the loss of power in the latter case exceeds what is gained in the former case; but we shall admit that it is so, and there will still be reason for maintaining the superiority of the 15 feet wheel. When the wheel has a diameter less than the height of the fall, any augmentation of the quantity of water discharged by the mill-course is of no use in increasing the effect of the wheel. The issuing water indeed acquires a velocity greater than it usually has, but this additional velocity is injurious to the motion of the wheel instead of being of any advantage. In the case of a 15 feet wheel, however, when the water rises 1 or 2 feet above its usual level, we have it in our power, by a particular form of the delivering sluice, to introduce this water upon the wheel 1 or 2 feet higher up the wheel, so that we are actually enabled to increase the height of the fall by this quantity.

On the velocity of overshot wheels.

From a series of experiments on overshot wheels, by M. Deparcieux, and published in 1754, he has concluded, that most work is performed by an overshot wheel when it moves slowly, and that the more we retard its motion by increasing the work to be performed, the greater will be the performance of the wheel. These experiments were made with a wheel 20 inches in diameter, and having 48 buckets. Cylinders of different diameters were placed upon the axle, and the effect of the wheel under different velocities was measured by the height to which it raised a weight suspended to a rope, which was wound round the different cylinders; and the general result was, that the slower the wheel turns, the greater is the effect, or the height to which the weight is raised.

In opposition to these results, the Chevalier D'Arcy maintained, that there is a certain velocity when the effect is a maximum; and he has shewn, from a comparison of Deparcieux's experiments with his own formulæ, that the wheel never moved with such a small velocity as would have given the maximum effect, and that if he had increased the diameter of his cylinders, he would have found that there was a velocity when the maximum effect began to diminish.

The experiments of Smeaton afford an excellent confirmation of the preceding reasoning. The wheel which he used was 25 inches in diameter. The depth of the buckets, or of the

shrouding, was 2 inches, and the number of buckets 36. When it made about 20 turns in a minute, the effect was nearly the greatest. When the number of turns was 30, the effect was diminished $\frac{1}{20}$th part. When the number was 40, the diminution was $\frac{1}{4}$th; when the number was less than $18\frac{1}{4}$, its motion was irregular; and when it was loaded so as not to be able to make 18 turns, the wheel was overpowered by its load.

Smeaton's experiments. "It is an advantage in practice," says Mr. Smeaton, "that the velocity of the wheel should not be diminished farther than what will procure some solid advantage in point of power; because, *cæteris paribus*, as the motion is slower the buckets must be made larger; and the wheel being more loaded with water, the stress upon every part of the work will be increased in proportion. The best velocity for practice, therefore, will be such, as when the wheel here used made about 30 turns in a minute; that is, when the velocity of the circumference is a little more than three feet in a second.

"Experience confirms, that this velocity of three feet in a second is applicable to the highest overshot wheels as well as the lowest; and all other parts of the work being properly adapted thereto, will produce very nearly the greatest effect possible; however, this also is certain from experience, that high wheels may deviate farther from this rule before they will lose their power by a given aliquot part of the whole, than low ones can be admitted to do; for a wheel of 24 feet high may move at the rate of six feet per second without losing any considerable part of its power; and, on the other hand, I have seen a wheel of 33 feet high, that has moved very steadily and well with a velocity but little exceeding two feet."

Bossut's experiments. The experiments of the Abbé Bossut afford the same results. He used a wheel three feet in diameter. The height of the buckets was three inches, their width five inches, and their number 48; and the canal which conveyed the water furnished uniformly 1194 cubic inches in a minute. When the wheel was unloaded, it made $40\frac{1}{4}$ turns in a minute. The following table, for which we have computed the fourth column, contains the results which he obtained.

Number of pounds raised.	Number of seconds in which the load was raised.	Number of revolutions performed by the wheel.	Effect of the wheel, or the product of the number of turns multiplied by the load.
11	60″	$11\frac{46}{48}$	$131\frac{32}{48}$
12	60	$11\frac{11}{48}$	$134\frac{36}{48}$
13	60	$10\frac{25}{48}$	$136\frac{37}{48}$
14	60	$9\frac{40}{48}$	$137\frac{32}{48}$
15	60	$9\frac{10}{48}$	$138\frac{6}{48}$
16	60	$8\frac{31}{48}$	$138\frac{16}{48}$
17	60	$8\frac{9}{48}$	$139\frac{9}{48}$
18	60	$7\frac{32}{48}$	138
19	The wheel turned very slowly.		
20	The wheel stopped, though first put in motion by the hand to make it catch the water.		

From this table it appears that the effect is a maximum when the number of turns is $8\frac{9}{48}$, or when the velocity of the circumference is 1 foot 4 inches per second. The effect diminished by diminishing the velocity, and the wheel was at last overpowered by its load, as in Smeaton's experiments, which ought always to happen when the resistance or load is equal to the effect of all the buckets when acting upon a semicircumference of the wheel with their respective quantities of water.

On the effect of overshot wheels.

In comparing the relative effects of water wheels, the Chevalier de Borda maintains, that an overshot wheel will raise through the height of the fall a quantity of water equal to that by which it is driven; while Albert Euler affirms that the effect is greatly inferior to this. The experiments of Mr. Smeaton shew, that when the heads and quantities of water are least, the ratio between the power and the effect at the maximum is nearly as 4 : 3; but when the heads and quantities of water were greater, it is as 4 : 2; and by a medium of the whole, it is as 3 : 2. When the powers of the water, computed for the height of the wheel only, are compared with the effects, they observe a more constant ratio, the variation being only between the ratio of 10 : 8.1 and 10 : 8.5. Hence the ratio of the power, computed upon the height of the wheel only, is to the effect, at a maximum, as 10 : 8, or as 5 : 4 nearly; and the effects, as well as the powers, are as the quantities of water and perpendicular heights multiplied together respectively.

The form of the delivering sluice, and the method of intro-

ducing the water into the buckets, will be best explained in the description of different overshot wheels.

Smeaton's Overshot Wheel.

Smeaton's overshot wheel.

The overshot wheel, as constructed by Mr. Smeaton for the upper paper-mill of Thornton, is shewn in Plate II, Fig. 6, where the diameter of the wheel is as nearly as possible equal to the height of the fall; and another wheel, which he considered as of an improved form, is represented in Fig. 7, where the diameter of the wheel exceeds the height of the fall. In both these figures *A B* is the wheel, and *M N* the extremity of the mill-course, where the water is delivered into the buckets. A vertical lever *a b c* turning round *b* as a centre, gives motion to the horizontal arm *c d*, and causes one of the shuttles *e f* to advance or recede; in consequence of which, the aperture on the right hand of *f* may be either increased or diminished, for the purpose of regulating the supply of water which the wheel may require. The iron bolt *g* goes through the bottom of the trough between the two shuttles, and is intended to prevent the bottom from sinking by the weight of the water. From the form of the aperture at *f*, it will be seen that the water will glide easily into the buckets without any waste. In both these machines, the water is turned back on the near half of the wheel; the consequence of which is, that the resistance of the lower water is removed, as it runs off in the same direction with the motion of the wheel. The wheel in Fig. 6 is made to fit its sweep and the sides of the conduit as if it were a breast wheel, so that the water does not get out of the buckets till it reaches the lowest point.

Improved overshot wheel.

An excellent overshot wheel, which we understand is used in Yorkshire, is represented in Fig. 8. It differs from the wheel in Fig. 6, in the construction of the extremity of the mill-course, and in the mode of delivering the water upon the wheel. A pinion *d*, turned with a handle, works in the teeth of a rack *c a*, having a roller *a*, whose breadth is equal to that of the mill-course, fixed at its extremity. Upon this roller is fixed a large piece of leather, which, after wrapping round part of the cylindrical circumference, extends downwards to *b*, where it is fixed, as seen in the figure, between two plates of iron or wood held together by screws. This leather forms the shuttle in the fol-

lowing manner. When the water stands so low in the mill-course *M N*, that none of it runs over the roller so as to fall into the buckets, the pinion *d* is made to move from right to left, so as to cause the rack *c a* and roller *a* to descend. The leather shuttle is thus wound up upon the roller, and the water is allowed to pass over the surface *a*, and fall into the buckets through the apertures made of iron bars, as shewn in the figure. When the water, on the contrary, rises in the mill-course, so that too much of it flows over the roller, the rack is made to move in the opposite direction, so as to diminish the supply. In this construction of the regulator, we see at once the advantages stated in p. 51, of having the diameter of the wheel *A B* greater than the height of the fall; for we are at liberty to take advantage of the additional head which is gained by any increase in the quantity of water which is conveyed to the wheel.

Description of Mr. Burns' overshot wheel without a shaft.

This very ingenious machine was invented and erected by the late Mr. Burns. It is represented in two different sections in Figs. 9 and 10, Plate II, and forms a large hollow cylinder by its buckets and sole, without having any shaft or axle-tree.

This wheel is $12\frac{1}{2}$ feet diameter, and 7 feet broad over all, and has 28 buckets. The gudgeon is 6 inches diameter, by 9 inches long. The flaunch is $1\frac{1}{4}$ inch thick at the extreme points. The arms are of red-wood fir, 6 inches square; one piece making two arms in length, where they cross one another at the wheel's centre, $1\frac{1}{4}$ inch of the wood remaining in each, connecting the two opposite arms as one piece. The wheel was made by first fitting the gudgeon into a large piece of hard wood, with the flaunch parallel to the horizon, and in that position the arms and rings were trained and bound fast to it. All the grooves for starts or raisers, and buckets, were cut out before it was removed; first one piece was bolted to the flaunch at *a a* (Fig. 9), and so of the others, leaving the distant openings for the cross bars that reach between each arm and its opposite arm. These bars, or pieces, were only 4 inches square, and were of good beech-wood, turned round in the body. They were 10 inches square at each end, in which was fitted a strong nut for a bolt, $1\frac{1}{4}$ inch thick, to go through *b*, and connect the two sides together.

After the arms were trained and fixed right upon the gudgeons, the innermost ring was completed; the tenons were train-

ed on the arms first, and the rings, $4\frac{1}{2}$ inches thick and 8 inches deep, put on by keys driven into the mortice. The remaining tenons were then reduced from $1\frac{1}{4}$ to 1 inch thick, and the outermost ring, only 3 inches thick by 6 inches deep, was firmly wedged thereon, and bound fast at the other ends by three strong wooden pins, as at *O C*, to the lower ring; the outside of the uppermost and undermost rings are flush, all the additional thickness of the lower ring projecting inside the buckets.

Some difficulty was found in laying the water properly into the buckets of this wheel, owing to the narrowness of the mouths of the buckets, by the high start or raiser, which was remedied by adopting the following plan.

The openings in the bottom of the troughing should be of iron, and so distant from each other that the water from them is thrown into two separate buckets. The iron curved parts should also be moveable, to adjust the openings to the quantity of water necessary for the wheel. Unless the head of water is 12 or 14 inches above these openings, it will be difficult to give it the proper direction into the buckets, especially if the openings are pretty wide for them; for then it deviates the more down from the line of direction, and tends to retard the wheel, by striking on the outside of the bucket.

The openings from which the buckets are filled, ought to be 10 inches less in length than the buckets, *i. e.* 5 inches at each side, otherwise the water is apt to jerk over on each side of the wheel, as the edge of the bucket passes by.

The mode of making and finishing the wheel at Cartside requires very little workmanship, compared to the usual method; and any good joiner will do it as well as a mill-wright. The joiner finished Cartside wheel in six or seven weeks. The construction will be better understood from the following reference to the figures.

Fig. 9 represents three distinct transverse views. The part marked *A*, supposes a part of the shrouding in section shewing the pins; the part marked *B*, is a section of the wheel through any part of the buckets, and shewing three of the ties, 1, 2, 3, in section. Part *C* shews the manner in which the exterior ends of the wheel are finished, also the gudgeons, flaunch, &c.

Fig. 10 is a longitudinal section of the wheel through one of the arms, shewing the projection of the shrouding—the manner in which the arms of the wheel are connected together—and likewise the manner in which the ties are connected to the gudgeon.

Description of a Double Overshot Wheel with a Chain of Buckets.

When there is a very small supply of water falling from a very great head, the overshot wheel which it is necessary to employ is so large and expensive, and so apt to be injured from its unwieldy size, that few persons would be disposed to erect one. We have seen at Coalbrook Dale a very excellent overshot wheel, of about fifty feet in diameter, which went remarkably well; and we understand that there are in Wales some wheels of nearly double this diameter. In circumstances like this, the double overshot wheel, with a chain of buckets, is a most invaluable machine, not merely from the small price at which it can be erected, but from the great power which it affords. A machine of this kind seems to have been first erected by M. Francini in 1668, in the garden of the king of France's old library. This machine of Francini's was driven by waste water, and raised water from a natural spring, by means of another chain of buckets fixed upon the same wheel.

M. Costar substituted a similar machine in place of the overshot wheel; and more recently Mr. Gladstanes, an ingenious millwright at Castle Douglas, without knowing that he had been anticipated in the invention, erected several in Galloway for the purpose of giving motion to threshing-mills.

The double overshot wheel is represented in Plate II, Fig. 11, where A and B are two rag wheels, as they are called, and $C\,D\,E\,F$ a series of buckets fixed to an endless chain, whose links fall into notches in the circumference of the rag wheels. The water issuing from the mill-course at $M\,N$, is introduced into the buckets on the side C. The descent of the loaded buckets on the side C puts the wheels A and B in motion, and the power is conveyed from the shaft of the wheel A to turn any kind of machinery. When the buckets reach F, they allow the water to escape, and ascending empty on the side E, they again return to the spout $M\,N$, to be filled as before. In this machine, the buckets have in every part of their path the same mechanical effect to turn the wheels, and they will not allow the water to escape till they have reached almost the lowest part of the fall.

This species of wheel possesses another advantage, which can be obtained from no other, namely, that by raising the wheel B, and taking out two or three of the buckets, it may be made to

work when there is such a quantity of back-water as would otherwise prevent it from moving.

Dr. Robison, in his Dissertation on Water Works, published in the second volume of his *System of Mechanical Philosophy*, has described a machine of this kind, in which plugs, or horizontal float-boards, are fixed to a chain. On the side *C* these plugs pass through a tube, a little greater in diameter than that of the floats, and the water acting by its pressure upon these floats, as it does in the case of a breast-wheel, gives motion to the wheels *A* and *B*.

The double overshot wheel is the best and the most economical which can be adopted for a small supply of water falling from a great height; but it is liable to get out of order, unless the chain which carries the bucket is made with great care and nicety.

On the method of computing the effective power of overshot wheels in turning machinery.

In overshot mills, where the wheel is moved by the weight of the water in the buckets, each bucket has a different power to turn the wheel; and this power is proportioned to the distance of the bucket from the top or bottom of the wheel; or more accurately, to the sine of the arch contained between the centre of the bucket and the top or bottom of the wheel, according as the bucket is above or below its centre. The bucket, for instance, placed upon the top of the wheel, has no power to turn it; the bucket next to this contributes but a little to turn the wheel, because it is virtually placed at the extremity of a very short lever; whereas the bucket, which is equally distant from the top and bottom of the wheel, and which is level with the centre, has the greatest power to turn it, because it acts at the extremity of a lever equal to the wheel's semidiameter. If we suppose, then, that each bucket contains *one* gallon of water, equal in weight to 10.2 lbs. avoirdupois; we may, by the simplest operations in trigonometry, compute, in pounds avoirdupois, the power which each bucket exerts in turning the wheel; and, by taking the sum of these, we will have the effective weight of the water[1] in the buckets,

[1] This phrase, which is used by practical mechanics, is very exceptionable; as every drop of water in the buckets, excepting the vertical bucket, is *effective*. By the effective weight of the water, therefore, we must understand that weight which, if suspended at the opposite extremity of the wheel, would keep it in *equilibrio*, or balance the loaded arch.

and, consequently, its proportion to the real weight of the water, with which the semi-circumference of the wheel is loaded. Those who choose to make this calculation, will find that the total weight of water upon the semi-circumference of an overshot wheel is to the effective weight as 1 to .637; but, as two or three of the buckets at the bottom of the arch are always empty, the proportion will rather be as 1 to .75 nearly. From these principles, we may deduce the following method, simpler than any hitherto given, of computing the effective weight of water upon overshot wheels of any diameter.

Rule for finding it.

Rule.—Multiply the constant number 6.12 by half the number of buckets, and this product by the number of gallons in each bucket, and the result will be the effective weight of the water upon the wheel, three buckets being supposed empty at the bottom. This rule is pretty accurate for wheels from 20 to 32 feet in diameter. But when the diameter of the wheel is less than 20 feet, the answer given by the rule must be diminished one pound avoirdupois for every foot which the wheel is less than 20.

Suppose that it is required to find the effective weight of water upon a wheel 18 feet in diameter, having 40 buckets, each containing two gallons ale measure. Then $6.12 \times 20 \times 2 = 244.8$. But as the diameter of the wheel is two feet below 20, we must deduct two pounds from the preceding answer, and the result will be 242.8 lbs. avoirdupois.

On the performance of Overshot and Undershot Mills.

Performance of overshot and under-shot wheels.

From a number of accurate experiments made by the ingenious Mr. Fenwick, upon a variety of excellent overshot mills, it appears, that when the water wheel is 20 feet in diameter, 392 gallons of water per minute (ale measure) will grind one boll of corn per hour (Winchester measure); 675 gallons per minute will grind 2 bolls; 945 gallons will grind 3 bolls; 1270 gallons will grind 4 bolls, and 1623 gallons will grind 5 bolls. From these data it will be easy to compute the performance of an overshot mill, whatever be the diameter of the wheel and the supply of water.

Examples.

Example 1.—Let it be required to find how many bolls of corn will be ground by an overshot mill, driven by a wheel 25 feet in diameter, upon which 1150 gallons of water are discharged in a minute. Say, as the

nearest number $\overset{\text{Galls.}}{1270} : \overset{\text{Bolls.}}{4} = \overset{\text{Galls.}}{1150} : \overset{\text{Bolls.}}{3.62}$, the quantity of corn ground by a wheel 20 feet in diameter. Then to find the quantity which a 25 feet wheel will grind, say,

As $\overset{\text{Feet.}}{20} : \overset{\text{Bolls.}}{3.62} = \overset{\text{Feet.}}{25} : \overset{\text{Bolls.}}{4.52}$, the answer required.

Example 2.—If it is required to grind $3\frac{1}{2}$ bolls of corn per hour, where the stream discharges 2220 gallons in a minute, what must be the diameter of the wheel? Find the number of gallons which a 20 feet wheel will require for grinding the given quantity of corn by the following proportion.

As $\overset{\text{Bolls.}}{4} : \overset{\text{Galls.}}{1270} = \overset{\text{Bolls.}}{3.5} : \overset{\text{Galls.}}{1111}$. Then, by inverse proportion, $\overset{\text{Galls.}}{1111} : \overset{\text{Feet.}}{20} = \overset{\text{Galls.}}{2220} : \overset{\text{Feet.}}{10}$, the diameter of the wheel required.

Performance of undershot mills.

In order to find the quantity of corn ground by an undershot mill, which is moved by a similar wheel, and a similar quantity of water, as an overshot mill; divide the quantity ground in an overshot mill by 2.4, and the quotient will be the answer. If it is required to know what size of wheel is necessary for making an undershot mill grind a certain quantity of corn, the supply of water being given; find the size of an overshot wheel necessary for producing the same effect, and multiply this by 2.4; the product will be the required diameter of the undershot wheel.

On Wheels driven by the Reaction or Counterpressure of Water.

Dr. Barker's mill.

The first mills which were driven by the reaction of water were called Barker's mill, and sometimes Parent's mill. We are not acquainted with the nature of M. Parent's claim to the invention; nor can we determine whether the priority is due to him or to Dr. Barker. Dr. Desaguliers, who seems to have been the first person who published an account of the machine, describes it as having been invented by Dr. Barker. "Sir George Saville says, he had a mill in Lincolnshire to grind corn, which took up so much water to work it, that it sunk his ponds visibly, for which reason he could not have constant work; but now, by Dr. Barker's improvement, the waste water only from Sir George's ponds keeps it constantly to work."

Dr. Barker's mill is shewn in Plate II, Fig. 12, where *C D* is a vertical axis, moving on a pivot at *D*, and carrying the upper millstone *m*, after passing through an opening in the fixed millstone *C*. Upon this axis is fixed a vertical tube *T T*

communicating with a horizontal tube *A B*, at the extremities of which *A B* are two apertures in opposite directions. When water from the mill-course *M N* is introduced into the tube *T T*, it flows out of the apertures *A B*, and by the reaction or counterpressure of the issuing water the arm *A B*, and consequently the whole machine, is put in motion. The bridge-tree *a b* is elevated or depressed by turning the nut *c* at the end of the lever *c b*. In order to understand how this motion is produced, let us suppose both the apertures shut, and the tube *T T* filled with water up to *T*. The apertures *A*, *B* which are shut up, will be pressed outwards by a force equal to the weight of a column of water whose height is *T T*, and whose area is the area of the apertures. Every part of the tube *A B* sustains a similar pressure; but as these pressures are balanced by equal and opposite pressures, the arm *A B* is at rest. By opening the aperture at *A*, however, the pressure at that place is removed, and consequently the arm is carried round by a pressure equal to that of a column *T T*, acting upon an area equal to that of the aperture *A*. The same thing happens on the arm *T B*; and these two pressures drive the arm *A B* round in the same direction. This machine may evidently be applied to drive any kind of machinery, by fixing a wheel upon the vertical axis *C D*.

Improvement on Barker's mill by M. Mathon de la Cour.

In the preceding form of Barker's mill, the length of the axis *C D* must always exceed the height of the fall *N D*, and therefore when the fall is very high, the difficulty of erecting such a machine would be great. In order to remove this difficulty, M. Mathon de la Cour proposes to introduce the water from the mill-course, or reservoir *F*, by means of the pipe *F G H*, entering at *D*, into the horizontal arms *A*, *B*, which are fixed to an upright spindle *C T*, but without any hollow tube *T T*. The water will obviously issue from the apertures *A B*, in the same manner as if it had been introduced at the top of a tube *T T* as high as the fall. Hence the spindle *C D* may be made as short as we please. The practical difficulty which attends this form of the machine, is to give the arms *A B* a motion round the mouth of the feeding pipe, which enters the arm at *D*, without any great friction, or any considerable loss of water. In a machine of this kind which M. Mathon de la Cour saw at Bourg Argental, *A B* was 92 inches, and its diameter three inches; the diameter of each orifice was $1\frac{1}{6}$ inch. The height of

the fall $F G$ was 21 feet; the internal diameter of D was two inches, and it was fitted into C by grinding. This machine made 115 turns in a minute when it was unloaded, and emitted water by one hole only. The machine, when empty, weighed 80 pounds, and it was half supported by the upward pressure of the water. This improvement which was published in Rozier's *Journal de Physique* for January and August 1775, appeared about 20 years afterwards as a new invention of Mr. Waring's in the *Transactions of the American Philosophical Society of Philadelphia*, who was probably not aware of the labours of M. Mathon de la Cour.

In the year 1747, Professor Segner of Gottingen published, in his *Excercitationes Hydraulicæ*, an account of a machine which differs only in form from Dr. Barker's mill. It consisted of a number of tubes arranged as it were in the circumference of a truncated cone; the water was introduced into the upper ends of these tubes, and flowing out at the lower ends, produced, in virtue of its reaction, a motion round the axis of the cone.

Another form of this machine has been suggested by Albert Euler. He proposes to introduce the water from the mill-course into an annular cavity in a fixed vessel of the shape nearly of a cylinder. The bottom of this vessel has several inclined apertures for the purpose of making the water flow out with a proper obliquity into the inferior and moveable vessel. This inferior vessel, which has the form of an inverted frustrum of a cone, moves about an axis passing up through the centre of the fixed vessel, and has a variety of tubes arranged round its circumference. These tubes do not reach to the very top of the vessel, and are bent into right angles at their lower ends. The water from the upper and fixed vessel being delivered into the tubes of the lower vessel, descends in the tubes, and issuing from their horizontal extremities, gives motion to the conical drum by its reaction.

History of this machine. The excellence of this method of employing the reaction of water, was first slightly pointed out by Dr. Desaguliers, and no further notice seems to have been taken of the invention till the appearance of Segner's machine in 1747. The attention of Leonhard Euler, John Bernoulli, and Albert Euler, was then directed to the subject, and it would appear, from the results of their investigations, that this is the most powerful of all hydraulic machines, and is therefore the best mode of employing water as a moving power.

Leonhard Euler published his theory of this machine in the *Memoirs of the Berlin Academy*, vol. vi, p. 311; and the application of the machine to all kinds of work, was explained in a subsequent paper in the seventh volume of the work, for 1752, p. 271. John Bernoulli's investigations will be found at the end of his *Hydraulics*.

Albert Euler concluded, that when the machine had the form given to it by Segner, the effect was equal to the power, and was a maximum when the velocity became infinite. Mr. Waring, in the paper which we have already quoted, makes the effect of the machine equal only to that of a good undershot wheel driven with the same quantity of water falling through the same height. The Abbé Bossut has likewise investigated the theory of this machine, and has found that an overshot wheel, and a wheel of the form given to it by Albert Euler, will produce equal effects with the same quantity of water, if the depth of the orifice below the mill-course in the latter machine is equal to the vertical height of the loaded arch in the overshot wheel; and he, upon the whole, recommends the overshot wheel as preferable in practice. The preceding result, however, proves the inferiority of the overshot wheel, as the height of the loaded arch must be always much less than that of the fall. A new and ingenious theory of this machine has lately been given by Mr. Ewart in the *Manchester Memoirs*.

Effect of Barker's mill.

Method of keeping off the Back-water from Water Wheels.

Mr. Burns of Cartside, in Renfrewshire, seems to have been the first who proposed and executed the method of keeping off the back-water from wheels in time of floods, by directing against it the force of the superabundant current. A B

This method is shewn in Fig. 8, where ~~*C D E*~~ is a current of water taken from the mill-lead, and acting against the back-water at *F*, so as to drive it back and keep it from the wheel. For this purpose, the water *C* is kept from the wheel by the boarding *D B E*, a channel being left at *E*, through which the back-water would rush upon the wheel if it were not driven back by the superior force of the current rushing down the channel *A*. Mr. Perkins makes the diameter of *B* $\frac{1}{5}$ larger than *E*. The current is made to act in a direction perpendicular to the plane of the wheel, when the wheel has been

already built. The water which comes from the buckets is also carried off through *B*. This method appears to have been adopted in America, and was recently submitted to the public by the ingenious Mr. Perkins, who was not aware of what had been done in Scotland, and published in the *Transactions of the Society of Arts*, vol. xxxviii, p. 109. See also the *Edinburgh Philosophical Journal*, vol. iv, p. 439, and vol. v, p. 222.

Sect. III.—*On the Force of Wind, and the mode of applying it to drive Machinery.*

Considering air as a fluid, it is obvious that its force, when in motion, may be applied to machinery, in the same way as moving water is applied to the float-boards of vertical or horizontal undershot wheels. As the current of air, however, is not limited in magnitude, we must direct it solely upon the float-boards on one side of the wheel, by screening the other side from its action. When the axis of a wheel of this kind is vertical, and consequently the motion of the vanes or float-boards horizontal, the machine is called a *horizontal wind-mill.* The most common method, however, of applying the force of wind, is to direct it against sails moving nearly in a vertical plane, as shewn in Plate II, Fig. 13. In this case, the machine is called a *vertical wind-mill.*

On Vertical Wind-Mills.

The vertical wind-mill, as improved by Mr. James Verrier, is represented in Plate II, Fig. 13, where *A A A* are the three principal posts, 27 feet $7\frac{1}{2}$ inches long, 22 inches broad at their lower extremities, 18 inches at their upper ends, and 17 inches thick. The column *B* is 12 feet $2\frac{1}{2}$ inches long, 19 inches in diameter at its lower extremity, and 16 inches at its upper end; it is fixed in the centre of the mill, passes through the first floor *E*, having its upper extremity secured by the bars *G G*. *E E E* are the girders of the first floor, one of which only is seen, being 8 feet 3 inches long, 11 inches broad, and 9 thick; they are mortised into the posts *A A A* and the column *B*, and are about 8 feet 3 inches distant from the ground floor. *D D D* are three posts, 6 feet 4 inches long, 9 inches broad, and 6 inches thick; they are mortised into the girders *E F* of the first and second floor, at the distance of 2 feet 4 inches from the posts *A*, &c. *F F F* are the girders of the second floor, 6 feet long, 11 inches broad, and 9 thick; they are mortised into the posts *A*, &c.

and rest upon the upper extremities of the posts *D*, &c. The three bars *G G G* are 3 feet $1\frac{1}{2}$ inches long, 7 inches broad, and 3 thick; they are mortised into the posts *D* and the upper end of the column *B*, 4 feet 3 inches above the floor. *P* is one of the beams which support the extremities of the bray-trees or brayers; its length is 2 feet 4 inches, its breadth 8 inches, and its thickness 6 inches. *I* is one of the bray-trees, into which the extremity of one of the bridge-trees *K* is mortised. Each bray-tree is 4 feet $9\frac{1}{2}$ inches long, $9\frac{1}{2}$ inches broad, and 7 thick; and each bridge-tree is 4 feet 6 inches long, 9 inches broad, and 7 thick, being furnished with a piece of brass on their upper surface to receive the under pivot of the millstones. *L L* are two iron screw-bolts, which raise or depress the extremities of the bray-trees. *M M M* are the three millstones, and *N N N* the iron spindles, or arbors, on which the turning millstones are fixed. *O* is one of three wheels, or trundles, which are fixed on the upper ends of the spindles *N N N;* they are 16 inches in diameter, and each is furnished with 14 staves. *f* is one of the carriage-rails, on which the upper pivot of the spindle turns, and is 4 feet 2 inches long, 7 inches broad, and 4 thick. It turns on an iron bolt at one end, and the other end slides in a bracket fixed to one of the joists, and forms a mortise, in which a wedge is driven to set the rail and trundle in or out of work; *t* is the horizontal spur-wheel that impels the trundles; it is 5 feet 6 inches diameter, is fixed to the perpendicular shaft *T*, and is furnished with 42 teeth. The perpendicular shaft *T* is 9 feet 1 inch long, and 14 inches in diameter, having an iron spindle at each of its extremities; the under spindle turns in a brass block fixed into the higher end of the column *B;* and the upper spindle moves in a brass plate inserted into the lower surface of the carriage-rail *C*.

The spur-wheel *r* is fixed on the upper end of the shaft *T*, and is turned by the crown-wheel *v* on the windshaft *c*; it is 3 feet 2 inches in diameter, and is furnished with 15 cogs. The carriage-rail *C*, which is fixed on the sliding kerb *Z*, is 17 feet 2 inches long, 1 foot broad, and 9 inches thick. *Y Y Q* is the fixed kerb, 17 feet 3 inches diameter, 14 inches broad, and 10 thick, and is mortised into the posts *A A A*, and fastened with screw-bolts. The sliding kerb *Z* is of the same diameter and breadth as the fixed kerb, but its thickness is only $7\frac{1}{2}$ inches; it revolves on 12 friction rollers fixed on the upper surface of

the kerb *Y Y Q*, and has 4 iron half staples, *Y*, *Y*, &c. fastened on its outer edge, whose perpendicular arms are 10 inches long, 2 inches broad, and 1 inch thick, and embrace the outer edge of the fixed kerb to prevent the sliding one from being blown off. The capsills, *X*, *V*, are 13 feet 9 inches long, 14 inches broad, and 1 foot thick; they are fixed at each end with strong iron screw-bolts, to the sliding kerb, and to the carriage-rail *C*. On the right hand of *w* is seen the extremity of a cross rail, which is fixed into the capsills *X* and *V*, by strong iron bolts; *e* is a bracket 5 feet long, 16 inches broad, and 10 inches thick; it is bushed with a strong brass collar, in which the inferior spindle of the windshaft turns, and is fixed to the cross-rail *w*: *b* is another bracket 7 feet long, 4 feet broad, and 10 inches thick; it is fixed into the fore-ends of the capsills, and, in order to embrace the collar of the windshaft, it is divided into two parts, which are fixed together with screw-bolts. The windshaft *c* is 15 feet long, 2 feet in diameter at the fore-end, and 18 inches at the other; its pivot at the back-end is 6 inches diameter, and the shaft is perforated to admit an iron rod to pass easily through it. The vertical crown-wheel *v* is 6 feet in diameter, and is furnished with 54 cogs, which drive the spur-wheel *r*. The bolster *d*, which is 6 feet 3 inches long, 13 inches broad, and half a foot thick, is fastened into the cross-rail *w*, directly under the centre of the windshaft, having a brass pulley fixed at its fore-end. On the upper surface of this bolster is a groove, in which the sliding bolt *R* moves, having a brass stud at its fore-end. This sliding bolt is not distinctly seen in the figure, but the round top of the brass stud is visible below the letter *h*: the iron rod that passes through the windshaft bears against this brass stud. The sliding bolt is 4 feet 9 inches long, 9 inches broad, and $\frac{1}{3}$ of a foot thick. At its fore-end is fixed a line, which passes over the brass pulley in the bolster, and appears at *a* with a weight attached to its extremity, sufficient to make the sails face the wind that is strong enough for the number of stones employed; and when the pressure of the wind is more than sufficient, the sails turn on an edge, and press back the sliding bolt, which prevents them from moving with too great velocity; and, as soon as the wind abates, the sails, by the weight *a* are pressed up to the wind, till its force is sufficient to give the mill a proper degree of velocity. By this apparatus, the wind is regulated and justly proportioned to the resistance or work to be

performed; an uniformity of motion is also obtained, and the mill is less liable to be destroyed by the rapidity of its motion.

That the reader may understand how these effects are produced, we have represented, in Fig. 14, the iron rod, and the arms which bear against the vanes; *a h* is the iron rod which passes through the windshaft *c*, in Fig. 13; *h* is the extremity, which moves in the brass stud that is fixed upon the sliding bolt; *a i*, *a i*, &c. are the cross arms, at right angles to *a h*, whose extremities *i*, *i*, similarly marked in Fig. 13, bear upon the edges of the vanes. The arms *a i* are 6½ feet long, reckoning from the centre *a*, 1 foot broad at the centre, and 5 inches thick; the arms *n*, *n*, &c. that carry the vanes or sails, are 18½ feet long, their greatest breadth is 1 foot, and their thickness 9 inches, gradually diminishing to their extremities, where they are only 3 inches in diameter. The four cardinal sails, *m*, *m*, *m*, *m*, are each 13 feet long, 8 feet broad at their outer ends, and 3 feet at their lower extremities; *p*, *p*, &c. are the four assistant sails, which have the same dimensions as the cardinal ones, to which they are joined by the line *S S S S*. The angle of the sails' inclination, when first opposed to the wind, is 45 degrees, and regularly the same from end to end.

Plate II. Fig. 14. Method of varying the angle of the sails' inclination.

It is evident, from the preceding description of this machine, that the windshaft *c* moves along with the sails; the vertical crown wheel *v* impels the spur wheel *r*, fixed upon the axis *T*, which carries also the spur wheel *t*. This wheel drives the three trundles *H*, one of which only is seen in the figure, which being fixed upon the spindles *N*, &c. communicate motion to the turning mill-stones.

That the wind may act with the greatest efficacy upon the sails, the windshaft or principal axis must always have the same direction as the wind. But as this direction is perpetually changing, some apparatus is necessary for bringing the windshaft and sails into their proper position. This is sometimes effected by supporting the machinery on a strong vertical axis, whose pivot moves in a brass socket firmly fixed into the ground, so that the whole machine, by means of a lever, may be made to revolve upon this axis, and be properly adjusted to the direction of the wind. Most wind-mills, however, are furnished with a moveable roof, which revolves upon friction rollers inserted in the fixed kerb of the mill; and the adjustment is effected by the assistance of

Method of turning the sails to the wind.

a simple lever. As both these methods of adjusting the windshaft require the assistance of men, it would be very desirable that the same effect could be produced solely by the action of the wind. This may be done, by fixing a large wooden vane, or weather-cock, at the extremity of a long horizontal arm, which lies in the same vertical plane with the windshaft. By this means, when the surface of the vane, and its distance from the centre of motion are sufficiently great, a very gentle breeze will exert a sufficient force upon the vane to turn the machinery, and will always bring the sails and windshaft to their proper position. This weathercock, it is evident, may be applied, either to machines which have a moveable roof, or which revolve upon a vertical arbor.

Wind-mills numerous in Holland.

Prior to the French revolution, wind-mills were more numerous in Holland and the Netherlands than in any other part of the world, and there they seem to have been brought to a very high state of perfection. This is evident, not only from the experiments of Mr. Smeaton, from which it appears, that sails weathered in the Dutch manner produced nearly a maximum effect, but also from the observations of the celebrated Coulomb. This philosopher examined above 50 wind-mills in the neighbourhood of Lisle, and found that each of them performed nearly the same quantity of work when the wind moved with the velocity of 18 or 20 feet per second, though there were some trifling differences in the inclination of their windshafts, and in the disposition of their sails. From this fact, Coulomb justly concluded, that the parts of the machine must have been so disposed as to produce nearly a maximum effect.

Form of Dutch wind-mills, according to Coulomb.

In the wind-mills on which Coulomb's experiments were made, the distance, from the extremity of each sail to the centre of the windshaft, or principal axis, was 33 feet. The sails were rectangular, and their width was a little more than 6 feet, 5 of which were formed with cloth stretched upon a frame, and the remaining foot consisted of a very light board. The line of junction of the board and the cloth, formed, on the side which faced the wind, an angle sensibly concave at the commencement of the sail, which diminished gradually till it vanished at its extremity. Though the surface of the cloth was curved, it may be regarded as composed of right lines perpendicular to the arm, or whip, which carries the frame, the extremities of these lines corre-

sponding with the concave angle formed by the junction of the cloth and the board. Upon this supposition these right lines at the commencement of the sail, which was distant about 6 feet from the centre of the wind shaft, formed an angle of 60° with the axis, or windshaft, and the lines, at the extremity of the wing, formed an angle, increasing from 78° to 84°, according as the inclination of the axis of rotation to the horizon increased from 8° to 15°; or, in other words, the greatest angle of weather was 30°, and the least varied from 12° to 6°, as the inclination of the windshaft varied from 8° to 15°.[1] A pretty distinct idea of the surface of wind-mill sails may be conveyed, by conceiving a number of triangles standing perpendicular to the horizon, in which the angle contained between the hypothenuse and the base is constantly diminishing: the hypothenuse of each triangle will then be in the superficies of the vane, and they would form that superficies if their number were infinite.

On the form and position of Wind-mill Sails.

Form and position of the sails.

M. Parent seems to have been the first mathematician who considered the subject of wind-mill sails in a scientific manner. The philosophers of his time entertained such erroneous opinions upon this point, as to suppose that the surface of the sails should be equally inclined to the direction of the wind and to the plane of their motion; or, what is the same thing, that the angle of weather should be 45°.[2] But it appears from the investigations of Parent, that a maximum effect will be produced when the sails are inclined $54\frac{3}{4}°$ to the axis of rotation, or when the angle of weather is $35\frac{1}{3}°$. In obtaining this conclusion, however, M. Parent has assumed data which are inadmissible, and has neglected several circumstances which must materially affect the result of his investigations. The angle, or inclination, assigned by Parent, is certainly the most efficacious for giving motion to the sails from a state of rest,[3] and for preventing them from

The inclination assigned by Parent erroneous.

[1] The *weather* of the sails is the angle which the surface of the sails forms with the plane of their motion, and is always equal to the complement of the angle which that surface forms with the axis.

[2] See Wolfii *Opera Mathematica*, tom. i, p. 680, where this angle is recommended.

[3] This may be demonstrated in the following manner. Let x be the cosine of the angle sought; then, since the sine, cosine, and radius of any arch form a right angled triangle, the square of the sine will be equal to the square of the cosine subtracted from the square of the radius, that is, $1 - x^2$ will be the square of the sine when the radius is unity. But the effect of the wind, on an oblique sail, is, in the com-

stopping when in motion; but he has not considered that the action of the wind upon a sail at rest is different from its action upon a sail in motion: for since the extremities of the sails move with greater rapidity than the parts nearer the centre, the angle of weather should be greater towards the centre than at the extremity, and should vary with the velocity of each part of the sail.[4] The reason of this is very obvious. It has been demonstrated by Bossut,[5] and sufficiently established by experience, that when any fluid acts upon a plain surface, the force of impulsion is always exerted most advantageously when the impelled surface is in a state of rest, and that this force diminishes as the velocity of the surface increases. Now, let us suppose, with Parent, that the most advantageous angle of weather for the sails of wind-mills is $35\frac{1}{3}$ degrees for that part of the sail which is nearest the centre of rotation, and that the sail has everywhere this angle of weather; then, since the extremity of the sail moves with the greatest velocity, it will, in a manner, withdraw itself from the action of the wind; or, to speak more properly, it will not receive the impulse of the wind so advantageously as those parts of the sail which have a less velocity. In order, therefore, to make up for this diminution of force, we must make the wind act more perpendicularly upon the sail, by diminishing its obliquity, that is, we must increase its inclination to the axis or the direction of the wind; or, what is the same thing, we must diminish its angle of weather. But, since the velocity of every part of the sail is proportional to its distance from the centre of motion, every elementary portion of it must have a different angle of weather diminishing from the centre to the extremity of the sail. The law or rate of diminution, however, is still to be discovered, and we are fortunately in possession of a theorem of Euler's, afterwards given by Maclaurin, which determines this law of variation.[6] Let a represent the velocity of the wind, and c the velocity of any given part of

Euler's theorem.

pound ratio of the square of the sine of its obliquity, and the breadth of the sail projected on a plane perpendicular to the direction of the wind. Now, this breadth is exactly x, the cosine of the sail's inclination; therefore $x \times \overline{1 - x^2}$, or $x - x^3$ will represent the effect of the wind upon the sail. And, as this is to be a maximum, let us take its fluxion, which will be $\dot{x} - 3x^2\dot{x} = 0$. Dividing by $\dot{x}$ we have $3x^2 = 1$, or $= \sqrt{\tfrac{1}{3}} = \frac{1}{1.7320508076} = .5773520$, which is the cosine of $54^\circ\ 44'\ 13''$.

[4] See vol. i, p. 67.

[5] *Traité d'Hydrodynamique*, § 772.

[6] See Maclaurin's *Fluxions*, art. 910–914.

the sail, then the effort of the wind upon that part of the sail will be greatest when the tangent of the angle of the wind's incidence, or of the sail's inclination to the axis, is to radius as $\sqrt{2+\frac{9cc}{4aa}}+\frac{3c}{2a}$ to 1.

In order to apply this theorem, let us suppose that the radius or whip $m\,s$ of the sail $\alpha\,\beta\,\delta\,i$, is divided into six equal parts, that the point n is equidistant from m and s, and is the point of the sail which has the same velocity as the wind; then, in the preceding theorem, we will have $c=a$, when the sail is loaded to a maximum; and therefore the tangent of the angle, which the surface of the sail at n makes with the axis, when Explanation and application of this theorem. Plate II. Fig. 14.

$a=1$ will be $\sqrt{2+\frac{9}{4}}+\frac{3}{2}=3.561=$ Tangent of 71° 19′,

which gives 15° 41′ for the angle of weather at the point n. Since, at $\frac{1}{2}$ of the radius $c=a$, and since c is proportional to the distance of the corresponding part of the sail from the centre, we will have, at $\frac{1}{6}$ of the radius $s\,m$, $c=\frac{a}{3}$; at $\frac{2}{6}$ of the radius $c=\frac{2a}{3}$; at $\frac{4}{6}$, $c=\frac{4a}{3}$; at $\frac{5}{6}$, $c=\frac{5a}{3}$; and at the extremity of the radius, $c=2a$. By substituting these different values of c, instead of c in the theorem, and by making $a=1$, the following Table will be obtained, which exhibits the angles of inclination and weather which must be given to different parts of the sails.

Table shewing the rate at which the Inclination varies.

Parts of the radius from the centre of motion at s.	Velocity of the sail at these distances, or values of c.	Angle formed with the axis.		Angle of weather.	
		Deg.	Min.	Deg.	Min.
$\frac{1}{6}$	$\frac{a}{3}$	63	26	26	34
$\frac{2}{6}$	$\frac{2a}{3}$	69	54	20	6
$\frac{3}{6}$ or $\frac{1}{2}$	a	74	19	15	4
$\frac{4}{6}$ or $\frac{2}{3}$	$\frac{4a}{3}$	77	20	12	40
$\frac{5}{6}$	$\frac{5a}{3}$	79	27	10	33
1	$2\,a$	81	0	9	0

Having thus pointed out an important error in Parent's theory, and shewn how to find the law of variation in the angle of weather, we have farther to observe, that, in order to simplify the calculus, Parent supposed the velocity of the wind to be infinite when compared with the velocity of the sail, and that its impulsion upon the sail was in the compound ratio of the square of its velocity and the square of the sine of incidence. The first of these suppositions is evidently inaccurate, and was shewn to be so by Daniel Bernouilli, in his Hydrodynamics. With regard to the force of impulsion on the sails, the proposition is perfectly true in theory, and has been demonstrated by Pitot,[7] and other philosophers; but it decidedly appears, from the experiments presented to the French Academy, in 1763, by M. le Chevalier de Borda, and from those made, in 1776, by M. d'Alembert, the Marquis Condorcet, and the Abbe Bossut,[8] that this proposition does not hold in practice. The first part of the proposition, indeed, that the force of impulsion is proportional to the square of the velocity of the surface that is impelled, is true in practice; but, when the angles of incidence are small, the latter part of the proposition must be abandoned, as it would afford very false results. In cases, however, where the angles of incidence are between 50 and 90 degrees, we may regard the impulsion as proportional to the square of the velocity multiplied by the square of the sine of incidence; but we must remember, that the force thus determined by the theory will be a little less than that which would be found by experiment, and that the difference increases as the angle of incidence recedes from 90°.

Force of impulsion on inclined surfaces.

Such being the circumstances which Parent has overlooked in his investigations, we need not be surprised to find, from the experiments of Smeaton, that when the angle which he recommends was adopted, the sails produced a smaller effect than when they were weathered in the common manner, or according to the Dutch construction.[9]

The theory of wind-mills has been treated at great length by M. Euler, the most profound and celebrated mathematician of

[7] *Mem. de l'Acad. Par.* 1729, p. 540.

[8] *Nouvelles Experiences sur la resistance des Fluides*, par M. M. d'Alembert, le Marquis de Condorcet, et l'Abbe Bossut, chap. v, § 35.

[9] Belidor has fallen into the same error as Parent, and observes, that the workmen at Paris make the angle of weather 18°, and thereby lose $\frac{2}{7}$ of the effect; whereas, this is nearly the most efficacious angle that can be adopted. See *Architecture Hydraulique*, par Belidor, tom. 2, B. iii, pp. 33–41.

his time. He has shewn, that the angle assigned by Parent is too small for a sail in motion, and that the angle of weather should vary with the velocity of the different parts of the sails; but, like Parent, he has supposed that the force of impulsion upon surfaces, with different obliquities, is proportional to the square of the sines of their inclination. As the angles of incidence, however, are sufficiently great, this circumstance will have but a trifling effect upon his conclusions. After Euler has shewn, in general, how to determine the force of impulsion upon the sails, whatever be their figure and disposition, and whatever be the celerity of their motion; he then investigates, by the method *de maximis et minimis*, what should be the inclination of the sails to the axis, and the velocity of their extremities, in order to produce a maximum effect; and he finds, that this inclination and velocity are variable, and are inversely proportional to the momentum of friction in the machine. That the reader may fully understand this important result, we may remark, that, in theory, the greatest effect will be produced when the velocity of the sails is infinitely great, and when their surfaces are perpendicular to the wind's direction; that is, when the angle of weather is nothing. But both these suppositions are excluded in practice; for though the sails receive the greatest possible impetus from the wind, when they are inclined 90° to the axis, yet this force has not the smallest tendency to put them in motion; and it is not difficult to perceive, that the friction of the machine, and the resistance of the air to the thickness of the sails, must always limit the velocity of their motion. In this case, theory does not accord with practice; but they may be easily reconciled, by making the angle of inclination 89° instead of 90, and supposing the sails to perform a finite, but a very great number of revolutions in a second, an hundred for example; then the sails, having still a very disadvantageous position, will receive but a small impetus from the wind, which may be called *one* pound. But this defect in the impelling power is made up by the great velocity of the sails; and since the effect is always equal to the product of the weight and the velocity, we will have, $1 \times 100 = 100$ for the effect of the machine. Now, let us take friction into the account, and suppose it to be so great as to diminish the rapidity of the sails, from 100 to 50 turns in a second; then, in order that the machine

Euler's observations on wind-mills.

may produce an effect equal to 100, as formerly, we must change the angle of the sail's inclination, till it receives from the wind an impetus equal to two pounds for $2 \times 50 = 100$. If the friction be still farther increased, the celerity of the machine will experience a proportional diminution, and the angle of inclination must undergo such a change, that the force of impulsion received from the wind may make up for the velocity that is lost by an increase of friction. From these observations it plainly appears, that the celerity of the sails, and their inclination to the axis, depend upon the momentum of friction; and as this is generally a constant quantity in machines, and can easily be determined experimentally, the position of the sails, the velocity of their motion, and the effect of the machine, may be found from the following Table, which is calculated from the formulæ of Euler, and adapted to different degrees of friction.

In this Table, F denotes the force of the wind upon all the sails; d is the radius of the sail, or the distance of its extremity from the centre of the axis or windshaft; v is the velocity of the wind; and s the velocity of the sail's extremity, which is equal to the numbers contained in the fourth column.

Table containing the Angle of Inclination and Weather of Wind-Mill Sails, the Velocity of their Extremities, and the Effect of the Machine, for any Degree of Friction.

Momentum of friction.	Angle of the sail's inclination to the axis.	Angle of weather.	Velocity of the sails at their extremities.	Effect of the machine.	Effect of the machine differently expressed.
0.235702 *Fd*	45°	45°	0.000000 *v*	0.000000 *Fv*	0.000000 *Fs*
0.175837 *Fd*	50	40	0.127686 *v*	0.004718 *Fv*	0.036950 *Fs*
0.122871 *Fd*	55	35	0.281334 *v*	0.017968 *Fv*	0.063869 *Fs*
0.079653 *Fd*	60	30	0.469882 *v*	0.037427 *Fv*	0.079653 *Fs*
0.047001 *Fd*	65	25	0.711154 *v*	0.060147 *Fv*	0.084576 *Fs*
0.024370 *Fd*	70	20	1.042160 *v*	0.083159 *Fv*	0.079795 *Fs*
0.010362 *Fd*	75	15	1.550395 *v*	0.103842 *Fv*	0.066978 *Fs*
0.003084 *Fd*	80	10	2.499421 *v*	0.120105 *Fv*	0.048053 *Fs*
0.000386 *Fd*	85	5	5.208606 *v*	0.130454 *Fv*	0.025046 *Fs*
0.000000 *Fd*	90	0	Infinite.	0.134001 *Fv*	0.000000 *Fs*
1	2	3	4	5	6

Explanation of the table.

The preceding table has been applied, by Euler, solely to that species of wind-mills in which the sails are sectors of an ellipse, and which intercept the whole cylinder of wind. This construction was recommended also by Parent;

but later and more accurate experiments have evinced, that when the whole area is filled up with sail, the wind does not produce its greatest effect, from the want of proper interstices to escape. On this account a small number of sails are generally used, and these are either rectangular, or a little enlarged at their extremities. It will be proper, therefore, to shew how the table can be applied to this description of sails, for the application is much more difficult than in the other case.

It is evident, from the first column of the table, that before we can use it, we must find the value of F, or the force of the wind upon all the sails. But as this force depends not merely upon the quantity of surface, and the velocity of the wind, which are always given, but also upon the angle of their inclination, which is unknown, some method of determining it, independently of this angle, must be adopted. Euler has shewn how to do this, in the case where the whole area is filled with elliptical sectors; but there is no direct method of determining the value of F in the case of rectangular sails, when the angle of inclination is unknown. We must find it therefore by approximation; that is, we must take any probable angle of inclination, 70° for example, and find the value of F suited to this angle, and thence the co-efficient of $F d$, in the first column. With this co-efficient enter the table, and take out the corresponding angle of inclination, which will be either less or greater than 70°. With this new angle of inclination find a more accurate value of F, and consequently a new co-efficient of $F d$. If this co-efficient does not differ very much from that formerly found, it may be regarded as true, and employed for taking out of the table a more accurate angle of inclination, along with the velocity of the sails, and the effect of the machine. We shall now illustrate both these methods by an example, after having shewn how to determine by experiment the momentum of friction, and the velocity of the wind.

To find the Momentum of Friction.

On the momentum of friction.

In a calm day, when the wind-mill is unloaded, or performing no work, bring two opposite sails into a horizontal position; and, having attached different weights to the extremities of their radius, find how many pounds are sufficient not only for impressing the smallest motion on the sails, but for continuing them in that state; and

the number of pounds multiplied into the length of the radius, will be the momentum of friction. When this experiment is made, it will always be found that a greater weight is necessary for moving the sails than for continuing them in motion; and, in order that the quantity of friction may be accurately estimated, the wind-mill should be put in motion immediately before the experiment is made; for the friction always increases with the time in which the communicating parts have remained in contact.

To find the Velocity of the Wind.

To find the wind's velocity.

Various instruments, denominated Anemometers, or Anemoscopes, have been invented for measuring the force and velocity of the wind.[1] The velocity of the wind has been deduced also from the motion of the clouds, and the change effected by the wind upon the motion of sound. The second of these methods is manifestly inaccurate, and the first takes for granted what is palpably erroneous, that the velocity of the wind is the same in the higher regions of the atmosphere, as at the surface of the earth.

Coulomb's method.

The most simple method of determining the velocity of the wind, is that which Coulomb employed in his experiments on wind-mills, and which requires neither the aid of instruments nor the trouble of calculation.[2] Two persons were placed on a small elevation, at the distance of 150 feet from one another, in the direction of the wind; and, while the one observed, the other measured the time which a small and light feather employed in moving through that space. The distance between the two persons, divided by the number of seconds, gave the velocity of the wind per second. Having thus shewn how to find the momentum of friction, and the velocity of the wind, we shall now explain the use of the table.

Explanation of the table.

Supposing the radius of the sails to be 20 feet, the velocity of the wind 10 feet per second, and that it requires a force of 10 pounds acting at the extremity of the radius to overcome the friction of the machine,—it is re-

[1] A full account of these different contrivances, illustrated with numerous drawings, will be found in the article *Anemometer*, in the *Edinburgh Encyclopædia*, vol. ii, p. 69.

[2] See *Mem. de l'Acad. Par.* 1781, p. 70.

quired to find the angle of weather, the velocity of the sails, and the effect of the machine.

Let d, the radius of the sails, be $= 20$ feet, then the momentum of friction will be $10 \times 20 = 200$ pounds. Let n, the number of sails, be $= 12$, while a represents the breadth of the sails at their extremities, and b the breadth into which they are projected, or the breadth which they would occupy if reduced into a plane perpendicular to the wind. Then, since the whole cylinder of wind is supposed to be intercepted, the effect produced upon all the oblique sails will be equal to the effect that would be produced upon a perpendicular surface, equal to the whole area of the polygon into which the oblique triangular sails are projected. The value of b, therefore, may be found by plane trigonometry, the length of the sail and the angle of the polygon being given, or by the following theorem: $b = 2d \times \text{tang.} \frac{180}{n}$, d being radius, and n the number of sails. In the present case, then, we shall have $b = 2 \times 20 \times \text{tang.} \frac{180}{n}$, or $b = 40 \times \text{tang.}\ 15°, = 10.717968$ feet. Now, since the area of any triangle is equal to its altitude multiplied by half its base, the area of a polygon will be equal to the altitude of one of the triangles which compose it, or to the radius of the inscribed circle, multiplied by half the number of its sides. The area of the polygon, therefore, into which the sails are projected, or the quantity of perpendicular surface impelled by the wind, will be $\frac{1}{2}\, n\, d\, b$, and, consequently, the force of impulsion F, upon this surface, will be $\frac{1}{2}\, n\, d\, b\, v\, v$, where $v\, v$ is the square of the wind's velocity, to which the force of impulsion is always proportional. In the present case, then, the force F, which impels the sails, will be $6 \times 20 \times 10.717968 \times v\, v$; and if $v\, v$ be the altitude which is due to the velocity of the wind, or the height through which a heavy body must fall in order to acquire that velocity, the force of impulsion F will be equal to the weight of a mass of air, whose volume is $1286.15616 \times v\, v$ cubic feet, or to $1\frac{3}{5}\, v\, v$ cubic feet of water; for water is about 800 times more dense than air; that is to 100 $v\, v$ pounds avoirdupois, $62\frac{1}{2}$ of which are equal to a cubic foot of water. But, in order that the machine may move, the momentum of friction 200 must be less than $0.235702 \times F\, d$, or $0.235702 \times 100\, v\, v \times 20$; for when it is

exactly this, the wind cannot move the machine, as appears from the first line of the table; or, what is the same thing, the height due to the velocity of the wind, viz. $v\,v$, must be greater than 0.424, or $\frac{3}{7}$ of a foot, which corresponds to a velocity of 5.222 or $5\frac{2}{9}$. Unless, therefore, the celerity of the wind exceeds $5\frac{2}{9}$ feet per second, it will not be able to move the machine. These things being premised, let us now proceed to determine the construction and effect of the machine, upon the supposition that the momentum of friction is 200 pounds, and the velocity of the wind 10 feet per second. Now, $v\,v$, the height due to this velocity, is $1\frac{3}{5}$ feet;[3] therefore the force of impulsion F is $= 100\,v\,v$ pounds, or $100 \times \frac{8}{5}$, or $= 160$ pounds avoirdupois; and $F\,d = 160 \times 20 = 3200$. But the momentum of friction, viz. $F\,d$, multiplied into its co-efficient, should be equal to 200 pounds; therefore, the co-efficient will be equal to $\frac{200}{Fd} = \frac{200}{3200} = 0.062500$, and the momentum of friction will be $0.062500\ F\,d$. With this number enter the first column of the table, and you will find the angle of inclination corresponding to it to be about 63°; the velocity of the sail's extremity $= \frac{3}{5}\,v$, or 6 feet per second; and the effect of the machine $= 0.05\ F\,v = 0.05 \times 160$ ℔ $\times 10 = 8$ ℔ $\times$ 10 feet, or 8 pounds raised through 10 feet in a second, which is equal to 1000 pounds raised through 288 feet in an hour. But the force of a man, according to Euler, is equal to 1000 pounds raised through 180 feet in an hour; therefore, the power of the machine, with a wind moving at the rate of 10 feet per second, is not equal to the power of two men.

Let us now suppose that the wind-mill is driven by means of four rectangular sails, 18 feet in length and four in breadth, and that the momentum of friction and the radius of the sails are the same as before. Then the area of each sail will be 18×4, and the whole surface that is acted upon by the wind will be $18 \times 4 \times 4 = 288$ square feet. But before we can determine the force which the wind exerts upon this surface, we must know its inclination to the wind: let us suppose this to be 70°, then the impetus of the wind upon the sails, or F, will

[3] The height answering to any velocity, and the velocity due to any height, may be found by the following theorems, in which v is the velocity, and h the height due to it; $v = 2\sqrt{16.087 \times h}$, hence $h = \frac{2vv}{129}$. See p. 33.

be $= 288 \times \text{Sin. } 70^2 \times v\,v$, in which case v is the wind's velocity, or $F = 254\ v\,v$ cubic feet of air. If $v\,v$ be the height due to the wind's velocity, dividing this quantity by 800, we will have $F = \frac{127}{400}\ v\,v$ cubic feet of water, and multiplying this by $62\frac{1}{2}$, we will have $F = 19.8\ v\,v$ pounds avoirdupois. Now, let the velocity of the wind be 30 feet per second, the height $v\,v$ due to this velocity will be 14 feet nearly; and, consequently, $F = 19.8 \times 14 = 276$ pounds avoirdupois. $F\,d$ will therefore be $= 5540$; and, since the whole momentum of friction is 200, the co-efficient of $F\,d$ will be $= \frac{200}{5540} = 0.036101$, and the momentum of friction, expressed as the table requires, will be $= 0.036101\ F\,d$. Having entered the table with this number, the proper angle of inclination will be found to be $67\frac{1}{2}$ degrees With this angle, instead of 70°, repeat the foregoing calculation, and after finding a new co-efficient to $F\,d$, enter the table with it a second time, and you will have the proper angle of inclination, differing but very little from the former, and likewise the velocity of the sails, and the effect of the machine.[4]

By comparing with the preceding theory the performance of the wind-mills examined by Coulomb and Lulofs,[5] it will be found that their power is almost double of that which is deduced from theory. This remarkable difference arises from a defect in the common hypothesis, which represents the force of impulsion as proportional to the square of the wind's velocity, and the square of the sine of the angle of incidence. When the wind impinges upon the sail, the air behind it is rarefied; this rarefaction increases with the velocity of the wind, and therefore the impulsion must be much greater than what is deduced from the common hypothesis. Euler supposes it to be twice as great; and, upon this supposition, has treated the subject more accurately in a subsequent memoir,[6] which, however, is too profound to be of any service to the practical mechanic.

[4] Those who wish to inquire farther into the theory of wind-mills, will find some excellent observations in D'Alembert's *Traite de l'Equilibre et du Mouvement des Fluides*, 1770, p. 396, § 368; or in his *Opuscules*, tom. v, p. 148, &c. and also by Lambert, in the *Mem. de l'Acad. Berlin*, 1775, p. 92.

[5] See page 82.

[6] *Recherches plus exactes sur l'effet des moulins à vent*, Mem. de l'Acad. Berlin, 1766, vol. xii, p. 164.

Results of Smeaton's experiments.

These theoretical deductions, however interesting they may be, must yield in point of practical utility to the observations of our countryman Mr. Smeaton. From a variety of well-conducted experiments, he found, that the common practice of inclining plane sails, from 72° to 75°, to the axis, was much more efficacious than the angle assigned by Parent, the effect being as 45 to 31. When the sails were weathered in the Dutch manner, that is, when their surfaces were concave to the wind, and when the angle of inclination increased towards their extremities, they produced a greater effect than when they were weathered either in the common way, or according to Maclaurin's theorem.[7] But when the sails were enlarged at their extremities, as represented at $\alpha\,\beta$ (Fig. 1), so that $\alpha\,\beta$ was one third of the radius $m\,s$, and $\alpha\,m$ to $m\,\beta$, as 5 to 3, their power was greatest of all, though the surface acted upon by the wind remained the same.[8] If the sails be farther enlarged, the effect is not increased in proportion to the surface; and, besides, when the quantity of cloth is great, the machine is much exposed to injury by sudden squalls of wind. In these experiments of Smeaton, the angle of weather varied with the distance from the axis; and he found, from several trials, that the most efficacious angles were those contained in the following Table:

Parts of the radius $m\,s$, which is divided into 6 parts.	Angle with the axis.	Angle of weather.
1	72	18
2	71	19
3	72	18 middle
4	74	16
5	$77\frac{1}{2}$	$12\frac{1}{2}$
6	83	7

Supposing the radius $m\,s$ of the sail to be 30 feet, then the sail will commence at $\frac{1}{6}\,m\,s$, or five feet from the axis, where the angle of inclination will be 72°. At $\frac{2}{6}\,m\,s$, or 10 feet from the axis, the angle will be 71°, and so on.

[7] See page 67.

[8] In the sails used in Portugal, the broad part is placed at the end of the arm. They are much more swollen than those of common wind-mills, and may be set to draw, in a manner similar to the stay sails of a ship.

On the Effect of Wind-mill Sails.

Effect of wind-mill sails, according to Smeaton.

The following maxims, deduced by Mr. Smeaton from his experiments, contain the best information which we have upon the effect of wind-mill sails, if we except a few experiments made by Coulomb.

Maxim 1. The velocity of wind-mill sails, whether unloaded or loaded, so as to produce a maximum effect, is nearly as the velocity of the wind, their shape and position being the same.

Maxim 2. The load at the maximum is nearly, but somewhat less than, as the square of the velocity of the wind, the shape and position of the sails being the same.

Maxim 3. The effects of the same sails at a maximum, are nearly, but somewhat less than, as the cubes of the velocity of the wind.

Maxim 4. The load of the same sails, at the maximum, is nearly as the squares, and their effect as the cubes of their number of turns in a given time.

Maxim 5. When sails are loaded, so as to produce a maximum at a given velocity, and the velocity of the wind increases, the load continuing the same; 1*st*, The increase of effect, when the increase of the velocity of the wind is small, will be nearly as the squares of those velocities; 2*dly*, When the velocity of the wind is double, the effects will be nearly as 10 to $27\frac{1}{2}$; but, 3*dly*, When the velocities compared are more than double of that when the given load produces a maximum, the effects increase nearly in the simple ratio of the velocity of the wind.

Maxim 6. In sails where the figure and position are similar, and the velocity of the wind the same, the number of turns, in a given time, will be reciprocally as the radius or length of the sail.

Maxim 7. The load, at a maximum, which sails of a similar figure and position will overcome, at a given distance from the centre of motion, will be as the cube of the radius.

Maxim 8. The effects of sails of similar figure and position are as the square of the radius.

Maxim 9. The velocity of the extremities of Dutch sails, as well as of the enlarged sails, in all their usual positions when

unloaded, or even loaded to a maximum, are considerably quicker than the velocity of the wind.[9]

On the absolute Effects of Vertical Windmills in manufacturing Oil of Colza.

In the wind-mills at Lisle, examined by Coulomb, which were used for the preparation of oil of Colza, an arbor was furnished with 14 wipers, or radii, for the purpose of raising seven stampers. Five of these stampers were made of oak, and were about 7 inches long, and 28 centimetres square. They were shod with iron, weighing about 25 or 30 kilogrammes, and served to bruise the seeds, the weight of each being about 500 kilogrammes. The other two stampers, which had the same length, but were only 19 or 20 centimetres square, were used to lock and unlock the wedges for extracting the oil by a strong compression, and weighed about 250 kilogrammes each. Of these two last only one acted at a time; but the other five all acted together.

Exp. 1. When the velocity of the wind was 2.27 metres in a second, the sails revolved $5\frac{1}{2}$ times in a minute when the mill was unloaded, and no stampers raised; but by putting into action one stamper weighing 510 kilogrammes, and striking two blows of 49 centimetres in height, the mill scarcely made three turns in a minute.

Exp. 2. When the velocity of the wind was 4.06 metres in a second, the arbor made from 7 to 8 turns in a minute; and when two of the stampers were put in action, that weighed 510 kilogrammes, and one of 250 kilogrammes, the mill prepared 1 ton, or 100 kilogrammes of oil in 24 hours.

Exp. 3. When the velocity of the wind was 6.5 metres in a second, the arbor made 13 turns in a minute; and when five of the stampers weighing 500 kilogrammes were put in action, and one of 250 kilogrammes, the mill prepared $3\frac{1}{2}$ tons of oil in 24 hours. With this velocity, the wind blew uniformly, the mill carried all the sails without straining the carpentry, and the

9 Mr. Smeaton found, when the radius was 30 feet, that for every *three* turns of the Dutch sails in ther common position (when the angle of weather at the extremity is nothing), the wind-mill moves at the rate of *two* miles an hour; for every *five* turns in a minute of the Dutch sails, in their best position, the wind moves *four* miles an hour; and for every *six* turns in a minute of the enlarged sails, at their best position, the wind moves *five* miles an hour.

velocity appeared to be that which was best suited to the machine.

Exp. 4. When the wind blew with force at the rate of about 9.1 metres in a second, it was necessary to take in an area of two metres of sail at the extremity of each sail. The arbor made from 17 to 18 turns in a minute, and the mill prepared, with all its six stampers in action, 5 tons of oil in 24 hours.

Exp. 5. On an average of several years, each wind-mill prepares annually 400 tons of oil. Each ton consumes from 14,000 to 15,000 dynamical units[10] of the force transmitted by the stampers. Coulomb supposes, that the blows of the stampers, and the action of the wipers and friction, absorb one-sixth of the force transmitted to the stampers, which reduces to 12,500 the number of dynamical units necessary for preparing 100 kilogrammes of oil.

The preceding result is confirmed by a very important experiment made by M. Halle at Lille, on a steam engine of 10 horse power, which he found to prepare in 24 hours 500 kilogrammes of oil. Now, the power of a horse has been reckoned at 5974 dynamical units; consequently, $10 \times 5974 = 59{,}740$, which, divided by 5, gives 11,948 for the number of units employed in preparing 100 kilogrammes of oil.

Effect of Wind-mills in grinding Corn.

When a wind-mill is employed to grind corn, the millstone makes five revolutions in the same time that the sails and the arbor make one.

Exp. 1. The mill does not begin to turn till the velocity of the wind is about 4 metres per second.

Exp. 2. When the velocity of the wind is 5.8 metres per second, the sails make from 11 to 12 turns in a minute, and the mill will grind from 400 to 450 kilogrammes in an hour, or about 100,000 kilogrammes in 24 hours.

Exp. 3. When the velocity of the wind is 9.1 metres in a second, the mill carries all her sails, makes 22 turns in a minute, and grinds 900 kilogrammes of flour in an hour, or 21,600 in 24 hours. With this velocity, the flour is heated to a considerable degree, and the millers change from time to time the kind of grain which is ground, in order, as they say, to refresh the mill.

[10] See page 7 of this Volume.

By comparing the preceding experiments, and considering that seven horses, or one man, can grind 167 kilogrammes of corn in a day, it will be found that the grinding of 100 kilogrammes of corn consumes 416, 525, or 538 dynamical units. The first of these numbers is obviously too small, and M. Hachette concludes, in general, that the preparation of 100 kilogrammes of corn corresponds to a dynamical effect of the arbor of a wind-mill equal to 500 or 550 units.

In examining the ratio between the velocity of the wind and the number of revolutions of the windshaft or arbor, Mr. Smeaton obtained the following results for Dutch sails in their common position, when their radius was 30 feet.

Number of revolutions of the windshaft in a minute.	Velocity of the wind *per hour.*	Ratio between the velocity of the wind and the revolutions of the windshaft.
3	2 miles.	0.666
5	4	0.800
6	5	0.833

The following results are deduced from the preceding experiments of Coulomb.

No. of experiments.	No. of turns in a minute.	Velocity of the wind in a second.	Velocity of the wind per hour, in miles.	Ratio of Col. 2 & 4.
1	3	2.27 metres	5.1	1.7
2	7 to 8	4.06	9.14	1.22
3	13	6.5	14.6	1.05
4	17 to 18	9.1	20.4	1.17
5	11 to 12	5.8	13.0	1.13
6	22	9.1	20.4	0.93

These results differ widely from those of Smeaton; but we must consider, that the mill was not doing its full work except in experiments 3, 4, 5, and 6. The mean of the ratios from these experiments will be 1.07, the ratio between the number of turns per minute, and the number of miles per hour. Hence, by multiplying the number of turns per minute by 1.07, we shall have an approximate measure of the velocity of the wind.

Mr. Lulofs of Leyden found, that a Dutch wind-mill employed to drain marshes was capable of raising 1500 cubic feet of water four feet high in a minute, when the wind moved at

the rate of 30 feet per second. It had four rectangular sails, and the mean angle of weather was 17°.

In the Dutch wind-mills of the dimensions formerly stated, Coulomb found, when the velocity of the wind was 20 feet per second, that the effect was equivalent to 1000 pounds raised through 218 feet in a minute. The force lost by the action of the wipers upon the stampers was equal to 1000 pounds raised through $16\frac{1}{2}$ feet in a minute, and the friction was equivalent to 1000 pounds raised through $18\frac{1}{2}$ feet in a minute. Hence the total force of the wind was equal to 1000 pounds raised through the height of 253 feet in a minute.

On Horizontal Wind-mills.

A variety of opinions have been entertained respecting the relative advantages of horizontal and vertical wind-mills. Mr. Smeaton, with great justice, gives a decided preference to the latter; but, when he asserts that horizontal wind-mills have only $\frac{1}{8}$ or $\frac{1}{10}$ of the power of vertical ones, he certainly forms too low an estimate of their power. Mr. Beatson, on the contrary, who has received a patent for the construction of a new horizontal wind-mill, seems to be prejudiced in their favour, and greatly exaggerates their comparative value. From an impartial investigation, it will probably appear, that the truth lies between these two opposite opinions; but before entering on this discussion, we must first consider the nature and form of horizontal wind-mills. Horizontal wind-mills.

In Fig. 15 of Plate II, CK is the perpendicular axis, or windshaft, which moves upon pivots. Four cross bars CA, CD, IB, FG, are fixed to this arbor, which carry the frames $APIB$, $DEFG$. The sails AI, EG, are stretched upon these frames, and are carried round the axis CK, by the perpendicular impulse of the wind. Upon the axis CK a toothed wheel is fixed, which gives motion to the particular machinery that is employed. In the figure only two sails are represented; but there are always other two placed at right angles to these. Now, let the sails be exposed to the wind, and it will be evident that no motion will ensue; for the force of the wind, upon the sail AI, is counteracted by an equal and opposite force, upon the sail EG. In order, then, that the wind may communicate motion to the machine, the force upon the returning sail EG Plate II. Fig. 15. Common method of bringing back the sails against the wind.

must either be removed by screening it from the wind, or diminished by making it present a less surface when returning against the wind. The first of these methods is adopted in Tartary, and in some provinces of Spain; but is objected to by Mr. Beatson, from the inconvenience and expense of the machinery and attendance requisite for turning the screens into their proper positions. Notwithstanding this objection, however, we are disposed to think that this is the best method of diminishing the action of the wind upon the returning sails, for the moveable screen may easily be made to follow the direction of the wind, and assume its proper position, by means of a large wooden weather-cock, without the aid either of men or machinery. It is true, indeed, that the resistance opposed to the returning sails is not completely removed; but it is at least as much diminished as it can be by any method hitherto proposed. Besides, when this plan is resorted to, there is no occasion for any moveable flaps and hinges, which must add greatly to the expense of every other method.

Beatson's method.

The mode of bringing the sails back against the wind, which Mr. Beatson invented, is, perhaps, the simplest and best of the kind. He makes each sail *A I* to consist of six or eight flaps, or vanes, *A P* 1 *b*, *b* 1 2 *c*, &c. moving upon hinges, represented by the dark lines *A P*, *b* 1, *c* 2, &c. so that the lower side *b* 1 of the first flap overlaps the hinge, or highest side, of the second flap, and so on. When the wind, therefore, acts upon the sail *A I*, each flap will press upon the hinge of the one immediately below it, and the whole surface of the sail will be exposed to its action. But when the sail *A I* returns against the wind, the flaps will revolve upon their hinges, and present only their edges to the wind, as is represented at *E G*, so that the resistance occasioned by the return of the sail must be greatly diminished, and the motion will be continued, by the superiority of force exerted upon the sails, in the position *A I*. In computing the force of the wind upon the sail *A I*, and the resistance opposed to it by the edges of the flaps in *E G*, Mr. Beatson finds, that, when the pressure upon the former is 1872 pounds, the resistance opposed by the latter is only about 36 pounds, or $\frac{1}{52}$ part of the whole force; but he neglects the action of the wind upon the arms *C A*, &c. and the frames which carry the sails, because they expose the same surface, in the position *A I*, as in the position

E G. This omission, however, has a tendency to mislead us in the present case, as we shall now see, for we ought to compare the whole force exerted upon the arms, as well as the sail, with the whole resistance which these arms and the edges of the flaps oppose to the motion of the wind-mill. By inspecting Fig. 15, it will appear, that if the force, upon the edges of the flaps, which Mr. Beatson supposed to be 12 in number, amounts to 36 pounds, the force spent upon the bars *CD*, *DG*, *GF*, *FE*, &c. cannot be less than 60 pounds. Now, since these bars are acted upon with an equal force, when the sails have the position *A I*, $1872 + 60 = 1932$ will be the force exerted upon the sail *A I*, and its appendages, while the opposite force, upon the bars and edges of the flaps, when returning against the wind, will be $36 + 60 = 96$ pounds, which is nearly $\frac{1}{20}$ of 1932, instead of $\frac{1}{52}$ as computed by Mr. Beatson. Hence we may see the advantages which will probably arise from using a screen for the returning sail, instead of moveable flaps, as it will preserve not only the sails, but the arms and the frame which support it, from the action of the wind.[11]

Comparison between vertical and horizontal wind-mills.

We shall now conclude this article with a few remarks on the comparative power of horizontal and vertical wind-mills. It was already stated, that Mr. Smeaton rather under-rated the former, while he maintained that they have only $\frac{1}{8}$ or $\frac{1}{10}$ the power of the latter. He observes, that when the vanes of a horizontal and vertical mill are of the same dimensions, the power of the latter is four times that of the former; because, in the first case, only one sail is acted upon at once; while, in the second case, all the four receive the impulse of the wind. This, however, is not strictly true, since the vertical sails are all oblique to the direction of the wind. Let us suppose that the area of each sail is 100 square feet; then the power of a horizontal sail will be 100,[1] as only one sail is acted upon, and as its surface is perpendicular to the wind, and the power of a vertical sail may be called 100 (Sine 70°) = 94 nearly (70° being the common angle of

[11] The sails of horizontal wind-mills are sometimes fixed, like float-boards, on the circumference of a large drum or cylinder. These sails move upon hinges, so as to stand at right angles to the drum, when they are to receive the impulse of the wind, and when they return against it, they fold down upon its circumferenee. See *Repertory of Arts*, vol. vi.

[1] This proceeds upon the supposition that the action upon two sails oblique to the wind, is nearly the same as one perpendicular to it.

inclination); but since there are four vertical sails, the power of them all will be $4 \times 94 = 376$; so that the power of the horizontal sail is to that of the four vertical ones, as 1. to 3.76, and not as 1 to 4 according to Mr. Smeaton. But Mr. Smeaton also observes, that if we consider the farther disadvantage which arises from the difficulty of getting the sails back against the wind, we need not wonder if horizontal wind-mills have only about $\frac{1}{8}$ or $\frac{1}{10}$ the power of the common sort. We have already seen, that the resistance occasioned by the return of the sails, amounts to $\frac{1}{20}$ of the whole force which they receive; by subtracting $\frac{1}{20}$, therefore, from $\frac{1}{3.76}$ we shall find that the power of horizontal wind-mills is only about $\frac{1}{5}$ less than that of vertical ones. This calculation proceeds upon a supposition, that the whole force exerted upon vertical sails is employed in turning them round the axis of motion; whereas, a considerable part of this force is lost in pressing the pivot of the axis, or windshaft, against its gudgeon. Mr. Smeaton has overlooked this circumstance, otherwise he could never have maintained that the power of four vertical sails was quadruple the power of one horizontal sail, the dimensions of each being the same. Taking this circumstance into the account, we cannot be far wrong in saying, that, in theory at least, if not in practice, the power of a horizontal wind-mill is about $\frac{1}{5}$ of the power of a vertical one, when the quantity of surface and the form of the sails is the same, and when every part of the horizontal sails have the same distance from the axis of motion as the corresponding parts of the vertical sails. But if the horizontal sails have the position *A I*, *E G* (Fig.), instead of the position *CA dm*, *CDon*, their power will be greatly increased, though the quantity of surface is the same; because the part *C P* 3 *m*, being transferred to *B I* 3 *d*, has much more power to turn the machinery. We would recommend it also to the mechanic, to furnish horizontal wind-mills with *six* or *eight sails;* for, as it happens in the analogous case of water-mills, the wind bends round their extremities, and impinges upon those parts of the sail immediately behind, which are not exposed to the direct action of the wind Having these methods, therefore, of increasing the power of horizontal sails, we would encourage every attempt to improve their construction, as not only laudable in itself, but calculated to be of essential utility in a commercial country.

A new and ingenious method of bringing back the sails of a

horizontal wind-mill against the wind has been invented by Mr. George Buchanan of Edinburgh, a full account of which, illustrated by drawings, will be found in the *Edinburgh Encyclopedia*, vol. xiii, p. 569, art. *Mechanics*. An account of Mr. Gray's method of regulating the velocity of wind-mills will be found in the same article, p. 589.

Sect. IV.—*On the Force of Steam, and the Method of applying it to drive Machinery.*

Importance of the steam-engine.

The superiority of inanimate power to the exertions of animals in turning machinery has been universally acknowledged. In the former, the power generally continues its action without the smallest intermission, but frequent and long relaxations are necessary for restoring the strength and activity of exhausted animals. There are many places, however, where a sufficient quantity of water cannot be procured, or where it cannot be employed for the want of proper declivities; and there are situations also which are highly unfavourable for the erection of wind-mills. But even when water and wind-mills can be conveniently erected, there is such a variation in the impelling power, arising from accidental and unavoidable causes, that sometimes, in the case of water, and often in the case of wind, there is not a sufficient force for putting the machinery in motion. In such circumstances, the discovery of steam as an impelling power may be regarded as a new era in the history of the arts. Wherever fire and water can be obtained, we can procure a quantity of steam capable of overcoming the most formidable resistance, and free from those accidental variations of power which affect every inanimate agent that has hitherto been employed as the first mover of machines.

The earliest notice[2] of the application of steam occurs in a work published by Solomon de Caus, in 1615, and entitled *Les Raisons des Forces mouvantes*. The author describes a machine consisting of a spherical venel placed in the fire, and having two apertures, one of which, furnished with a stopcock, receives the water, while the other receives a tube, which descends through

[2] In the 50th and 71st propositions of Hero of Alexandria's *Spiritalia*, two toys are described, which move by the reaction of steam issuing from an eolipile moving round an axis, like Barker's mill.

the water, and nearly touches the bottom. The author observes, that when the vessel is sufficiently heated, the water rises and issues out of the tube ; and he also remarks, that the aqueous vapour may be condensed into the same weight of water.

In 1629, the mechanical impulse of steam was employed by Giovani Branca as the impelling power of a stamping engine. The steam was formed in a vessel resembling the human figure, and after issuing out of its mouth, it struck the vanes of a wheel, which, by means of some intermediate wheel-work and wipers, elevated the stamper of a mill. This contrivance is described in a work entitled *Le Machine diverse del Signior Giovani Branca*.[3]

In 1663, the Marquis of Worcester published his *Century of Inventions*, the 68th of which contains the following description of his method of raising water by means of steam :—

" This admirable method which I propose of raising water by the force of fire has no bounds if the vessels be strong enough : for I have taken a cannon, and having filled it three-fourths full of water, and shut up its muzzle and touch-hole, and exposed it to the fire for 24 hours, it burst with a great explosion. Having afterwards discovered a method of fortifying vessels internally, and combined them in such a way that they filled and acted alternately, I have made the water spout in an uninterrupted stream 40 feet high ; and one vessel of rarefied water raised 40 of cold water. The person who conducted the operation had nothing to do but turn two cocks ; so that one vessel of water being consumed, another begins to force, and then to fill itself with cold water, and so on in succession."

In a work entitled the " *Miner's Friend*," published in 1696, Captain Savary describes several steam-engines which were actually constructed, and in which the steam was condensed by injection, a contrivance of which he seems to have been the sole inventor. Papin, to whom the French have ascribed the invention of the steam-engine, does not appear to have employed steam in raising water till 1698, but the contrivances which he used for this purpose, as described in the *Acta Eruditorum Lipsiæ* and in Leupold's *Theatrum Machinarum*, are clumsy and impracticable.

The steam-engine received great improvements from the

[3] See the *Journal des Mines*, tom. xxxiii, 1813, p. 321.

hands of Newcomen, Beighton, Blakey,[4] and other ingenious men; but it was brought to its present state of perfection by the late celebrated Mr. James Watt of Glasgow, one of the most accomplished engineers of the last century. Hitherto the steam-engine had been used merely as a hydraulic machine for draining mines or for raising water; but in consequence of Mr. Watt's improvements, it has for a series of years been employed as the impelling power or first mover of almost every species of machinery.

It would be unsuitable to the nature of a popular work, to enter into any detailed account, either of the history of this engine, or of the various forms which it has received. We shall content ourselves, therefore, with describing, as clearly as we can, one of the engines which Mr. Watt erected for the Albion Mills in 1787.

Description of Mr. Watt's steam-engine. Plate III. Fig 1.

In Fig. 1 of Plate III, is represented a section of one of Mr. Watt's best engines erected at the Albion Mills: *n* is the boiler in which the water is converted into steam by the heat of the furnace *p q*. It is sometimes made of copper, but more frequently of iron; its bottom is concave, and the flame is made to circulate round its sides in the flues *o o*, and is sometimes conducted through the very middle of the water, through a tube *o'*, so that as great a surface as possible may be exposed to the action of the fire. In some of Mr. Watt's engines, the fire contained in an iron vessel was introduced into the very middle of the water, and the outer boiler was formed of wood, as being a slow conductor of heat. When the furnaces are constructed in the most judicious manner, 8 square feet of the boiler's surface must be acted upon by the fire or the flame, in order to convert one cubic foot of water into steam in the space of an hour, and *one bushel* of Newcastle coals, or *one cwt.* of Wednesbury coals, thus applied, will boil off from 8 to 12 cubic feet. When fire is applied to the boiler, the water does not evaporate into steam till it has reached the temperature of 212° of Farenheit, or the boiling point. This arises from the superincumbent weight of the asmosphere; for when the water is heated in a vessel exhausted of air, steam is generated even below the temperature of 96°, or blood heat. When the water, however, is pressed by air or steam more condensed

[4] An account of Blakey's contrivance will be found in Vol. I, p. 312.

than the atmosphere, a temperature greater than 212° is necessary for the production of steam; but the heat requisite for this purpose increases in a less ratio than the pressures to be overcome. The steam which is produced in the boiler *n* is about 1800 times rarer than water, and is conveyed through the steam-pipe *F* into the cylinder *A*, where it acts upon the piston *B*, and communicates motion to the great beam *a a*.

Method of supplying the boiler with water.

But before we proceed to consider the manner by which this motion is conveyed, we shall point out the very ingenious method which Mr. Watt has employed for supplying the boiler regularly with water, and preserving it at the same height $a' b'$. This is absolutely necessary in order that the quantity and elasticity of the steam in the boiler may be always the same. The small cistern c', placed above the boiler, is supplied with water from the hot well *t t*, by means of the pump *V*, and the feed-pipe $f' f''$. To the bottom of this cistern is fitted the pipe $c' d'$, which is immersed in the water *a*, *b*, and is bent at its lower extremity in order to prevent the entrance of the rising steam. A crooked arm $c' f'$, attached to the side of the cistern c', supports the small lever $e' g'$, which moves upon f' as a centre. The extremity g' of this lever carries, by means of the wire $g' p'$, a stone or piece of metal p', which hangs just below the surface of the water in the boiler, and the other extremity e' is connected by the wire $e' d'$ with a valve at the bottom of the cistern c', which covers the top of the pipe $c' d'$. Now, it is a maxim in hydrostatics,[5] that when a heavy body is suspended in a fluid, the body loses as much of its weight as the quantity of water which it displaces. When the water $a' b'$, therefore, is diminished by part of it being converted into steam, the upper surface of the body p' will be above the water, and its weight will consequently be increased in proportion to the quantity of the body that is out of the water; or, to speak more precisely, the additional weight which the body p' receives by a diminution of the water in the boiler, is equal to the weight of a quantity of the fluid, whose bulk is the same as the part of the body p' which is above the water. By this addition to its weight, the stone p' will cause the extremity g' of the lever to descend, and by elevating the arm $c' f'$, will open the valve at the top of the pipe $c' d'$ and thus gradually introduce a quantity of

[5] See Vol. I, p. 116.

water into the boiler, equal to that which was lost by evaporation. This process is continually going on while the water is converting into steam; and it is evident that too much water can never be introduced; for as soon as the surface of the water coincides with the upper surface of the body p', it recovers its former weight, and the valve in c' shuts the top of the pipe $c'\, d'$. In order to know the exact height of the water in the boiler, two gage-pipes with cocks, shewn at $u\, u'$, are employed, one of which, u, reaches to within a little of the height at which the water should stand, and the other, u', reaches a very little below that height. If the water stands at the desired height, one of the cocks u being opened will give out steam, and the other cock u' will emit water, in consequence of the pressure of the superincumbent steam on the water $a'\, b'$; but if water should issue from both cocks, the water will stand too high in the boiler, and if steam issues from both, the water will stand too low. As there would be great danger of the boiler's bursting if the steam should become too strong, it is furnished with the safety-valve v, which is loaded in such a manner, that its weight, added to that of the atmosphere, may exceed the pressure of the interior steam when of a sufficient strength. As soon as the steam becomes so elastic as to endanger the boiler, its pressure preponderates over the pressure of the safety-valve and the atmosphere. The valve, therefore, opens, and the steam escapes from the boiler, till its strength is sufficiently diminished, and the safety-valve shuts by the predominance of its pressure over that of the interior steam. By opening the safety-valve, the engine may be stopped at pleasure. A small rectangular lever, with equal arms, is fixed upon the side of the valve, and connected with its top. To one of these arms a chain is attached, which passes over a pulley from a horizontal to a vertical direction, and by pulling which, the safety-valve is opened, and the machine stopped.

Safety-valve.

From the dome of the boiler proceeds the steam-pipe F, which conveys the steam into the top of the cylinder A, 34 inches in diameter, by means of the upper steam nozle valve G, and into the bottom of the cylinder by means of the lower steam nozle valve K. The branch of the pipe which extends from G to K is cut off in Fig. 1, in order to shew the valves. The cylinder G is sometimes inclosed in a wooden case, in order to prevent it from being cooled by the ambient air; and

Construction of cylinder.

sometimes in a metallic case, that it may be surrounded and kept warm by a quantity of steam, which is brought from the steam-pipe *F*, through the small pipe *f*, by turning a cock. The water is carried off from this case by means of a syphon. It is generally thought, however, that little benefit is obtained by encircling the cylinder with steam, as the quantity thus lost is almost equal to what is destroyed by the coldness of the cylinder. After the steam, which was admitted above the piston *B* by the valve *G*, and below it by the valve *K*, has performed its respective offices of depressing and elevating the piston *B*, and consequently the great beam *a a*, it escapes by the eduction-valves *H* and *L* into the condenser *M*, immersed in a cistern of cold water, where it is converted into water by means of a jet playing in the inside of it. The water thus collected in the condenser is carried off, along with the air which it contains, into the hot well *t*, by the air pump *P*, which is wrought by the piston rod from *g*, attached to the great beam *a a*. From the hot well *t*, this water is conveyed by the hot water pump *T*, and the pipe *V*, into the cistern, for the purpose of supplying the boiler. The water *C′* which renders air-tight the pump *P*, and supplies the jet of water in the condenser, is furnished by the cold water pump *U*, which is worked by the great beam. The steam and eduction-valves *G*, *H*, *K*, *L*, are opened and shut by the spanners near *Y*, whose handles are moved by the plugs 1, 2, fixed to the piston rod of the air pump. This part of the machinery has been called the *working geer*; and is so constructed that the steam and eduction-valves can be worked either by the hand or by the piston of the air pump. Its mode of operation, which is minutely shewn in Figs. 2 and 3, will be afterwards more particularly described. The piston rod *C*, which moves the piston *B*, and makes a stroke of 8 feet, passes through a box or collar of leathers fixed in a strong metallic plate on the top of the cylinder. The rod is turned perfectly cylindrical, and is finely polished, and greased with beef or mutton tallow, in order to prevent any air from passing by its sides.

In the steam-engines of Newcomen and Beighton, where the piston was raised merely by a counterweight at one extremity of the great beam, the piston rod was connected with its other extremity by means of a chain bending round the arch of a circle fixed at that extremity; but in Mr. Watt's improved engines with

a double stroke, in which the piston receives a strong impulse upwards as well as downwards, the chain would slacken, and could not communicate motion to the beam. An inflexible rod, therefore, must be employed for connecting the piston with the beam, or the piston must be suspended by double chains, like those of engines for extinguishing fire. In some of Mr. Watt's engines, the latter of these methods was adopted. He then employed a toothed rack working in a toothed sector fixed at the end of the beam, but as this was very subject to wear, and even when perfect, did not move with the requisite smoothness, he was led to invent the mechanism of the *Parallel Motion*, which will afterwards be described.

Mode of converting a reciprocating into a rotatory motion.

All the engines which were constructed before the time of Mr. Watt were employed merely for raising water, and were never used as the first movers of machinery. Mr. R. Fitzgerald, indeed, published in the Transactions of the Royal Society, a method of converting the irregular motion of the beam into a continued rotatory motion, by means of a crank and a train of wheel work, connected with a large and massy fly, which, by accumulating the pressure of the machine during the working stroke, urged round the machinery during the returning stroke, when there was no force pressing it forward. For this new and ingenious contrivance, Mr. Fitzgerald took a patent, and proposed to apply the steam-engine as the moving power of every kind of machinery; but it does not appear that any mills were erected under this patent.[6] In order to convert the reciprocating motion of the beam into a circular motion, Mr. Watt fixed a strong and inflexible rod *h*, called the connecting rod, to the extremity of the great beam. To the lower end of this rod, a toothed wheel *i* is fastened by bolts and straps, so that it cannot move round its axis. This wheel is connected with another toothed wheel *j* of the same size, by means of iron bars, which permit the former to revolve round the latter, but prevent them from quitting each other. This apparatus is called the *Sun and Planet wheels*, from the similarity of their motion to that of the two luminaries. On the axis of the wheel *j* is placed

[6] Mr. Jonathan Hull, in his steam tow-boat, proposed in 1736, converted the reciprocating motion of the piston into a rotatory motion, by ropes and wheels. See p. 113.

the large and heavy fly-wheel *m* which regulates the desultory motion of the beam. When the extremity of the great beam rises from its lowest position, it will bring along with it the wheel *i*, and cause it to revolve upon the circumference of the wheel *j*, so that the interior part of the former, or the part next the cylinder, will act upon the exterior part of the latter, or the part farthest from the cylinder, and put it in motion along with the fly *m*. After the wheel *i* has got to the top of the wheel *j*, the end of the beam will have reached its highest position, and the wheel *j*, along with the fly, will have performed one complete revolution. When the wheel *i* passes from the top of *j* into its former position below it, the extremity of the beam will also descend from its highest to its lowest position; so that for every ascent or descent of the piston or the great beam, the planet wheel *i* will make one turn, while the sun wheel *j* and fly *m* will perform two complete revolutions.

On account of the weight of the fly, and the great celerity of its motion, there is much friction between its gudgeons and the sockets in which they move. In order to diminish the heat which is thus generated, it is customary to add to the engine a small pump, which conveys a gentle stream of water to the gudgeons of the fly.

When the steam-engine is employed to drive machinery, as in cotton-spinning, where the resistance is very variable, and where a determinate velocity cannot properly be dispensed with, Mr. Watt has applied what he calls a *Governor*, or a *Conical Pendulum*, which is represented at *W*, for procuring an uniform velocity. This regulator consists of two heavy balls m', n', suspended by two iron rods which cross each other in a mortise formed in the top of the vertical axis p' q'. These two rods, after crossing, are bent outwards in a contrary direction, and two short pieces, 3, 4, are jointed to them, the other ends of which are united to a collar sliding up and down on the upper part of the axis q', and is put in motion by the rope p' r', which passes over the pulleys p', r', and round the axis of the fly. Since the velocity of the fly and sun-wheel increases and diminishes with the quantity of steam that is admitted into the cylinder, let us suppose that too much is admitted; then the velocity of the fly will increase, but the velocity of the vertical axis p' q' will also increase, and the balls m' n' will recede from the axis by the augmentation of their centrifugal force. By

Conical pendulum.

this recess of the balls, the extremity q' of the lever q' X moving upon y' as a centre, is depressed; its other extremity X rises, and by acting on the double ended lever Y, which is connected with an arm fixed on the spindle t of the throttle-valve, closes it a little, and diminishes the supply of steam. When the velocity of the engine becomes too small, the opposite effect is produced. The impelling power being thus diminished or increased, the velocity of the fly and the axis p' q' decrease or increase in proportion, and the balls m' m' resume their former position. The throttle-valve is shewn in three different views in No. 1, 2, 3 of Fig. 4, Plate III, and consists of a circular plate of metal A A, with a spindle B, fixed across its diameter. The plate A is accurately fitted to an aperture in a thick metallic ring C C, through the edge of which the spindle is fitted so as to be steam-tight. The end of the spindle has a spanner fixed upon it, by which it can be turned in either direction, and more or less steam admitted in proportion to the degree to which it is opened.

Construction of the valves. Plate III. Fig. 2.

In Mr. Watt's improved engine, the steam and eduction nozle-valves are all puppet clacks. Two of these valves, and the method of opening and shutting them, is represented in Fig. 2 and 3 of Plate III.

As the piston approaches the top of the cylinder, the slider a, Fig. 2, fastened upon the plug-tree Z, raises the handle b, which is fixed upon the lower Y-shaft, or axis c, carrying the detent d, which takes hold of the double-ended catch e; but, in doing this, the upper end of the catch allows the detent f to escape, and a weight hung to the rod g turns the axis h. The arm i, and rod j, are moved out of their former rectilineal position at l, and by a lever k turn the spindle m in the upper nozle, which, by means of a toothed sector n, and rack o, raises the valve p, and admits steam into the cylinder *above* the piston through the horizontal pipe A. At the same time, another arm u, fixed upon the same shaft, by means of the rod w, acts upon a spindle, &c. in the lower nozle, opens the exhaustion-valve L, Fig. 2, and thereby forms a communication between the cylinder *below* the piston and the condenser. The piston now descending, another slider q moves the handle r into the position s; this raises the weight g, while i and k are brought back to the position l, and the valves p and L are

shut. The detent *f*, in acting upon the catch at the upper end of *e*, disengages *d* from the catch at the lower end of *e*, Fig. 3, where the parts are distinguished by the same letters; the lower *Y*-shaft turns upon its axis, and two arms attached to it (similar to those upon the upper *Y*-shaft, which are omitted to avoid confusion) by means of the rod *x* and *y*, open the lower steam-valve *K*, Fig. 1, and the upper exhaustion-valve *t*. The cylinder above the piston becomes exhausted, and the steam rushing in beneath it causes the piston to reascend. In the plan, Fig. 3, only one of the *Y*-shafts is shown, and the levers which open and shut the steam-valve of the upper nozle.

Mode of operation.

Having thus described the different parts of one of the most improved steam-engines, we shall now follow Mr. Watt in his description of its mode of operation. By means of the steam-pipe *F*, Fig. 1, proceeding from the boiler *n o*, steam is conveyed to the cross-pipe, or upper steam-nozle *G*, and by the perpendicular steam-pipe *I*, to the lower steam-nozle *K*. In the nozle *G* is a valve, which, when open, admits steam into the cylinder *above* the piston *B*, through the horizontal square pipe at its top; and in the *lower* steam-nozle *K* there is another valve, which, when open, admits steam into the cylinder *below* the piston. In the upper exhaustion-nozle *H* is a valve, which, when open, admits steam to pass from the cylinder *above* the piston into the exhaustion-pipe *J*, which conveys it to the condensing-vessel *M*, where it meets the jet of the injection water from the cock *N*, and the steam reduced to water; and, in the *lower* exhaustion-nozle *L*, there is also a valve, which, when open, admits steam to pass out of the cylinder *below* the piston, by the eduction-pipe, into the condenser *M*.

The engine being at rest, the cylinder quite cold, and the condenser-cistern full of water, when the water in the boiler begins to boil, steam will enter by a small pipe into the space between the cylinder and the steam or heating-case *E*, which will expel the air contained in that space, and between the two bottoms of the cylinder, at a cock fixed in the outer bottom, which, when all the air is expelled, and the cylinder thoroughly warmed, is to be shut, and the water which may be formed in these spaces during the working of the engine will issue by the inverted syphon *c*.

Things being in this situation, to produce a commencement of motion, the first operation is to open all the four valves, *G*, *H*,

K, *L* ; (the injection-cock being shut) the steam will drive the air out of the steam and exhaustion-pipes *I* and *J*, and out of the condenser *M*, through the blow-pipe and its valve *O* (behind *M*), and as soon as this is succeeded by a sharp crackling noise in the little cistern *O* (behind *M*), the valves are to be shut until it is thought that the steam which has entered is nearly condensed.

The same operation is to be repeated, giving a longer time to cool between the times of blowing, until it is found that, upon opening the injection-cock, some water will enter, and the barometer, to be afterwards described, shall shew some degree of exhaustion, after which, the repetition of blowing will soon empty the cylinder of air.

The piston being then at the top of its stroke, the valves *G* and *L* are to be opened, and the fly-wheel *m* turned by hand about one-eighth of a revolution, or more, in the direction in which it is intended to move ; the steam which is then in the cylinder will pass by *L* into the condenser, when, meeting the jet of water from the injection-cock, it will be converted into water, and the cylinder thus becoming exhausted, the steam, entering the cylinder by the valve *G*, will press upon the piston and cause it to descend, while, by its action upon the working-beam through the piston-rod, &c. it pulls down the cylinder end of the beam, and raises up the outer-end and the connecting rod *h*, which causes the planet-wheel *i* to tend to revolve round the sun-wheel *j* ; but the former of these wheels, being fixed upon the connecting rod so that it cannot turn upon its own axis, and its teeth being engaged in those of the sun-wheel, the latter, and the fly-wheel, upon whose axle or shaft it is fixed, are made to revolve in the desired direction, and give motion to the mill-work.

As the piston descends, the plug-tree Z also descends, and a clamp or slider *q*, fixed upon the side of the plug-tree, presses upon the handle 1 of the upper *Y* shaft or axis, and thereby shuts the valves *G* and *L*, and the same operation, by disengaging a detent, permits a weight suspended to the arm of the lower *Y*-shaft to turn the shaft upon its axis, and thereby to open the valves *K* and *H*. The moment previous to the opening these valves, the piston had reached the lowest part of its stroke, and the cylinder *above* the piston was filled with steam ; but as soon as *H* is opened, that steam rushes by the eduction-pipe *J*, into the condenser, and the cylinder *above* the piston becomes exhausted.

The steam from the boiler entering by *I* and *K*, acts upon the *lower* side of the piston, and forces it to return to the top of the cylinder. When the piston is very near the upper termination of its stroke, another slider *a* raises the handle 2, and, in so doing, disengages the catch which permits the upper *Y*-shaft to revolve upon its own axis, and open the valves *G* and *L*, and the downward stroke recommences as has been described.

When the piston descends, the buckets *R*, *T* of the air-pump *P* and hot-water pump *T* also descend. The water which is contained in these pumps passes through the valves of their buckets, and is drawn up and discharged by them through the lander or trough *t*, by the next descending stroke of the piston. Part of this water is raised up by the pump *V*, for the supply of the boiler, and the rest runs to waste.

History of the Parallel Motion.

Before we proceed to explain the beautiful contrivance shewn at *c d f g*, and called the *Parallel Motion*, it may be proper to notice the steps by which it was invented. In single engines, the vertical descent of the piston was formerly produced by a chain which lapped upon a sector or arched head at the end of the beam; and in double engines, where the end of the beam required to be pushed upwards, a toothed rack and sector were employed. The chains being in many respects objectionable, and the racks and sectors being subject to wear, Mr. Watt set himself to contrive some mechanism moving upon centres, which would keep the piston-rods vertical both in pushing and pulling. He conceived two levers *A B*, *C D* (Fig. 5), of equal length, placed in the same vertical plane, and moveable on *B* and *C* as centres; and he saw that if their ends *A*, *D* were connected with a rod *A D*, the middle point *E* would describe nearly a straight and perpendicular line when the ends *A*, *D* described segments of circles *F G* and *I H*, provided the circle *F G* did not much exceed 40°. By attaching, therefore, the piston-rod to the point *F*, it would be guided in a vertical direction by the above mechanism. If the lever *C D*, which Mr. Watt called the *regulating radius*, were made only half the length of the lever *A B* (which represents half the length or radius of the working beam), a point *E′*, so taken that *A E′* is $\frac{1}{3}$ of *A D*, would then move nearly in a vertical direction.[6]

[6] This first idea of Mr. Watt was afterwards brought forward in France as a new invention.

From these ingenious ideas the Parallel Motion was converted into the form shewn in Plate III, Fig. 1 and 6, where *c c* are the perpendicular links of the parallel motion, *d d* the parallel bars, *e e* the regulating radii, *f* the small perpendicular links, and *g* the second parallel motion for the air-pump. It is obvious that the joint *x* in Fig. 1, and also *y*, must rise and fall vertically, for the same reason that the point *E* does in Fig. 5; for *e* corresponds with *C D* in Fig. 5; *b f* with *B A*, and *f x w* with *A E D*. In order to understand how the point *z* and the piston-rod *c* are kept in a vertical direction by the bars *e* and *d*, we must consider that the point *w* moves in a circle round *B*, as a fixed centre, and the point *z* round the moveable centre *w*. The beam *a a* being in its highest position, as in the figure, the point *z*, in its descending motion round *w*, would be carried out of the perpendicular to the left hand if *w* were fixed; but the point *w*, in its descending motion round *β*, is carried to the right hand as much as *z* would be carried to the left; and *z* being compelled by the rod *d* to follow *w*, is thus kept in a vertical direction by its equal and opposite tendencies to the right and left.

Besides these contrivances, Mr. Watt enriched the steam-engine with the barometer, the steam-gage, and the indicator.

The barometer is used to indicate the degree of exhaustion in his engines. It is made of an iron tube, in the form of an inverted syphon, one leg being about half the length of the other. A pipe and cock, communicating with the condenser, are joined to the upper end of the long leg, and when a proper quantity of mercury was poured into the short leg of the syphon, it stood at the same level in both legs. A light float with a slender stem was then placed in the short leg, and indicated, upon a scale divided into half inches, the number of inches which the mercury rose in the long leg, or the exhaustion of the engine.

The steam-gage, for ascertaining the elasticity of the steam, is a short tube of glass, having its lower end immersed in a cistern of mercury, placed within an iron box, screwed to the boiler, or the steam-pipe, or any other part which communicates freely with the steam. The rise of the mercury in the tube (which is open to the air at the upper end), arising from the pressure of the steam on the surface of the mercury, indicates the elastic power of the steam over that of the atmosphere.

The object of the indicator is to ascertain the state of ex-

haustion of the cylinder at the different periods of the stroke of the piston. It consists of a truly bored cylinder, about 1 inch in diameter, and 6 inches long, with a solid piston accurately fitted to it, and sliding easily by the aid of some oil. A cock and small pipe are joined to the bottom of this cylinder. The pipe having a conical end, may be inserted in a hole drilled in the cylinder of the engine, near one of its ends, in order that the opening of the cock may make a communication between the indicator and the interior of the cylinder.

"The cylinder of the indicator," says Mr. Watt, "is fastened upon a wooden or metal frame, more than twice its own length; one end of a spiral steel spring, like that of a spring steelyard, is attached to the upper part of the frame, and the other end of the spring is attached to the upper end of the piston-rod of the indicator. The spring is made of such a strength, that when the cylinder of the indicator is perfectly exhausted, the pressure of the atmosphere may force its piston down within an inch of its bottom. An index being fixed to the top of its piston-rod, the point where it stands, when quite exhausted, is marked from an observation of a barometer communicating with the same exhausted vessel, and the scale divided accordingly."

Value of Mr. Watt's improvements.

From this brief description of the steam-engine, the reader will be enabled to perceive the nature, and appreciate the value of Mr. Watt's improvements. It had hitherto been the practice to condense the steam in the cylinder itself, by the injection of cold water; but the water which was injected acquired a considerable degree of heat from the cylinder, and being placed in air highly rarified, part of it was converted into steam, which resisted the piston, and diminished the power of the engine. When the steam was next admitted, part of it was converted into water by coming in contact with the cylinder, which was of a lower temperature than the steam, in consequence of the destruction of its heat by the injection-water. By condensing the steam, therefore, in the cylinder itself, the resistance to the piston was increased by a partial reproduction of this elastic vapour, and the impelling power was diminished by a partial destruction of the steam which was next admitted. Both these inconveniencies Mr. Watt has in a great measure avoided, by using a condenser separate from the cylinder, and encircled with cold water;[7] and

[7] Even in Mr. Watt's best engines, a very small quantity of steam remains in

by surrounding the cylinder with a wooden case, and interposing light wood ashes, in order to prevent its heat from being abstracted by the ambient air; or by means of an iron steam case filled with steam, and surrounding the cylinder.

The greatest of Mr. Watt's improvements consists in his employing the steam both to elevate and depress the piston. In the engines of Newcomen and Beighton, the steam was not the impelling power; it was used merely for producing a vacuum below the piston, which was forced down by the pressure of the atmosphere, and elevated by the counterweight at the other extremity of the great beam. The cylinder, therefore, was exposed to the external air at every descent of the piston, and a considerable portion of its heat being thus abstracted, a corresponding quantity of steam was of consequence destroyed. In Mr. Watt's engines, however, the external air is excluded by a metal plate at the top of the cylinder, which has a hole in it for admitting the piston-rod; and the piston itself is raised and depressed merely by the force of steam.

When these improvements are adopted, and the engine constructed in the most perfect manner, there is not above $\frac{1}{4}$ part of the steam consumed in heating the apparatus; and, therefore, it is impossible that the engine can be rendered $\frac{1}{4}$ more powerful than it is at present. It would be very desirable, however, that the force of the piston could be properly communicated to the machinery without the intervention of the great beam. This, indeed, has been attempted by Mr. Watt, who has employed the piston-rod itself to drive the machinery; and Mr. Cartwright has, in his engine, converted the perpendicular motion of the piston into a rotatory motion, by means of two cranks fixed to the axis of two equal wheels which work in each other. Notwithstanding the simplicity of these methods, none of them have come into general use, and Mr. Watt still prefers the intervention of the great beam, which is generally made of hard oak, with its heart taken out, in order to prevent it from warping. A considerable quantity of power, however, is wasted by dragging, at every stroke of the piston, such a mass of matter from a state of rest to a state of motion, and then from a

the cylinder, having the temperature of the hot well *h*, or of the water, into which the ejected steam is converted. Its pressure is indicated by the barometer already described.

state of motion to a state of rest. To prevent this loss of power, a light frame of carpentry[8] has been employed by several engineers, instead of the solid beam; but after being used for some time, the wood was generally cut by the iron bolts, and the frame itself was often instantaneously destroyed. In some of the engines lately constructed Mr. Watt, he has formed the great beam of cast iron, and while he has thus added to its durability, he has at the same time diminished its weight, and increased the power of his engine.

Encouraged by Mr. Watt's success, several improvements upon the steam-engine have been made by other engineers of this country. But it does not appear that they have either materially increased the power of the engine, or diminished its expense.

Account of Mr. Woolf's Steam-Engines.

Woolf's double cylinder expansion engines.

In the year 1804, Mr. Arthur Woolf announced to the public a discovery respecting the expansibility of steam, which promises to be of very essential utility. Mr. Watt had formerly ascertained, that steam, which acts with the expansive force of 4 pounds per square inch, against a safety valve exposed to the weight of the atmosphere, after expanding itself to four times the volume it thus occupies, is still equal to the pressure of the atmosphere. But Mr. Woolf has gone much farther, and has proved, that quantities of steam, having the force of 5, 6, 7, 8, 9, 10, &c. pounds on every square inch, may be allowed to expand 5, 6, 7, 8, 9, 10, &c. times its volume, and will still be equal to the atmosphere's weight, provided that the cylinder in which the expansion takes place has the same temperature as the steam before it began to expand. It is evident, however, that an increase of temperature is necessary both to produce and to maintain this augmentation of the steam's expansive force above the pressure of the atmosphere. At 212° of Fahrenheit, the force of steam is equal only to the pressure of the atmosphere, and, in order to give it an additional elastic force of 5 pounds per square inch, the temperature must be increased to about 227½°, as is evident from the following Table.

[8] The great beam in Mr. Hornblower's engine, is constructed in this manner and is formed upon truly scientific principles. Dr. Robison observes, that it is stronger than a beam of the common form, which contains twenty times its quantity of timber.

Table of the Pressures, Temperatures, and Expansibility of Steam, equal to the Force of the Atmosphere.

Elastic Force of Steam predominating over the Pressure of the Atmosphere, and acting upon a Safety Valve.	Degrees of Temperature requisite for bringing the Steam to the different Expansive Forces in the preceding Column.	No. of times its Volume that Steam of the preceding Force and Temperature will expand, and still continue equal to the pressure of the Atmosphere.
Pounds per Square Inch.	Degrees of Heat.	Expansibility.
5	$227\frac{1}{2}$	5
6	$230\frac{1}{4}$	6
7	$232\frac{3}{4}$	7
8	$235\frac{1}{4}$	8
9	$237\frac{1}{2}$	9
10	$239\frac{1}{2}$	10
15	$250\frac{1}{2}$	15
20	$259\frac{1}{2}$	20
25	267	25
30	273	30
35	278	35
40	282	40

In this manner, by small additions of temperature, an expansive power may be given to steam, which will enable it to expand 50, 100, 200, 300, &c. times its volume, and still have the same force as the atmosphere.

Upon this principle Mr. Woolf has taken out a patent for various improvements on the steam-engine, a short account of which we shall subjoin in the words of the specification, adding the letters of reference to a sketch of the valves and cylinders of the engines actually erected by Mr. Woolf at the Wheal Vor and Wheal Abraham Mines in Cornwall, for pumping water. See Plate IV, Fig. 1.

" If the engine be constructed originally with the intention of adopting the preceding improvement, it ought to have two steam vessels *A*, *B*, of different dimensions, according to the expansive force to be communicated to the beam ; for the smaller steam cylinder *B* must be a measure for the larger. For example, if steam of 40 pounds the square inch is fixed on, then the smaller steam-vessel should be at least $\frac{1}{40}$ part the contents of the larger one. Each steam vessel

should be furnished with a piston *P*, *p*, and the smaller cylinder should have a communication both at its top and bottom, with the boiler which supplies the steam, which communications, by means of cocks or valves, are to be alternately opened and shut during the working of the engine. The top of the small cylinder should have a communication at *G* with the bottom of the larger cylinder, and the bottom of the smaller one with the top of the larger at *H*, with proper means to open and shut these alternately by cocks, valves, or any other contrivance. And both the top and bottom of the larger cylinder should, while the engine is at work, communicate alternately by a valve *I*, with a condensing vessel, into which a jet of water is admitted to hasten the condensation. Things being thus arranged, when the engine is at work, steam of high temperature is admitted from the boiler to act by it's elastic force on one side of the smaller piston, while the steam which had last moved it has a communication with the larger cylinder, where it follows the larger piston now moving towards that end of its cylinder which is open to the condensing vessel. Let both pistons end their stroke at one time, and let us now suppose them both at the top of their respective cylinders ready to descend; then the steam of 40 pounds the square inch, entering above the smaller piston *A*, will carry it downwards, while the steam below it, instead of being allowed to escape into the atmosphere, or applied to any other purposes, will pass into the larger cylinder by the valve *G*, above its piston, which will take its downward stroke at the same time that the piston of the smaller cylinder is doing the same thing; and while this goes on, the steam which last filled the larger cylinder, in the upward stroke of the engine, will be passing into the condenser, there to be condensed in the downward stroke. When the pistons *P*, *p*, in the smaller and larger cylinder have thus been made to descend to the bottom of their cylinders, then the steam from the boiler is to be shut off from the top, and admitted to the bottom of the smaller cylinder, and the communication *G*, between the bottom of the smaller and the top of the larger cylinder is also to be cut off, and the communication *H* to be opened between the top of the smaller and the bottom of the larger cylinder; the steam which, in the downward stroke of the engine, filled the larger cylinder, being now open to the condenser, and the communication between

the bottom of the larger cylinder and the condenser cut off; and so on alternately, admitting the steam to the different sides of the smaller piston, while the steam last admitted into the smaller cylinder passes alternately to the different sides of the larger cylinder, the top and bottom of which are made to communicate alternately with the condenser.

" In an engine where these improvements are adopted, that waste of steam which arises in other engines, from steam passing the piston, is totally prevented, for the steam which passes the piston in the smaller cylinder is received into the larger."

In the sketches given in Fig. 1 of Plate IV, the steam is admitted from the boiler into the steam-case of *A*, by a communication at *C*, and this steam-case communicates with that of the small cylinder. The water produced by condensation in the steam-case, before the engine is properly heated, is carried to the boiler through *D*. The pipe which supplies the engine from the steam-case is shewn at *E*, and *F* is the valve-box of the small cylinder.

Mr. Woolf has also shewn how the preceding arrangement may be altered, and has pointed out various other modifications of his invention, and the method of applying his improvements to steam-engines which are already constructed.

The boiler used by Mr. Woolf for producing steam of high elasticity consists of 8 or more cylindrical tubes of cast iron, filled with water, and exposed in a horizontal position to the flame. Two safety-valves are always added to the boiler.

Mr. Woolf took out new patents for his improvements in 1805 and 1810, and has added to his engine various contrivances of great ingenuity and utility. It would be inconsistent, however, both with the plan and limits of a popular work, to enter into more minute details respecting these inventions.

High Pressure Steam-Engines.

High pressure engines are those in which the steam is raised to a very high degree of elasticity, so as to exert a pressure upon every square inch of 3, 4, 5, 6, 7, 8, 9, or even 10 times greater than that which is exerted in the low pressure engines, such as those made by Mr. Watt. In Mr. Watt's engines, the pressure on every square inch of the piston is from 9 to 12 lbs. on every square inch. In one of Mr. Woolf's at Wheal Vor, it is 14 lbs. and in another 24 lbs. on every square inch of the

great piston ; but in one of the *high pressure* engines erected in Wales in 1804, it is so high as 65 lbs. on every square inch.

In his patent for 1769, Mr. Watt proposed a high pressure engine, but he does not appear to have carried it into execution. They were first actually introduced by Messrs. Trevethick and Vivian in 1802, chiefly in reference to their application to drive carriages on rail-roads. In order to convey to the reader some idea of these engines, we have given a representation of one in Fig. 2 of Plate IV, where *A A* is a large horizontal and cylindrical boiler of cast iron, *D* a cylindrical tube of wrought iron, which is double, returning back at *B*, and communicating with the flue *T*. The fire-grate occupies the upper half of the double tube, and the ash-pit the lower half; *Z* is a door for removing the soot. The safety-valve *n* is kept down by a lever *p v*, with a weight *p*. The cylinder shewn at *A′* is nearly inclosed within the boiler. The piston-rod *H*, carrying the piston *G*, is fastened to the middle of a cross-bar *I*, placed at right angles to the length of the boiler. At the ends of the cross bar are two connecting-rods *L*, the lower ends of which drive two cranks on the axis of the fly-wheel *M*. The *four passage* cock for admitting the steam to the cylinder is shewn at *i f g k*. The passage *g* rises directly from the boiler, and brings steam to the cock so as to be carried either to the top or bottom of the cylinder, according to the position of the cock. The second passage *f* carries the steam above the piston. The third passage *k* conveys steam below the piston, and the fourth passage *i* allows the steam to escape into the flue *T*, along the waste-pipe *I F*.

When the steam enters by *f* above the piston, it is forced down, and the steam below it escapes by *i* to the waste-pipe. When the piston reaches the bottom of the cylinder, the cock is turned one-fourth round by means of pins on a rod joined to the cross bar. The steam thus enters below the piston, and raises it, while that above the piston is drawn off by the waste-pipe *I F*. The cold water is conveyed to the boiler along a pipe *r*, surrounding the heated waste-pipe *I F*, and it receives a considerable quantity of heat, which would otherwise be lost.

The high pressure engines now described, though extremely simple in their construetion, and very convenient in particular cases, are attended with such danger that they should never be

used but for urgent reasons, and the strength of the boiler should always be subjected to an experimental examination before it is actually used.[9]

On the Power of Steam-Engines, and the Method of Computing it.

Power of steam-engines.

From the account which has been given of the steam-engine, and the mode of its operation, it must be evident that its power depends upon the breadth and heighth of the cylinder, or, in other words, on the area of the piston and the length of its stroke. If we suppose that no force is lost in overcoming the inertia of the great beam, and that the lever by which the power acts is equal to the lever of resistance; then, if steam of a certain elastic force is admitted above the piston *B* (Plate III, Fig. 1), so as to press it downwards with a force of a little more than 100 pounds, it will be able to raise a weight of 100 pounds hanging at the end of the great beam. When the piston has descended to the bottom of the cylinder, through the space of 4 feet, the weight will have risen through the same space; and 100 pounds raised through the height of 4 feet, during one descent of the piston, will express the mechanical power of the engine. But if the area of the piston *B*, and the length of the cylinder are doubled, while the expansive force of the steam, and the time of the piston's descent remain the same, the mechanical energy of the engine will be quadruple, and will be represented by 200 pounds raised through the space of 8 feet during the time of the piston's descent. The power of steam-engines, therefore, is, *cæteris paribus*, in the compound ratio of the area of the piston, and the length of the stroke. These observations being premised, it will be easy to compute the power of steam-engines of any size.

Method of computing it.

Thus, let it be required to determine the power of steam-engines, whose cylinder is 24 inches diameter, and which make 22 double strokes in a minute, each stroke being 5 feet long, and the force of the steam being equal to a pressure of 12 pounds avoirdupois upon every square inch.[1]

[9] A very interesting account of the explosion of one of these steam-boilers will be found in the *Edinburgh Philosophical Journal*, vol. v, p. 147.

[1] The working pressure is generally reckoned at 10 pounds on every circular inch,

The diameter of the piston being multiplied by its circumference, and divided by 4, will give its area in square inches; thus, $\frac{24 \times 3.1416 \times 24}{4} = 452.4$, the number of square inches exposed to the pressure of the steam. Now if we multiply this area by 12 pounds, the pressure upon every square inch, we shall have $452.4 \times 12 = 5428.8$ pounds, the whole pressure upon the piston, or the weight which the engine is capable of raising. But since the engine performs 22 *double* strokes, 5 feet long, in a minute, the piston must move through $22 \times 5 \times 2 = 220$ feet in the same time; and therefore the power of the engine will be represented by 5428.8 pounds avoirdupois, raised through 220 feet in a minute, or by 10.4 hogsheads of water, ale measure, raised through the same height in the same time. Now this is equivalent to $5428.8 \times 220 = 1{,}194{,}336$ pounds, or $10.4 \times 220 = 2288$ hogsheads raised through the height of 1 foot in a minute. This is the most unequivocal expression of the mechanical power of any machine whatever, that can possibly be obtained. But as steam-engines were substituted in the room of mill-horses, it has been customary to calculate their mechanical energy in *horse powers*, or to find the number of horses which could perform the same work. This indeed is a very vague expression of power, on account of the different degrees of strength which different horses possess. But still, when we are told that a steam engine is equal to 16 horses, we have a more distinct conception of its power, than when we are informed that it is capable of raising a number of pounds through a certain space in a certain time.

Horse powers. Messrs. Watt and Boulton suppose a horse capable of raising 33,000 pounds avoirdupois 1 foot high in a minute, while Dr. Desaguliers makes it 27,500 pounds, and Mr. Smeaton only 22,916. If we divide, therefore, the number of pounds which any engine can raise 1 foot high in a minute, by these three numbers, each quotient will represent the number of horses to which the engine is equivalent. Thus, in the present example $\frac{1194336}{33000} = 36\frac{1}{5}$ horses according to Watt and Boulton; $\frac{1194336}{27500} = 43\frac{1}{3}$ horses, according to Desaguliers; and $\frac{1194336}{22916} = 52\frac{1}{7}$ horses, according to Smeaton. In this cal-

and Smeaton makes it only 7 pounds on every circular inch. The effective pressure which we have adopted is between these extremes, being equivalent to 9.42 pounds on every circular inch.

culation, it is supposed that the engine works only eight hours a-day; so that if it wrought during the whole 24 hours, it would be equivalent to thrice the number of horses found by the preceding rule. We cannot help observing, and it is with sincere pleasure that we pay that tribute of respect to the honour and integrity of Messrs. Watt and Boulton which has everywhere been paid to their talents and genius,—that in estimating the power of a horse, they have assigned a value the most unfavourable to their own interests. While Mr. Smeaton and Dr. Desaguliers would have made the engine in the preceding example equivalent to 52 or 43 horses, the patentees themselves state that it will perform the work only of 36.

Performance of Watt's steam-engines.

Before concluding this article, we shall state the performance of some of these engines, as determined by experiment, and in Mr. Watt's own words:— "The burning of one bushel of good Newcastle or Swansea coals in Mr. Watt's reciprocating engines, working more or less expansively, was found, by the accounts kept at the Cornish mines, to raise from 24 to 32 millions of pounds of water one foot high; the greater or less effect depending upon the state of the engine, its size, and rate of working, and upon the quality of the coal.

"In engines upon the rotative double construction, one having a cylinder of $31\frac{1}{2}$ inches diameter, and making $17\frac{1}{2}$ strokes of 7 feet long per minute, called 40 horses' power, meaning the constant exertion of 40 horses (for which purpose, supposing the working to go on night and day, 3 relays, or at least 120 horses, must be kept), consumed about 4 bushels of good Newcastle coal per hour, or 400 weight of good Wednesbury coal. A rotative double engine, with a cylinder of $23\frac{3}{4}$ inches in diameter, making $21\frac{1}{2}$ strokes of 5 feet long per minute, was called 20 horses' power; and an engine, with a cylinder of $17\frac{1}{2}$ inches diameter, making 25 strokes of 4 feet long per minute, was called 10 horses' power; and the consumption of coals by these was nearly proportional to that of the 40 horses' power.

"A bushel of Newcastle coals, which thus appears to be the consumption of a 10-horse engine for one hour, grinds and dresses about 10 bushels, Winchester measure, of wheat."

In order to determine the power of different steam-engines used in Cornwall, several of the proprietors of the tin and copper mines appointed Messrs. Thomas and J. Lean to keep

regular accounts of the performance of their engines. These reports have been published in the *Philosophical Magazine.*

The following are the results of the reports on three of Messrs. Boulton and Watt's engines, two of Mr. Woolf's, and one high pressure engine.

	Messrs. Boulton and Watt's.			Mr. Woolf's.		High Pressure Engine.
	1.	2.	3.	1.	2.	
Diameter of cylinder,	63	63	70	53	45	8
Length of stroke	$7\frac{3}{4}$ feet.	$8\frac{1}{4}$ feet.	$8\frac{1}{2}$ feet.	9 feet.	7 feet.	$4\frac{1}{2}$ feet.
Pressure on sq. inch, ..	9.4 lbs.	9 lbs.	9.9 lbs.	14.1 lbs.	24 lbs.	65 lbs.
Millions of lbs. of water raised 1 inch, on an average of 4 months, ...	$29\frac{2}{3}$	$27\frac{2}{3}$	$26\frac{3}{4}$	$46\frac{1}{3}$	52	$17\frac{1}{2}$

We have given the preceding results more as historical statements than as scientific facts, as we are entirely unacquainted with the condition of the engines whose performance was subjected to examination.

On the Application of Steam to the propelling of Boats, &c.

The great advantages which navigation has derived from the application of the steam-engine to boats and ships, have rendered the introduction of steam-boats an era in the history of the useful arts.

Many candidates have accordingly arisen to claim the honour of this invention, and no small portion of national feeling has been roused in favour of the different competitors.

We are not acquainted with any disputed question in the arts or sciences where it is so difficult to come to a decision. Admitting even all the allegations of the contending parties, it is not easy to discover any general principle upon which a decision can be founded, or by means of which we may allot to each claimant his due share of approbation.

It is quite obvious that neither the inventors or improvers of the steam-engine, nor those intelligent individuals who hazarded their capital in the construction of steam-vessels, can have any share of the honour which we are attempting to award. Their merit was of a distinct kind. The noblest laurels which fame

can confer have rewarded the former, and the results of successful speculation have demonstrated the sagacity and remunerated the enterprise of the latter.

In some works which were published several centuries ago, we have seen drawings of vessels with paddle-wheels, which were driven by the application of the strength of men. The merit, therefore, of propelling vessels by means of mechanical power applied to two paddle-wheels cannot be claimed by any modern inventor. The substitution of the power of horses, or of steam, or of heated air, in place of the strength of men, appears to us no invention at all, for if it were, we should have numerous rivals contending for the honour of applying the steam-engine to the thrashing-machine. When Mr. Jonathan Hulls, therefore, in the year 1736, took out a patent for the application of one of Newcomen's steam-engines to a vessel for towing ships in and out of harbour, he merely proposed to substitute the power of steam in place of the power of men. His proposal was characterised neither by sagacity nor inventive genius, and the intermediate mechanism by which the reciprocating motion of the piston was converted into the rotatory motion of the paddle-wheel or *fans* as he calls them, was clumsy and imperfect. The paddles were fixed to radii without any connecting rim; they were placed a considerable way behind the two boats, upon two projecting arms, and were driven by means of ropes. The steam tow-boat of Mr. Hulls, in short, we consider as impracticable. It appears even never to have been put to the test of experiment, and certainly never excited the notice of that class of persons who were either likely to improve upon the suggestion, or derive commercial advantage from it.[2]

The late Patrick Miller, Esq. of Dalswinton, a gentleman of acknowledged ingenuity and talents, had long directed his

[2] "Up inland rivers," says Mr. Hulls, "where the bottom can possibly be reached, the fans may be taken out, and cranks placed at the hindmost axis to strike a shaft to the bottom of the river, which will drive the vessel forward with the greater force."—P. 47.

"It is my opinion," adds Mr. Hulls, "it will not be found practicable to place the machine here recommended in the vessel itself that is to be taken in or out of the port," &c.—P. 41.

"If it should be said that this is not a new invention, because I make use of the same power to drive my machine that others have made use of to drive theirs for other purposes, I *answer*, the application of this power is no more than the application of any common or known instrument used in mechanism for new invented pur-

attention to the improvement of naval architecture. His speculations were not of that visionary cast which terminate in the communication of a new thought either in conversation or in writing. He embodied his ideas in the substantial form of working models. He subjected his views to the rigid test of direct experiment, and the failure of any of his schemes became only a new incentive to higher and more vigorous efforts. Mr. Miller was familiar with all the details of seamanship and ship-building. He was thoroughly acquainted with the causes which rendered navigation both insecure and uncertain, and his mind was bent upon removing, in so far as he could, the imperfections of an art which had raised his country above all other nations. The idea, therefore, of actually impelling vessels by steam, when they must either be becalmed or be driven about by opposing winds, an idea very different from the limited proposal of towing vessels in and out of harbour, was one which sprung directly from his own active pursuits, and which he never for a moment abandoned till his speculations were realised in the actual construction of a steam-boat. Mr. Miller was engaged in these occupations before the year 1787, when he wrote a treatise on the subject, entitled *The Elevation, Section, Plans, and View of a Triple Vessel, and of Wheels*, copies of which he presented to the different sovereigns of Europe.[3] The boat which he fitted up with a steam-engine was repeatedly tried on the Forth and Clyde Canal, and though it was not used for any specific commercial purpose, yet the experiment was successful, and established the practicability of his invention. Mr. Miller's in-

poses."—P. 45. See "*Description and Draught of a new invented Machine for carrying Vessels or Ships out of or into any Harbour, Port, or River, against Wind and Tide, or in a Calm.* By Jonathan Hulls." Lond. 1737.

[3] "The first, or principal property," Mr. Miller remarks, "of vessels constructed upon the plan here communicated, is derived from the wheels, the mechanism of which is simple and obvious. From the experiments I have made in different vessels with the wheels wrought by cranks, as shewn in the plan, it appears to me that ships, however great their burden, if there be no wind, and the water is smooth, may be made to pass through it at the rate of from three to four miles an hour. When the movement of the wheels comes to be aided by mechanical powers, so as to accelerate their revolutions, the before-mentioned rate of a ship's going through the water will be in proportion to the power used. I have also reason to believe that the power of the steam-engine may be applied to work the wheels, so as to give them a quicker motion, and consequently to increase that of the ship. In the course of this summer I intend to make the experiment, and the result, if favourable, shall be communicated to the public."—P. 9–10. Mr. Miller's triple vessel with wheels was put upon the stocks at Leith on the 17th January 1786, and launched on the 14th October 1787.

vention consisted of two boats placed together, with paddle-wheels between them, which were driven by a steam-engine; and it is singular enough that this very construction has been recently adopted as a ferry-boat on the Firth of Tay. Mr. Miller of Dalswinton was therefore the first individual who actually constructed a steam-boat of his own invention, and established its utility by actual experiment. That he was the inventor of the steam-boat in the strict sense of the word, we will not venture to affirm; but we have no hesitation in stating it as our decided opinion, that he is more entitled to this distinction than any other individual who has yet been named.[4]

In the year 1795, the late Lord Stanhope tried the model of a steam-boat at his seat at Cheveney.

The experiments of Mr. Miller having been either witnessed by Mr. Fulton, or communicated to him by those who had witnessed them, this able engineer immediately perceived the advantage which America would derive from the introduction of the steam-boat upon its extensive rivers and lakes. Mr. Symington, a Scotch engineer, who assisted Mr. Miller in his trials on the Clyde, is said to have erected the first steam-engine in America; but Mr. Fulton had the great merit of introducing them on a great scale in the United States, before they had actually been used for navigation in Great Britain.

In 1813, the first steam-boat used on the Clyde was built by Mr. Bell of Helensburgh, and no fewer than *thirty-six*[5] are now employed in the navigation of the Firth of Clyde and its adjoining inlets. In the passage from Holyhead to Dublin, from Glasgow to Belfast, from Calais to Dover, from Brighton to Dieppe, from Inverness to Fortwilliam, along the line of the Caledonian Canal, and from Leith to Aberdeen, Inverness, and London, they have also been successfully introduced. They are used on the Russian rivers, in the Baltic, and in the Gulf of Venice; and we have, not long ago, witnessed the rare sight of a steam-vessel crossing the Atlantic to Liverpool, and afterwards navigating the Baltic to Stockholm and St. Petersburg. The name of this fine American vessel was the *Savannah*, Captain Rogers, which arrived at Liverpool on the 20th June 1819, after a passage of 21 days, from land to land.

[4] We are not acquainted either with the dates or the particulars of the Marquis de Geoffroi's experiments with a steam-boat on the Seine.

[5] See *Edin. Phil. Journal*, vol. vii, p. 80.

The vessel was of 350 tons burthen, and was fitted up in a superior style, with 32 state-rooms. The engine was in use during eighteen days, and by taking out some of the spokes of the cast iron wheel, and turning the blank segment downwards, the vessel was able to sail by the direct action of the wind, without any immersed floatboards to retard her motion.[5] The steam-vessels called the *City of Edinburgh* and the *James Watt*, which ply between Leith and London, are the only ones in Great Britain which rival the Savannah in magnitude and splendour.

In order to convey to the reader a general idea of the construction of a steam-boat, we have given a representation of one in Figs. 3 and 4 of Plate IV. Fig. 3 is a representation of the *City of Edinburgh* steam-boat, where *W W* is one of the paddle wheels, driven by the steam-engine within. Fig. 4 represents a cross section of the Royal George steam-ship, with an elevation of the engine, as at present executing by Mr. Gutzmer, civil engineer, Edinburgh. At the two ends of the two horizontal axles *A P*, passing through the steam-boat, are fixed the paddle wheels, *one* of which is shewn at *W W*. The two cylinders of the engine are placed behind the steam-chests *F*, *F*, which contain the valves; and by the alternate ascent and descent of their piston rods *B E*, attached to the cranks *C C*, a rotatory motion is communicated to the horizontal axles *A P*, *A P*, and consequently to the paddle wheels. The cranks which work the steam-valves are shewn at *K K*. The air-pump *E* of the engine is wrought by means of the crank *D*, which receives its motion from the inner branch *H* of each of the cranks. The boiler *G G* extends across the whole breadth of the ship. The pillars *h*, *h*, *h*, *h* support the top frame *i i* of the engine, and the rest of the machinery. The sides of the ship are shewn at *l*, *l*, *l*, *l*, the deck at *m*, *m*, *m*, *m*, and the funnel at *k*.

The advantages which are to be derived from any improvement in the steam-boat have occasioned many attempts to perfect its machinery. The ingenuity of inventors has been applied principally to some substitute for the paddle-wheels. It has been proposed to construct compound wheels, in which the

[5] A fuller account of this vessel will be found in the *Edinburgh Philosophical Journal* 1820, vol. ii, p. 197.

float-boards shall always act with their full force in the water; but these, though advantageous in theory, have always proved defective in practice. Some have proposed to apply the steam of the engine to a long axis, with spiral floatboards, and others have employed it as the impelling agent of a current of water, or air issuing from the middle of the boat, and acting against the water beneath.[6] Mr Gladstones has recently proposed two wheels on each side, with a chain of floatboards between them, as in Plate II, Fig. 11, floatboards being substituted in place of buckets, and the chain of them being inclined a little to the horizon. Among the various forms of the water wheel, by which it has been proposed to impel steam-boats, that of an undershot wheel, constructed like a wind-mill, and moving in a a plane perpendicular to the direction of the current, does not seem to have been suggested. This water wheel, of which we have given some account in p. 45, has great power, and might be placed to most advantage in the middle of the vessel. Two or even three of these wheels might be fixed longitudinally on the same axis, lying in the direction of the length of the boat.

Proposed improvements on steam-boats.

Various attempts have been recently made to render steam-boats fit for a tempestuous voyage. Messrs. Redhead and Parry have proposed to extend two horizontal channels through the whole length of the vessel, with apertures by which the water can enter and escape. The water reaches nearly to the top of these channels, and two or more pair of paddle wheels are mounted, with their lower parts immersed about one foot under the water in the channel. In stormy weather the apertures of the channels may be closed by sliding shutters, and the water pumped out of the channels. The wheels are therefore entirely closed in, and the boat may be navigated by sails, like any ordinary vessel. Mr. Wight of Edinburgh has proposed an analogous contrivance with only one wheel.

On the Application of Steam to drive Carriages.

The first person who suggested the idea of driving carriages by means of steam was our celebrated countryman Dr. Ro-

[6] An account of some of these contrivances, which are very ingenious, will be found in the *Edinburgh Philosophical Journal* 1821, vol. v, p. 116.

bison; but it does not seem to have been put in practice till 1802, when Messrs. Trevethick and Vivian took out their patent for high pressure engines, for the express purpose of driving carriages. They executed a steam-carriage in 1804, and tried it upon the rail-roads at Merthyr Tidvil. The trials, however, which were made of steam-carriages, were principally of an experimental nature till 1811, when Mr. Blinkinsop, proprietor of the Middleton coal works, which supply the town of Leeds, introduced them for the purpose of conveying coal along his rail-roads. The boiler in Mr. Blinkinsop's steam-carriage was supported by four wheels without teeth; but it received its motion by a crank connected with the piston, which moved other two wheels in the centre of the carriage, having strong teeth on their circumference. These teeth acted upon teeth in the rail-road, and in this manner the carriage with its train of 30 coal-waggons was moved along.

In the year 1816, Messrs. Losh and Stephenson of Newcastle took out a patent for improvements in the steam-carriage, of which we have given a representation in Plate IV, Figs. 5, 6, 7, and 8, Fig. 5 being a section of the engine and carriage, and Fig. 6 being a view of the same, dragging after it the carriage containing coals *E*, a cistern of water *F*, and one of the coal-waggons *G*, &c. The steam-engine, which has already been described, has two cylinders, whose pistons *A*, *B* drive the crank-rods *A C*, *B D*, which give a rotatory motion to the two wheels *C*, *D*, to which they are fixed, the two opposite wheels, which are not seen in the figure, being driven by two similar rods. The rail-rods *R R*, which are new and ingenious, are acted against by the circumferences of the wheels merely by friction. The middle pair of wheels receive their motion from the other two pair by means of a chain passing over two rag-wheels *m*, *n*, placed in the centre of each axle, as seen at *c* in Fig. 5. The chain communicates its motion to a third rag-wheel *c*, and thus drives the middle pair of wheels. In Mr. Blinkinsop's carriage, the engine rested directly upon the axles; but Messrs. Losh and Stephenson connect the boiler with the axles by the intervention of six floating pistons *b*, *b*, moveable within cylinders *a*, *a*, *a*, *a*, into which the steam or the water of the boiler is allowed to enter. These cylinders are best seen in Fig. 5, where *b*, *b* are the floating pistons connected with wrought iron rods below, the ends of which rest upon the bearing-brasses

of the axles of the wheels C, D. These pistons press equally on all the axles, and cause each wheel to bear with equal force upon the rail-road $R R$, and to act upon them with an equal degree of friction, although the rails should not be all in the same plane, for the bearing-brasses have the liberty of moving in a perpendicular direction in a groove or slide; and, carrying the axles and wheels along with them, force the wheels to accommodate themselves to the irregularities of the railway. By means of these pistons a steadiness of motion is obtained, and shocks are prevented in a way which could not be effected by suspending the engine on the finest steel springs. Figs. 7 and 8 represent the construction of the rail-road and the wheels.

CHAPTER II.

ON THE FORMATION OF THE TEETH OF WHEELS, THE LEAVES OF PINIONS, AND THE WIPERS OF STAMPERS, &c.

Though nothing is more essential to the perfection of machinery than the proper formation of the teeth of wheels, and those parts of engines, by which their force and velocity are conveyed to other parts; yet no branch of mechanical science has been more overlooked by the speculative and practical mechanics of this country. In vain do we search our systems of experimental philosophy for information on this point. Their authors seem either to reckon it beneath their notice, or to be unacquainted with the labours of De la Hire, Camus, Kaestner, Euler, and other foreign academicians, who have written very ingenious dissertations on the teeth of wheels. It is in the writings, indeed, of these philosophers, that almost all our knowledge upon this subject is contained, if we except a few general remarks, by Dr. Robison and Dr. Young, and a recent practical work by the late Mr. Robertson Buchanan.[1]

[1] Two ingenious memoirs have also been written upon this subject, by A. G. Kaestner, entitled, *De Dentibus Rotarum*, and published in the Comment. Reg. Soc. Gotting. vol. iv, and v, 1781, &c. The celebrated Euler has likewise treated this subject with great ability, in his memoir *De aptissima Figura Rotarum Dentibus tribuenda, Nov. Comment. Petropol.* 1754, 1755, tom. v, p. 299.

The curves which it is necessary to give to the teeth of wheels, in order that they may drive one another with uniform force and velocity, are called *Cycloids* and *Epicycloids.* A *Cycloid* is a curve which is described by any point in the circumference of a wheel or circle when rolling along a straight line; and an *Epicycloid* is a curve described by any point in the circumference of a circle rolling either upon the *interior* or *exterior* circumference of another circle. The circle which rolls is called the *Generating Circle.* When the radius of the generating circle is infinitely great, a part of its circumference may be considered as a straight line, and in this case the epicycloid which would be formed by such a circle rolling upon another, is the same curve that is produced by any point of a string wrapped round the circumference of a circle and uncoiled, or by any point in the straight edge of a ruler which is made to roll upon the circumference. Such a curve is called an *Involute*, and the circle from which the string is uncoiled the *Evolute.* These curves are easily formed by methods which will afterwards be explained, and the practical mechanic can have no difficulty in understanding their general properties. There is one epicycloid, however, which has an interesting character. If a *circle* rolls on the inside of another of twice its diameter, the epicycloid described is a straight line. This curious property is not only of great advantage, as will be presently seen; but has, in the hands of an ingenious mechanic, been embodied into a beautiful contrivance for at once converting the rectilineal motion of a piston into a rotatory motion.

This property of the epicycloid discovered by Roemer.

For the discovery of the mechanical properties of the epicycloid, which Dr. Robison has ascribed to De la Hire, or Dr. Hook, we are indebted to the Danish astronomer Olaus Roemer, the discoverer of the progressive motion of light; and Wolfius, upon whose authority this fact is stated,[2] laments, that the mechanics of his time did not avail themselves of the discovery.

In order to insure an uniformity of pressure and velocity in the action of one wheel upon another, it is not necessary

[2] Ex eodem fonte *Olaus Romerus*, cum Parisiis commoraretur, quamvis non sine subsidio Geometriæ sublimioris, deduxit figuram dentium in rotis epicycloidalem esse debere: id quod post eum quoque ostendit *Philippus de la Hire;* sed quod dolendum hactenus in praxin recepta non est. *Wolfii Opera Mathemat.* tom. i, p. 684. The same fact is stated by Leibnitz, in the *Miscellan. Berolinens*, 1710, p. 315.

that the teeth, either of one or both wheels be exactly epicycloids. If the teeth of one of them be either circular, or triangular, with its sides converging to the wheels' centre, or, indeed, of any other form, this uniformity of force and motion will be attained, provided that the teeth of the other wheel have a figure which is compounded of that of an epicycloid and the figure of the teeth of the first wheel.[3] But, as it is often difficult to describe this compound curve, and sometimes impossible to discover its nature, we shall endeavour to select such a form for the teeth as may be easily described by the practical mechanic, while it ensures an uniformity of pressure and velocity.

In pursuing this subject, we shall call the largest of the two wheels M, N, viz. M, the *wheel*, and the smaller, N, the *pinion*. The acting parts of the wheel are called the *teeth*, and the acting parts of the pinions *leaves*. When the pinions have the form shewn at $E\,F$, $G\,H$, and $B\,B$ in Fig. 4, Plate III, Vol. I, they are called *lanterns*, and the cylindrical teeth *trundles* or *spindles*. Plate V. Fig. 1.

The line $B\,F$ joining the centres B and F of the wheel and pinion, is called the *line of centres;* and when this line of centres is divided into two parts $B\,A$, $F\,A$, which are to one another as the number of leaves in the pinion is to the number of teeth in the wheel, $F\,A$ is called the *primitive radius* of the wheel, and $B\,A$ the *primitive radius* of the pinion; while the lines $F\,f$ and $B\,b$ are called the *true radii* of the wheel and pinion. The circles $X\,A\,X$, $R\,A\,R$, are called the *primitive circumferences*, or *pitch lines*.

Case I. If the acting faces of the teeth of a *Wheel* are epicycloids, whose generating circle has the same radius as the primitive radius of the *Pinion*, and whose base has the same radius as the primitive radius of the wheel; and if these teeth drive the pinion by acting upon infinitely small pins in its circumference, the motion of the wheel and pinion will be uniform.

Let $m\,m'\,m''$ (Fig. 2) be the primitive circumference of the wheel M, and $a\,b\,c$ that of the pinion N. Let $m\,n$, $m'\,n'$, $m''\,n''$, be equidistant epicycloidal lines generated by the points $a\,b\,c$ of the wheel N rolling upon M, or successive positions of the

[3] M. de la Hire has shewn, in a variety of cases, how to find this compound curve.

same curve line $m\,n$; and a, b, c small equidistant pins in the rim of the pinion or successive positions of the pin a, by which the epicycloidal lines were generated. Now, since the epicycloids $m\,n$, $m'\,n'$, $m''\,n''$ are formed by the circumference $c\,b\,a$ rolling over $m''\,m'\,m$, the arch $a\,b$ must be equal to $m\,m$, and the arches $a\,b$, $a\,c$, to $m\,m'$, $m\,m''$ respectively, for as the epicycloids $m\,n$, $m'\,n'$, $m''\,n''$, are described by the points a, b, c, every point of the arch $a\,b$ must have been in contact with every corresponding point of $m\,m''$; and therefore $a\,b = m\,m'$, and, for the same reason, $a\,c = m\,m''$. Hence, if the pinion N is driven by the wheel M, by means of the epicycloidal teeth $m\,n$, &c. acting upon the pins a, b, c, &c. each of them will describe in the same time arcs of equal length, and therefore their motion will be uniform.

Case II. If the acting faces of the teeth of a wheel are epicycloids, whose generating circle is of any magnitude whatever, and whose base is the *convex* circumference of the wheel, and if the acting faces of the leaves of the pinion are epicycloids, described by the same generating circle, and whose base is the *concave* circumference of the pinion, the motion of the wheel and pinion will be uniform.

Let $m\,m'$ (Fig. 3) be the convex circumference of the wheel M, and $m\,n$ an epicycloid generated by a circle of any diameter $A\,m$ rolling upon it; and let $a\,\nu$ be another epicycloid formed by the same circle $A\,m$, rolling upon the concave circumference $c\,a\,b$ of the pinion N; then if the teeth of the wheel have the acting faces of the form $m\,n$, and the leaves of the pinion their acting faces of the form $a\,\nu$, the one will drive the other uniformly. Let the epicycloids $m\,n$, $a\,\nu$, be in the positions $m'\,n'$, $b\,\nu'$; then since in the formation of the epicycloids $m'\,n'$, $b\,\nu'$, the two bases $m\,m'$, $a\,b$, coincide at m, the two epicycloids must also coincide and touch each other at the point d in the circle $A\,m$, so taken that $a\,d = m\,m'$. But $a\,b$ is also equal to $a\,d$, as the epicycloid $b\,\nu'$ is formed by the developement of $a\,d$ from $b\,a$; hence $a\,b = m\,m'$; that is, the wheel and pinion will describe arcs of equal length in the same time, and therefore their motion will be uniform.

Case III. If the acting faces of the teeth of a wheel are epicycloids, whose generating circle has a radius equal to half the radius of the pinion, and a base of the same radius as the wheel,

and if the acting faces of the leaves of the pinion are straight lines directed to the centre of the pinion, the motion of the wheel and pinion will be uniform.

This mode of action, which we have represented in Fig. 4, is a particular case of the preceding mode; for when the radius of the generating circle is one half of the radius of the concave base on which it revolves, the *interior epicycloid* $a\,v$ (Fig. 3), which it describes, then becomes a straight line.[4] The demonstration is therefore the very same as in Case II.

Case IV. If a wheel M drives a wheel N (Fig. 5), by means of teeth $m\,n$, $m'\,n'$, acting upon the leaves $a\,a'$, $b\,b'$, which are respectively involutes of circles $m\,m'\,C$, $b\,a\,D$, to which any line $C\,D$, passing through A, is a common tangent, the motion of the wheel and pinion will be uniform.

Having drawn through A a right line $C\,D$, touching both the circles $N\,b\,a\,D$, $C\,m'\,m$, let fall upon it from the points F and B the perpendiculars $F\,C$ and $B\,D$; then if we form the involutes $m\,n$, $a\,a'$, by evolving $C\,A$ from the circumferences $C\,m$ and $D\,a$, it is obvious that they will touch one another at the point A, and that the common tangent $C\,D$ will be perpendicular to both at that point. Let $m\,n$ have the position $m'\,n'$, when $a\,a'$ has the position $b\,b'$, then it is manifest that the point A', in which they touch one another, will always be in the straight line $C\,D$; for A' will be in both the involutes formed by evolving $C\,A'$, $D\,A'$ from the arches $C\,m'$, $D\,b$. Now, since

$m\,m' = C\,m - C\,m' = C\,A - C\,A' = A\,A'$, and

$a\,b = D\,b - D\,a = D\,A' - D\,A = A\,A'$,

we have $m\,m = a\,b$; that is, the wheel and pinion will describe arcs of equal length in the same time, and therefore move uniformly.

The following curious results respecting the teeth of wheels have been recently given by M. Clapeyron:[5]—

In order to communicate the motion of one wheel to an external wheel, without friction, the teeth must be terminated with equal arcs of equal ellipses, having for their greater axis the distance of the centres, and a small rotatory axis.

Let A, C, B, D, (Fig. 6), be the foci of two equal ellipses placed beside one another so as to touch in the ex-

[4] See Art. *Epicycloid*, *Edinburgh Encyclopædia*, vol. ix, p. 184, col. 1, § 3.

[5] *Annales des Mines*, 1820, tom. v, p. 113.

tremities of their greater axes. Let us suppose that one of the ellipses $A\,C$ is fixed, and that the other, $B\,D$, rolls upon it. Then if it is demonstrated that the epicycloid described by the focus B of the moveable ellipse is a *circle*, having A for its centre, and $A\,B$ for its radius, it will follow that if we suppose A and B fixed in the two ellipses, and make them moveable, the first round A and the second round B, the ellipse $B\,D$ will communicate its motion to the ellipse $A\,C$, without any friction on the touching surfaces.

Let $D'\,B'$ be one of the positions of the ellipse $B\,D$, revolving round $A\,C$, supposed fixed; then it will be found that $A\,B' = A\,B$. For since the moveable ellipse has rolled upon the fixed ellipse, we have $R\,s = R\,m$, and consequently $R\,B' = R\,C$, since the ellipses are equal. For the same reason, the angle of the *radii vectors* $B'\,R$ and $C\,R$, with the common tangent $P\,q$, will be equal; and as the angle $P\,R\,A$ and $C\,R\,q$ in an ellipse are equal, it follows that the points A, R, B are in a straight line; and since $A\,R + R\,B = A\,R + R\,C = A\,B$, it follows that $A\,B' = A\,B$, and that $B\,B'$ is the arch of a circle.

In order to communicate the motion of one wheel to an interior wheel without friction, the teeth may be terminated with equal arcs of equal hyperbolas, having for their greater axis the distance of the centres of the wheels.

If we suppose two equal hyperbolas, AmB, $A'\,m\,B'$ (Fig. 7), touching at their summit m, and one of them, viz. $A\,m\,B$ revolving round the other $A'\,m\,B'$, when fixed, then the curve described by the focus R of the moveable hyperbola will be a circle, having its centre in the focus T of the fixed hyperbola. Let n be the primitive position of the focus of the moveable hyperbola $A\,m\,B$ and let $a\,s\,b$ be one of its positions; then, since the one has rolled upon the other, we have $R\,q = P\,q$, and therefore the points T, R, q are in a straight line. Hence, $R\,T = T\,q - P\,q =$ a constant quantity, since the curve $A\,m\,B$ is a hyperbola. If we suppose, therefore, the foci n and T fixed, and each hyperbola turning round its focus, the one will move the other without friction.

On the Teeth of Rackwork.

Case V. If the teeth of a rack, as shewn at O, P, I, K in Plate XIII, Fig. 4, Vol. I, drive a pinion, then when the acting faces of the teeth of the rack are cycloids, whose ge-

nerating circle has the same radius as the pinion, and when their teeth drive the pinion by acting upon infinitely small pins placed in its circumference, the motion of both will be uniform.

Since this is a variety of Case I, where the radius of the wheel is infinitely great, and where the *epicycloids* pass into *cycloids*, it does not require any farther demonstration.

Case VI. If the acting faces of the teeth of the rack are cycloids, whose generating circle has a radius equal to half the radius of the wheel, and if they drive the wheel by acting upon rectilineal teeth, which are a continuation of the radii, the motion of both will be uniform.

As this is a variety of Case III, where the radius of the wheel, or the base of the epicycloids, is infinitely great, it requires no demonstration.

Case VII. When the teeth of a wheel driving a rack have their acting faces formed of the involutes of a circle, having the same radius as the pinion, and drive the rack by acting upon infinitely small pins fixed in its rectilineal edge, the motion will be uniform.

This being a variety of Case I, where the radius of the pinion is infinitely great, and where the *epicycloid* becomes an *involute*, from the radius of its generating circle becoming infinitely great, it requires no demonstration.

M. Clapeyron, in extending his reasoning to the case where the external wheel has a radius infinitely great, which is the case of a rack, has shewn that the *hyperbolic arcs* become *parabolic*, and hence it follows that for rackwork, or cases where a rod rises or moves in a rectilineal direction, the faces of the acting parts may be parabolic.

On the Form of Spindles or Trundles for driving Lanterns.

Case VIII. If the acting faces of the teeth of a wheel are curves drawn parallel to an epicycloid, formed by the revolution of a circle equal to the lantern upon the wheel as a base, so that the distance between the epicycloid and the curve is equal to the radius of the spindles, this wheel will drive the spindles with an uniform motion.

Let the figure of the teeth of the wheel M (Fig. 8), whose primitive radius is $F\,m$ (F being supposed at the centre of the arch $m'\,m$), be required, in order to drive uniformly the lantern

N, whose primitive radius is $B\,m$. Let $p\,m$ be the radius of the spindles, then with a generating circle, whose radius is $B\,m$, and on a base, whose radius is $F\,m$, describe the *exterior epicycloid* $m\,n$. This epicycloid ould be the proper form of the teeth, if the spindles $p\,m$ were infinitely small. (See Case I.) Upon any number of points in the epicycloid $m\,n$, as centres viz. 1, 2, 3, 4, with $p\,m$, the semidiameter of the spindles, as a radius, describe arches of circles, seen between m' and n' the curve $m'\,n'$, drawn so as to touch all these circles, will be parallel to the epicycloid, and will be the form of the teeth required. The same form must be given to the teeth when they are placed on the concave or interior side of the wheel, and drive the lantern in the inside of the wheel. The epicycloids are of course in that case interior ones, as the generating circle rolls upon the concave side of the base.

Since the epicycloid $m\,n$ drives the infinitely small spindle at m uniformly by Case I, so that m is at μ when the spindle is at a, we have $a\,m = m\,\mu$, and hence the equidistant curve $m'\,n'$ will drive the spindle $p\,m$ uniformly. For as the point where $m'\,n'$ touches the spindle must always be distant from m by the radius of the spindle, the point m' will want a quantity $= m\,p$ of the point μ, and the corresponding point of the spindle which the point m' touches will want the same quantity $m\,p$ of the point a; and, therefore, since $a\,m = \mu\,m$, $a\,m - m\,p = \mu\,m - m\,p$; that is, the wheel and lantern will move uniformly.

If it is required, that the teeth do not act upon the spindles till the former reach the line of centres $F\,B$, the curve $m'\,n'$ should not touch the spindles till the point m' has arrived at m. The tooth $m'\,n'$, therefore, will begin to act upon the spindle at the point o, where the primitive circumference of the lantern cuts the circumference of the spindle. The unshaded parts, therefore, of the spindles, might be cut away as superfluous; but as it is obviously better that they retain their cylindrical form, the hollows between the teeth of the wheel should be arches of a circle described by a radius a little greater than $m\,p$.

On the Form of the Wipers of Stampers, and the lifting Cogs or Cams of Forge Hammers.

In machinery, where large weights are to be raised, such as in fulling-mills, mills for pounding ore, &c. or where large pistons are to be elevated by the arms of levers, it is of the greatest

consequence that the power should raise the weight with an uniform force and velocity; and this can be effected only by giving a proper form to the wipers. A certain class of mechanics generally excuse themselves for not attending to the proper form of the teeth of wheels, by alleging that the scientific form differs but little from theirs, and that teeth, however badly formed, will, in the course of time, work into the proper shape. This excuse, however, will not apologize for their negligence in the present case. The scientific form of the wipers of stampers and the arms of levers are so widely different from the form which is generally assigned them, as to increase very much the performance of the machine, and preserve its parts from that injury which is always occasioned by the want of an uniform motion.

Case IX. If the wipers for raising stampers have their acting faces involutes of a circle, whose radius is that of the wheel upon which they are to be fixed, and if they elevate the stamper, by acting upon a plain surface perpendicular to the vertical direction in which the stamper rises, the motion will be uniform.

This kind of action is shewn in Fig. 9, where M is the wheel furnished with wipers, and N the stamper. The curve $b\,c$ of the stamper is an involute, formed by evolving the arch $a\,b$, which must always be equal to the height $n\,n'$ through which the stamper is to be raised. This is the same case as that in Case VII.

Case X. If the wiper is to raise the stamper, as in Fig. 10, by means of a curve fixed upon the stamper, the curve must be a cycloid, the radius of whose generating circle is the length of the arm $D\,H$.

Thus, in Fig. 10, let $C\,D$ be an axis, moved by any power, in which are fixed the arms $D\,H$, $M\,R$, having rollers, H, R, at their extremities, which act upon the curved arm $o\,p$. When the piston $E\,F$ is raised to the proper height, by the action of the roller H upon $o\,p$, it then falls, and is again elevated by the arm $M\,R$. In order that its motion may be uniform, the arm $o\,p$ must be part of a cycloid, the radius of whose generating circle is equal to the length of the arm $D\,H$, reckoning from its extremity H, or the centre of the roller, to the centre of the axle $D\,C$. But, when a roller is fixed upon the extremity H, we must draw a curve parallel to the cycloid, and without it, at

the distance of the roller's semidiameter; and this curve will be the proper form for the arm *o p*. It is evident, that, when this mode of raising the piston is adopted, the arm *D H* must be bent, as in the figure, otherwise the extremity *p* would prevent the roller *H* from acting upon the arm *o p*.

Case XI. If the wiper is to raise the stamper in the manner shewn in Fig. 11, it will be raised uniformly if the acting face of the wipers has the form of the spiral of Archimedes.

Plate V. Fig. 11. Let *A H* be a wheel moved by any power which is sufficient to raise the weight *M N* by its extremity *O*, from *O* to *e*, in the same time that the wheel moves round one-fourth of its circumference it is required to fix upon its rim a wing *O B C D E H*, which shall produce this effect with an uniform effort. Divide the quadrant *O H* into any number of equal parts *O m*, *m n*, &c. the more the better, and *O e* into the same number *O b*, *b c*, *c d*, &c. and through the points *m*, *n*, *p*, *H*, draw the indefinite lines *A B*, *A C*, *A D*, *A E*, and make *A B* equal to *A b*, *A C* to *A c*, *A D* to *A d*, and *A E* to *A e*; then, through the points *O*, *B*, *C*, *D*, *E*, draw the curve *O B C D E*, which is a portion of the spiral of Archimedes, and will be the proper form for the wiper, or wing *O H E*.[8] It is evident, that when the point *m* has arrived at *O*, the extremity of the weight will have arrived at *b*; because *A B* is equal to *A b*; and, for the same reason, when the points *n*, *p*, *H*, have successively arrived at *O*, the extremity of the weight will have arrived at the corresponding points *c*, *d*, *e*. The motion, therefore, will be uniform; because the space described by the weight is proportional to the space described by the moving power, *O b* being to *O c* as *O m* to *O n*.

If it be desired to raise the weight with an accelerated or a retarded motion, we have only to divide the line *O e* according to the law of acceleration and retardation, and form the curve *O B C D E* as formerly.

Case XII. If the weight is to move round a centre, and rise in the same plane as that in which the wheel moves, the acting faces of the wipers must be formed in the following manner:—

The celebrated Deparcieux, of the Academy of Sciences of Paris, has given an ingenious and simple method of tracing me-

[8] For a different way of forming this spiral, see Wolfii *Opera Mathematica*, tom. i, p. 399.

chanically the curves which are necessary for this purpose. Though this method was published about fifty years ago in the Memoirs of the Academy, it does not seem to be at all known to the mechanics of this country. We shall therefore lay it before the reader in as abridged and simplified a form as the nature of the subject will permit. Let *A B* (Fig. 12), be a lever lying horizontally, which it is required to raise uniformly through the arch *B C* into the position *A C*, by means of the wheel *BFEH*, furnished with the wing *BNOP*, which acts upon the extremity *C* of the lever; and let it be required to raise it through *B C* in the same time that the wheel *B F E H* moves through one half of its circumference; that is, while the point *M* moves to *B*, in the direction *M F B*. Divide the chord *C B* into any number of equal parts, the more the better, in the points 1, 2, 3, and draw the lines 1 *a* 2 *b* 3 *c* parallel to *A B*, or a horizontal line passing through the point *B*, and meeting the arch *C B* in the points *a*, *b*, *c*. Draw the lines *C D*, *a D*, *b D*, *c D*, and *B D*, cutting the circle *B F H* in the points *m*, *n*, *o*, *p*. Plate V. Fig. 12.

Having drawn the diameter *B M*, divide the semicircle *BFM* into as many equal parts as the chord *C B*, in the points *q*, *s*, *u*. Take *B m* and set it from *q* to *r*; take *B n* and set it from *s* to *t*; take *B o* and set it from *u* to *v*; and lastly, set *B p* from *M* to *E*. Through the points *r*, *t*, *v*, *E*, draw the indefinite lines *D N*, *D O*, *D P*, *D Q*, and make *D N* equal to *D c*; *D O* equal to *D b*; *D P* equal to *D a*; and *D Q* equal to *D C*. Then through the points *Q*, *P*, *O*, *N*, *B*, draw the spiral *B*, *N*, *O*, *P*, *Q*, which will be the proper form for the wing of the wheel when it moves in the direction *E M B*.

That the spiral *B N O* will raise the lever *A C* with an uniform motion, by acting upon its extremity *C*, will appear from the slightest attention to the construction of the figure. It is evident, that when the point *q* arrives at *B*, the point *r* will be in *m*, because *B m* is equal to *q r*, and the point *N* will be at *c*, because *D N* is equal to *D c*; the extremity of the lever, therefore, will be found in the point *c*, having moved through *B c*. In like manner, when the point *s* has arrived at *B*, the point *t* will be at *n*, and the point *O* in *b*, where the extremity of the lever will now be found; and so on with the rest, till the point *M* has arrived at *B*: The point *E* will then be in *p*, and the

point Q in C; so that the lever will now have the position $A\ C$, having moved through the equal heights $B\ c$, $c\ b$, $b\ a$, $a\ C$,[1] in the same time that the power has moved through the equal spaces $q\ B$, $s\ q$, $u\ s$, $M\ u$. The lever, therefore, has been raised uniformly, the ratio between the velocity of the power, and that of the weight, remaining always the same.

If the wheel D turns in a contrary direction, according to the letters $M\ H\ B$, we must divide the semicircle $B\ H\ E\ M$, into as many equal parts as the chord $C\ B$, viz. in the points c, g, i. Then, having set the arch $B\ m$ from e to d, the arch $B\ n$ from g to f, and the rest in a similar manner, draw through the points d, f, h, E, the indefinite lines $D\ R$, $D\ S$, $D\ T$, $D\ Q$, make $D\ R$ equal to $D\ c$; $D\ S$ equal to $D\ b$; $D\ T$ equal to $D\ a$, and $D\ Q$ equal to $D\ C$; and through the points B, R, S, T, Q, describe the spiral $B\ R\ S\ T\ Q$, which will be the proper form for the wing, when the wheel turns in the direction $M\ H\ B$. For, when the point c arrives at B, the point d will be in m, and R in c, where the extremity of the lever will now be found, having moved through $B\ c$ in the same time that the power, or wheel, has moved through the division $e\ B$. In the same manner it may be shewn, that the lever will rise through the equal heights $c\ b$, $b\ a$, $a\ C$, in the same time that the power moves through the corresponding spaces $e\ g$, $g\ i$, $i\ M$. The motion of the lever, therefore, and also that of the power, are always uniform. Of all the positions that can be given to the point B, the most disadvantageous are those which are nearest the points F, H; and the most advantageous position is when the chord $B\ C$ is vertical, and passes, when prolonged, through D, the centre of the circle.[2] In this particular case the two curves have equal bases, though they differ a little in point of curvature. The farther that the centre A is distant, the nearer do these curves resemble each other; and if it were infinitely distant, they would be exactly similar, and would be the spirals of Archimedes, as the extremity C would, in this case, rise perpendicularly.

[1] The arches $B\ c$, $c\ b$, &c. are not equal; but the perpendiculars let fall from the points, c, a, b, &c. upon the horizontal lines, passing through a, b, &c. are equal, being proportional to the equal lines c 1; I, 2, Eucl. vi, 2. Had it been required to raise the lever through equal arches, instead of equal heights, in equal times, then the arch $B\ C$, instead of its chord, would have been divided into equal parts.

[2] In the figure we have taken the point B in a disadvantageous position, because the intersections are in this case most distinct.

The intelligent reader will easily perceive, that 4, 6, or 8 wings may be placed upon the circumference of the circle, and may be formed by dividing into the same number of equal parts as the chord *B C*, $\frac{1}{4}$, $\frac{1}{6}$, or $\frac{1}{8}$, of the circumference, instead of the semicircle *B F M*.

That the wing *B N O* may not act upon any part of the lever between *A* and *C*, the arm *A C* should be bent; and that the friction may be diminished as much as possible, a roller should be fixed upon its extremity *C*. When a roller is used, however, a curve must always be drawn parallel to the spiral described, according to the method already explained, the distance between it and the spiral being everywhere equal to the radius of the roller.

When two or more wings are placed upon the circumference of the wheel, it has been the custom of practical mechanics to make them portions of an ellipse whose semi-transverse axis is equal to *Q D*, the greatest distance of the curve from the centre of the circle. But it will appear, from a comparison of an elliptical arch with the spiral *N*, that it will not produce an uniform motion. If it should be required to raise the lever with an accelerated or retarded motion, we have only to divide the chord *B C*, according to the degree of retardation or acceleration required, and the circle into the same number of equal parts as before, and then describe the curve by the method already illustrated.

Case XIII. If the weight is to move round a centre, and in the same plane as that in which the wiper moves, it may be done by a constant radius acting upon a curvilineal surface on the body to be raised, which may be found in the following manner.

Let *A B* (Fig. 13), be a lever, which it is required to raise uniformly through the arch *B C*, into the position *A C*, by means of the arm or constant radius *D E*, moving upon *D* as a centre, in the same time that the extremity *E* describes the arch *E e F*. From the point *C* draw *C H* at right angles to *A B*, and divide it into any number of equal parts, suppose three, in the points 1, 2; and through the points 1, 2, draw 1 *a*, 2 *b*, parallel to the horizontal line *A B*, cutting the arch *C B* in the points *a*, *b*, through which draw *a A*, *b A*. Upon *D* as a centre, with the distance *D E*, describe the arch *E i c F*; and upon *A* as a centre, with the distance *A D*, describe the arch *c O D*, cutting the arch

E i e F in the point *e*. Divide the arches *E i e*, and *F s e*, each into the same number of equal parts as the perpendicular *c H*, in the points *k*, *i*, *s*, *m*, and through these points, about the centre *A*, describe the arches *k z*, *i g*, *q r*, *m n*. Take *z x* and set it from *k* to *l*, and take *g f*, and set it from *i* to *h*. Take *r q* also, and set it from *s* to *t*, and set *n m* from *o* to *p*, and *d c* from *e* to *O*. Then through the points *E*, *l*, *h*, *O*, and *O*, *t*, *p*, *F*, draw the two curves *E l h O*, and *O t p F*, which will be the proper form that must be given to the arm of the lever. If the handle *D E* moves from *E* towards *F*, the curve *E O* must be used, but if in the contrary direction, we must employ the curve *O F*.

It is evident, that when the extremity *E* of the handle *D E*, has run through the arch *E k*, or rather *E l*, the point *l* will be in *k*, and the point *z* in *x*, because *x z* is equal to *k l*, and the lever will have the position *A b*. For the same reason, when the extremity *E* of the handle has arrived at *i*, the point *h* will be in *i*, and the point *g* in *f*, and the lever will be raised to the position *A a*. Thus it appears, that the motion of the power and the weight are always proportional. When a roller is fixed at *E*, a curve parallel to *E O*, or *O F*, must be drawn as formerly.

It is upon these principles that the detent levers of clocks, and those connected with the striking part should be formed. In every machine, indeed, where weights are to be raised or depressed, either by variable or constant levers, its performance depends much on the proper form of the communicating parts.

Case XIV. If the wheel which carries the wipers moves at right angles to the plane in which the lever moves, the wipers ought to be made in the following manner:—

Let *A B C* (Fig. 14), be the lever which is to be raised round the axis *A B*, by the action of the wing *m n* of the wheel *D*, upon the roller *C*, fixed at the extremity of the lever;—it is required to find the form which must be given to the wiper *m n*. It is evident from Fig. 15, where *C B* is a section of the lever and roller, and *BA* the arch through which it is to be raised, that the breadth of the wiper must always be equal to *m n*, or *r B*, the versed sine of the arch *BA*, through which the roller moves, so that the extremity *n* of the wiper may act upon the roller *B* at the commencement of the motion, and that the other

extremity m may act upon the roller A, when the lever arrives at the required position $C A$. It is easy to perceive, however, that, if the acting surface $m n$ of the wiper is always parallel to the horizon, or perpendicular to the radii of the wheel D, or the plane in which it moves, it will act disadvantageously, except at the commencement of the motion, when $m n$ is parallel to $C B$. For, when $m n$ has arrived at the position $o p$, the extremity o will act upon the roller A, but in such an oblique and disadvantageous manner, that it will scarcely have any power to turn it upon its axis, or move the lever round the fulcrum C. The friction of the roller upon its axis, therefore, will increase, and the power of the wiper to turn the lever will diminish, in proportion to the length of the arch $B A$; and if $C A$ arrives at a vertical position, the power of the wiper will be solely employed in wrenching the lever from its fulcrum.

In order to avoid this inconvenience, we must endeavour to give such a form to the wiper, that its acting surface may always be parallel to the lever, or axis of the roller, having the position $m n$ when the roller is at B, and the position $o b$ when the roller is at A.

Having stated the peculiarities of this construction, let us now attend to the method by which the acting surface of the wiper must be formed. Since the lever $C B$ is to be raised perpendicularly through the equal spaces $r c$, $c a$, $a A$ in equal times, the acting surface of the wiper must evidently be part of the spiral of Archimedes, (see page 128), the method of describing which is shewn in Fig. 11; but the difficulty lies in giving different degrees of inclination to the acting surface, in order that the part in contact with the roller may be parallel to the direction of the lever. Let $A D$, Fig. 17, be the wheel, which is to be furnished with wings, and let $C q$ the perpendicular height, through which the lever is to rise, be equal to $A r$, in Fig. 15. Divide the quadrant $D b$ into any number of equal parts, the more the better, suppose three, in the points c and r, and describe the spiral of Archimedes $D i n C$, as formerly directed. Divide $A r$ (Fig. 15) the sine of the arch $B A$, into the same number of equal parts, in the points c, a; and draw $a f$, $c g$ parallel to $C B$, and cutting the circle in the points d, e, and the tangent $B b$ in the points $f g$; and through the points C and d draw $C k i$. The line $d f$ is equal to the difference between radius and the cosine of the

arch $d B$; $f i$ is equal to the difference between the tangent and the sine of the same arch; $i B$ being the tangent, and $f B$ equal to the sine of the arch $d B$, or angle $d c B$; $a d$ is equal to $a f - d f$, or to the difference between $d f$ and the versed sine of the whole arch $A B$; and $a k$ is equal to $\frac{f i \times d a}{d f}$, for on account of the similar triangles $d f i$, $d a k$, we have $d f : f i = d a : a k$;[3] and consequently $a k = \frac{f i \times d a}{d f}$.

Since then the points r, c, a, A (in Fig. 15) correspond respectively with the points D, i, n, C of the spiral (in Fig 17), take $f i$ and set it from n to m, and $a k$ from n to o; take also $g h$, and set it from i to h (Fig. 17), set $c q$ from i to k, and make $c B$ (in Fig. 17), equal to $p b$ (in Fig. 15), or the difference between the tangent and the sine of the arch $A B$, and through the points D, k, o, C, and D, h, m, B, draw the curves $D o C$, $D m B$, which will be the proper form for the sides $O N$, $M P$ of the spiral wiper $M O N P$ (Fig. 16), the acting surface $M O N P$ must then be wrought in such a manner as to consist of a variety of planes, differently inclined to the plane $B O N$ of the wiper, the angle of inclination being a right angle at O and M, but increasing gradually till the inclination at $N P$ becomes equal to the angle $D C E$, or $A C B$, in Fig. 15.[4] From the construction of Fig. 15, it is evident, that the arches $B e$, $e d$, $d A$ are not equal, nor are they aliquot parts of $A B$. But since the arch $A B$, and its sine $A r$ are known, and since the sines of the other arches are known, viz. $b c$, $b a$, the arches themselves may be easily found by a table of natural sines.

In Fig. 16, we have a perspective view of a wheel, furnished with two wipers, formed according to the preceding directions. $F C$ and $L N$ correspond with $b C$ and $r A$, in Fig. 17 and 15. The curves $A n m C$, and $O N$ correspond with $D k o C$, in Fig. 17, and $M P$ with $D h m B$. The diagonal curve $M N$ corresponds with the diagonal curve $D i n C$, and $O M$, the breadth of the wiper with $m n$, or $r B$, the versed sine of the

[3] The lines $d f$, $f i$, $a d$, $a k$, may also be found mechanically, by making $A C$ equal to the real length of the lever.

[4] The curves, which must be employed in practice, should be curves drawn parallel to those formed by the preceding method, at the distance of the semidiameter of the roller.

arch $A B$, in Fig. 15. The breadth $O M$, however, should always be a little greater than the versed sine of the arch through which the lever is to be raised, since $M N$ is the path of the roller over the wiper's surface.

Having thus described the different methods of raising weights, whether perpendicularly, or round a centre, with an uniform velocity and force, it would be unnecessary to apply the principles of construction to those machines which are formed for the elevation of weights. The practical mechanic can easily do this for himself. There is one case, however, which deserves peculiar attention, because the wipers, formed according to the preceding rules, will not produce the intended effect. This happens in the case of the large sledge hammer which is employed in forges, and which is moved round as a centre, by means of a wiper acting upon its extremity. The hammer must be tossed up with a sudden motion, so as to strike an elastic oaken spring, which, being compressed, drives back the hammer, with great force, upon the anvil. Now, if spiral wipers, constructed according to the directions already given, are employed, the hammer will indeed be raised equably without the least jolting, but it will rise no higher than the wiper lifts it, and will, therefore, fall merely with its own weight. But, if the wipers are constructed in the common way, and the hammer elevated with a motion greatly accelerated, it will rise much higher than the wiper lifts it,—it will impinge against the oaken beam, which is often used as a spring, and be repelled with great effect against the iron on the anvil. In any of the preceding constructions, this accelerated motion may be produced, merely by dividing $B C$ according to the law of acceleration, and proceeding as already directed.

On the Teeth of Bevelled or Conical Wheels.

In bevelled or conical wheels, a portion of a fluted or toothed cone is made to drive a portion of another fluted or toothed cone, as shewn in Figs. 18 and 19 of Plate V, for the purpose of changing the direction of any motion into any other direction, inclined at any angle to the first direction.

In order to determine the relative size of the wheels for changing a motion into a direction inclined 30°, for example,

to its first direction, and in which the new axle shall move with four times the velocity of the first,—Let $A\,B$ (Fig. 20) be the original direction of the motion; through any point ~~O O~~ draw O $C\,O\,D$, inclined 30°, to $A\,B$; then since the axle $C\,D$ is to move four times more rapidly than $A\,B$, the wheel which it carries must have one-fourth the number of teeth, and one-fourth the diameter. Draw $c\,d$ at any convenient distance from $C\,D$, and parallel to it, and draw $a\,b$ parallel to $A\,B$, so that $A\,a = B\,b = 4\,C\,c = 4\,D\,d$, and join the points of intersection i and O. Draw $O\,m$, so that the angle $B\,O\,m = B\,O\,i$, and draw $O\,n$, so that $D\,O\,n = D\,O\,i$, and these lines will mark out the size and situation of the cones of which the wheels are to be portions. By attending to the preceding construction, it is obvious that it is nothing more than to divide the angle $B\,O\,D$ into two angles, whose sines are to one onother as the number of the revolutions of the one wheel is to the number of revolutions of the other, and as Mr. Simpson has shewn, (*Select Exercises*, p. 138), $2 \sin. \frac{1}{2} B\,o\,r = 2 \sin. \frac{1}{2} B\,O\,D \times \frac{m}{m \times n}$, m and n representing the number of revolutions of the wheels.

Spherical Epicycloids.

If we suppose the primitive circumference of the pinion N, Fig. 19, or the circumference of the base of the cone $C\,N$, to roll upon the primitive circumference of the wheel M, or the circumference of the base of the cone $C\,M$, any point will obviously describe a curve; and as the point N is always at the same distance $N\,C$ from the centre C, it must be always in the surface of a sphere, whose centre is C, and therefore the curve which it describes will also be in the surface of a sphere, and has hence been called a *Spherical Epicycloid.*

Case I. If the acting faces of the teeth of the wheel M have the form of a spherical epicycloid, whose generating circle is the primitive circumference of the pinion, and whose base is the primitive circumference of the wheel, and if they drive the pinion N by acting upon infinitely small pins in its circumference at N, the motion of both will be uniform.

Let $a\,b$ (Fig. 21, Plate V.) be part of the primitive circumference of the pinion, whose axis is $A\,N$, and $m\,m'$, part of the primitive circumference of the wheel, whose axis is $A\,M$. Let $m\,n$ be a portion of a spherical epicycloid formed by the generating circle $a\,b$ rolling upon the base $m\,m'$, and let it act

upon the pin a, and have the position $m'\ n'$, when the pin a has reached b. Had the point b been placed on m', and the generating circle rolled from m' to m, the point b would have formed the spherical epicycloid $m'\ n'$, and the arch $b\,m$ would have been equal to $m'\ m$; but as the tooth $m\ n$ has driven a to b in the same time that m has advanced to m', and as $a\,b = m'\,m$, the wheel M has driven the pinion N uniformly.

Case II. If the acting faces of the teeth of the wheel M have the form of spherical epicycloids, whose generating arch has a radius equal to half the primitive radius of the pinion, and whose base is the primitive circumference of the wheel, and if they drive the pinion N, by acting upon rectilineal leaves directed to its centre, the motion will be uniform.

Let $N\,m$, (Fig 22), be now the radius of pinion, and $m\,n$ a spherical epicycloid, whose generating circle has its diameter $N\,m$ equal to the radius $N\,m$ of the pinion $m\,o$. Let $N\,m$ and $N\,o$ be the rectilineal faces of its leaves directed to N, and let the leaf $N\,m$ be driven into $N\,o$, when the tooth $m\,n$ has reached $m'\,n'$. Now the arc $m\,p$ is equal to $m\,m'$, because $m'\,n'$ is a spherical epicycloid formed by the point p; and since the straight line $N\,o$ is formed by the circle $m\,p$, revolving on the concave side of $m\,o$, (see Case III, p. 122) the point p must have coincided with o, and therefore the arc $m\,o$ is equal to $m\,p$, and $m\,o = m\,m'$; and consequently the wheel and pinion have moved through equal spaces in equal times.

Case III. If a bevelled wheel drives a conical lantern with conical spindles, it will be done uniformly when the acting faces of the teeth of the bevelled wheel have the form of a curve drawn parallel to the spherical epicycloid in Case I, p. 136, and distant from it by the radius of the spindle.

This is demonstrated in the same way as in Case VIII, p. 125, and the curve is drawn in the manner there described.

In forming the teeth of bevelled wheels, the external spherical section a, Fig. 19, must be different from the internal spherical section b, the spherical epicycloid at b being generated by a circle, which is the circumference of the conical pinion at n, and the base being the circumference of the conical wheel at b. When one of these spherical epicycloids is given, the other may be found by drawing straight lines from C to every point

of the one which is given, and these lines will pass through the corresponding points of the other at all distances from *C*.

M. Clapeyron has assigned new curves for bevelled wheels. He supposes *O B* and *O D* (Fig. 20), the axes of two cones, having their common summit in *O*, and demonstrates that the teeth ought to have the form of a cone, having its summit in *O*, and having for its base a curve analogous to the ellipse traced on a sphere having its centre also in *O*. Before demonstrating this, he gives the following method of describing the curve mechanically. Let *A* (Fig. 23), be the centre of a sphere; *B* and *C* any two points taken on its surface. Fix the two ends of a loose thread at *B* and *C*, and by means of a point describe a curve in the surface of the sphere in the same manner as an ellipse is described on a plane. This curve is the one required. Then, if two of these equal curves like *D C* and *A B* (Fig. 24), are disposed in the surface of a sphere so as to touch at the summit of their greater axis, as at *m*, and if we make the one roll upon the other, supposed to be fixed, so that the moveable curve never goes out of the surface of the sphere, then *D*, *C*, *A*, *B* being the foci of the curves, the focus *A* will remain at a constant distance from the point *D*, reckoned on a great circle; and consequently the radius of the sphere, passing by the point *A*, will describe a right cone round the radius passing through the focus *D*. This may be proved by the same reasoning that was formerly applied to the ellipse in p. 124.

When the wheels are interior, the curves are analogous to hyperbolas traced on the surface of the sphere.

On the Formation of Exterior and Interior Epicycloids, and on the Disposition of the Teeth on the Wheel's Circumference.

Mechanical method of forming epicycloids. Plate V. Fig. 25.

Nothing can be of greater importance to the practical mechanic, than to have a method of drawing epicycloids with facility and accuracy; the following ,we trust, will be found a very simple mechanical method:—Take a piece of plain wood *G H* (Fig. 25), and fix upon it another piece of wood *E*, having its circumference *m b* of the same curvature as the circular base upon which the generating circle *A B* is to roll: when the generating circle is large, the shaded segment *B* will be sufficient.

In any part of the circumference of this segment, fix a sharp pointed nail *a*, sloping in such a manner that the distance of its point from the centre of the circle may be exactly equal to its radius; and fasten to the board *G H* a piece of thin brass, or copper, or tinplate *a b*, distinguished by the dotted lines. Place the segment *B* in such a position that the point of the nail *a* may be upon the point *b*, and roll the segment towards *G*, so that the nail *a* may rise gradually, and the point of contact between the two circular segments may advance towards *m*; the curve *a b* described upon the brass plate will be an accurate *exterior epicycloid.* In order to prevent the segments from sliding, their peripheries should be rubbed with rosin or chalk; or a number of small iron points may be fixed in the circumference of the generating segment. Remove, with a file, the part of the brass on the left hand of the epicycloid, and the remaining concave arch or gage *a b* will be a pattern tooth, by means of which all the rest may be easily formed. When an *interior epicycloid* is wanted, the concave side of its circular base must be used. The method of describing it is represented in Fig. 26, where *C D* is the generating circle, *F* the concave circular base, *M N* the piece of wood on which this base is fixed, and *c d* the interior epicycloid formed upon the plate of brass, by rolling the generating circle *C*, or the generating segment *D*, towards the right hand. The *cycloid*, which is useful in forming the teeth of *rack-work*, is generated precisely in the same manner, with this difference only, that the base on which the generating circle rolls must be a straight line.

Both sides of the teeth should be properly shaped.

Although, in general, it is necessary to give the proper curvature only to one side of the teeth, yet it may be proper to form both sides with equal care, that the wheels may be able to move in a retrograde direction. This is particularly necessary when a reciprocating power is employed. In the case of a mill moving by the force of a single-stroke steam-engine, the direction of the pressure on the communicating parts of the machinery is changed twice every stroke. During the working stroke, the teeth of the wheels which convey the motion from the beam to the machinery are acting with one side of their teeth, but during the returning stroke the wheels act with the other side of their teeth.[5]

[5] See Dr. Robison's *Treatise on Machinery*, in his *Works*, vol. ii, p. 38.

In order that the teeth may not embarrass one another before their action commences, and that one tooth may begin to act upon its corresponding leaf of the pinion, before the preceding tooth has ceased to act upon the preceding leaf, the height, breadth, and distance, of the teeth must be properly proportioned. For this purpose the pitch line or circumference of the wheel must be divided into as many equal spaces as the number of teeth which the wheel is to carry. Divide each of these spaces into 16 equal parts; allow 7 of these for the greatest breadth of the tooth, and 9 for the distance between each, or the distance of the teeth may be made equal to their breadth. If the wheel drives a trundle, each space should be divided into 7 equal parts, and 3 of these allotted for the thickness of the tooth, and $3\frac{2}{3}$ for the diameter of the cylindrical stave of the trundle. If each of the spaces already mentioned, or if the distance between the centres of each tooth, be divided into 3 equal parts, the height of the teeth must be equal to 2 of these. These distances and heights, however, vary according to the mode of action which is employed.[6] The teeth should be rounded off at the extremities, and the radius of the wheel made a little larger than that which is deduced from theory. But when the pinion drives the wheel, a small addition should be made to the radius of the pinion.

On the formation of Cycloids and Epicycloids by Means of Points, and the Method of drawing Lines parallel to them.

Method of forming epicycloids by means of points.

As the preceding mechanical method of forming epicycloidal curves may be regarded by some as too difficult in practice, and too liable to error, we shall point out a method of describing epicycloids by means of points, and a more accurate way of drawing lines parallel to them than that which is described in the preceding pages.

Plate V. Fig. 27.

Let the radius AB, Fig. 27, Plate V, of the large wheel be called a, and the radius BC of the lesser one, or generating circle, be called b, and let the variable quantity x be equal to $\frac{a}{b} \times z$, z being any number of degrees taken at pleasure, and equal to the variable angle BAO. Then having drawn the chord BO we shall have ABO, or $AOB =$

[6] Wolfii *Opera Mathematica*, tom. i, pp. 696–7.

$90° - \frac{z}{2}$; the chord $B\,O = 2\,a \times \sin.\frac{z}{2}$; and $A\,O\,D = 90°$ $\times \frac{x}{2}$. Whence $B\,O\,D = \frac{x \times z}{2}$; and $O\,D = 2\,b \times \sin.\frac{x}{2}$. The line $O\,D$ being thus determined, we have one point D of the epicycloid $B\,D$. If the angle $B\,A\,O$, or the variable quantity z be gradually diminished, and $O\,D$ determined anew, we shall have other points of the epicycloid between D and B: or if z be increased, other points of the epicycloid beyond D will be determined. Since a very small arch of any curve may be represented by the arch of a circle equicurve to it in the same point, we may describe a small portion of the epicycloid at D with a radius equal to $\frac{a+b}{a+2\,b} \times 2\,O\,D$. This radius being reckoned from D, on the line $D\,O$, which is perpendicular to the epicycloid at D, will give the centre from which the elementary arc at D may be described. In finding the different points D, d of the epicycloid $B\,D$, we determine at the same time the lines $D\,O$, $d\,O$, perpendicular to the epicycloid in the respective points D, d; hence it will be an easy matter to draw a curve parallel to the epicycloid $B\,D$ at any given distance. Thus let M be the given distance, then take the line M in the compasses, and set it from D to F on the perpendicular $D\,F$, and also from d to f, on $d\,f$, and so on for the other points. A number of points F, f, &c. will therefore be determined, through which we can describe the curve $E\,f\,F\,G$, which will be parallel to the epicycloid $B\,D$, and distant from it by the given quantity M.

In order to illustrate this method by an example, let $A\,B$, the radius of the large wheel, be 42.991 inches, and $B\,C = 25.7946 = A\,B \times 0.6$, then $a \,.\, b$ as $10 : 6$. Let us suppose $z = 12°$. Then $x = \frac{10}{6} \times 12$, or $x = 20°$; consequently $\frac{z}{2} = 60$; $B\,O\,D = 16°$. Since $B\,O$ is equal to $2\,a \times \sin.\frac{z}{2}$ we shall have

Logarithm $2\,a$	$=$	1.9344123
Log. Sine $\frac{z}{2}$ or $6°$	$=$	9.0192346
Therefore $B\,O = 8.9876$		0.9536469 Log.

In order to find $O D = 2 b \times \sin. \frac{x}{2}$ we have

$$\text{Logarithm} \quad 2 b = 1.7125636$$

$$\text{Log. Sine } \frac{x}{2} \text{ or } 10° = 9.2396702$$

$$\text{Therefore } O D = 8.9584, \quad 0.9522338$$

The radius of curvature at the point D, consequently, will be $= \frac{16}{11} \times O D$, when $z = 12°$,[7] that is, the radius of curvature will be 13.030. If z be successively diminished to 6°, 4°, and 2°, we shall have the results contained in the following table, which are found in the same way as when $z = 12°$.

z	x	$E B O$	$B O D$	$B O$	$O D$	Radius of Curvature.
2°	3° 20′	1°	2° 40′	1.5006	1.5004	2.1824
	6 40	2	5 20	3.0007	2.9996	4.3631
6	10 0	3	8 0	4.5000	4.4962	6.5400
12	12 0	6	16 0	8.9876	8.9584	13.0300

By means of this table, four points of the epicycloid may be found. Make the angle $B A O = 12°$, $B O D = 16$, and $O D = 8.9584$, which will determine the point D; and so on with the rest.

As it would be extremely difficult to project the wheels C and A upon paper, when they are very large, we shall shew how to describe the epicycloid without using the centres C and A. Draw $B E$ perpendicular to the line $C A$ that joins the centres of the wheels, and make the angle $E B O$ equal to one half of z, viz. 6 degrees. Make $B O$, as before found, equal to 8.9876; the angle $B O D = 16°$, and $O D = 8.9584$, and the point D will be determined when the line $C A$ is only given in position.

In the cycloid let the line $B O$ (Fig. 28, Plate V.) be equal to $b \times z$, b being the radius of the generating circle c, and z any number of degrees taken at pleasure. Then $D O = 2 b \times \text{Sine} \frac{z}{2}$,

[7] The radius of curvature being always $= \frac{a + b}{a + 2 b} \times 2 O D$, it will be equal, in the present example, to $\frac{10 + 6}{10 + 12} \times 2 O D$, or $\frac{16}{22} \times 2 O D$, or $\frac{16}{11} \times O D$.

and $D\,O\,B = \frac{z}{2}$. From D let fall the perpendicular $D\,K$, and let $D\,K = y$, and $B\,K = x$; then $D\,o \times \text{sine}\,\frac{z}{2} = D\,K$, or $y = 2\,b \times \text{sine}\,\frac{z^2}{2} = b \times \text{versed sine}\ z = b \times \overline{1 - \cos.\ z}$. Likewise we have $K\,O = 2\,b \times \cos.\,\frac{z}{2} \times \text{sine}\,\frac{z}{2} = b \times \text{sine}\ z$. Whence $B\,K$, or $x = b \times \overline{z - \text{sine}\ z}$. Wherefore $B\,K$ and $D\,K$ being thus found, the point D in the cycloid will be determined; and by diminishing z continually, we shall then have other points of the cycloid between D and B, and by increasing it we shall have points beyond D.

Example.

To illustrate this by an example, let $b = 1$ and $z = 120° = 180° - 60°$, then since $b = 1$ we shall have $y = \text{versed sine}\ 120° = 2 - \text{versed sine}\ 60° = 1.500$. To find x, which is $= b \times \overline{z - \text{sine}\ z}$, or, in the present case, $= z - \text{sine}\ z$, since b is equal to 1. The arch z, or 120°, being $\frac{1}{3}$ of the circumference of a circle whose radius is 1, and whose circumference is 3.1415927×2, or 6.2831854, will be equal to 2.0943950, and the sines of 120°, or its supplement 60°, is 0.8660254. Therefore

$$\begin{array}{r} 120° = 2.0943951 \\ \text{sine}\ 120° = 0.8660254 \\ \hline x = 1.2283697 \end{array}$$

If z be made 5°, x will be $= 0.0001108$, and $y = 0.0038053$. The nnmbers x and y being thus determined, we have only to make $B\,K$ equal to x, and $K\,D$ to y, in order to find the point D. It may be proper to observe, that the variable number z should be taken pretty small both for the cycloid and epicycloid, as it is only a little portion of these curves that is required for the teeth of wheels; and when several points of the curve are determined, the intervening space may be made arches of a circle equicurve to the epicycloid at the same point.[8]

In the preceding observations, we have given a general view of the methods of shaping the acting faces of the communicating parts of machines, which is all that we can attempt to do in a work like the present. That the engineer, however, may

[8] See Kæstner's Memoir *de Dentibus Rotarum*, in the Comment. Soc. Reg. Gotting. 1782, vol. v, pp. 9, 24.

have some idea of the subordinate details to which he must attend in the actual construction of the teeth of wheels, we shall point them out in the case of an ordinary wheel and pinion.

Plate V. Fig. 29. Let $F A$, $B A$, be the primitive radii of the wheel and pinion, which must always be in the ratio of the number of teeth in the one to the number of teeth in the other, and these again as the velocities of the axles upon which they are fixed. With these radii describe the primitive circumferences $C' C D A$, $c' c d a$. As the distances between the teeth should always be a little greater than the thickness of the teeth, and as the number of the teeth must always be a whole number, we shall have the following formulæ, calling

T, t = thickness $C D$ of the teeth,
D, d = distance $C' C'$ between the teeth,
N, n = the number of teeth in the wheel and pinion.

In the wheel
$$\begin{cases} T = \dfrac{2 \times 3.1416\, F A}{N} - D \\[2ex] D = \dfrac{2 \times 3.1416\, F A}{N} - T \end{cases}$$

In the pinion
$$\begin{cases} t = \dfrac{2 \times 3.1416\, B A}{n} - d \\[2ex] d = \dfrac{2 \times 3.1416\, B A}{n} - t \end{cases}$$

Upon the radius $B A$ describe the circle $B P A$, with which, as a generating circle rolling upon $C' C D$ as a base, describe the epicycloid $C E$, which will be the form of that part of the tooth. The point E, where the epicycloid $C E$ cuts the same epicycloid $D E$ reversed, determines the summit E of each tooth. As the leaves of the pinion must have their epicycloids $d e$, $c e$ formed by a generating circle, whose diameter is $F A$, rolling upon a base whose radius is $B A$, these epicycloids must, by Case III, p. 122, act upon rectilineal or plane surfaces, in order to give a uniform motion. Hence the parts $C M$, $D N$ of the wheel, and $c m$, $d m$ of the pinion, must be straight lines directed to the centres F, B. In order to determine the points m, n, describe a circle $E E$ from F as a centre passing through the summits of the teeth, and let P be the point where this circle cuts the primitive circumference $c' c d$, and Q the point where it cuts the generating circle $B Q C$, and we shall have $c m = d n = B P - B O$; or a circle $B Q R$, described with the radius $B Q$, will

mark out all the points m, n for every tooth of the pinion, and therefore determine the size of what has been called the *flanks* $c\,m$, $d\,n$ of the teeth. The epicycloidal face $A\,G$ of the tooth, in conducting the flank $A\,R$ of the pinion into $Q\,A$, passes from the position $A\,G$ to the position $Q\,Q'$, so that $A\,Q = A\,Q'$, and then the radius $B\,Q\,A'$ will (by Case III, p. 122) be a tangent to the point Q. The tooth will still act upon the flank beyond this position, and will finally quit it at the point P. When the flank $B\,A$ has arrived at $B\,A'$, another tooth of the wheel should begin to act upon another flank of the pinion.

The size of the flanks $C\,M$, $D\,N$, of the wheel $F\,A$, are determined in the very same manner.

The form of the curve $m\,s$ must obviously be the line which the point G of the tooth $A\,G$ describes upon the plane of the wheel B, when both are in motion. This curve may be shewn to be a prolate epicycloid, whose generating circle has a radius equal to $F\,A$, and a base whose radius is $B\,A$. In like manner, the curve $M\,N$ will be a prolate epicycloid, whose generating circle has $B\,A$ for its radius, and whose base is a circle, with a radius $F\,A$.[9] The application of this curve to the hollows of the teeth, is not of any importance with regard to their mutual action. It determines only the path of the point of the teeth; and in this way we know exactly how great a hollow to cut out, in order that the teeth may have the greatest possible degree of strength, no more being excavated from their base than is absolutely necessary.

The following Table of pitches of wheels in actual use in millwork, was drawn up by the late Mr. Robertson Buchanan, civil engineer, and will be found of great use to the practical mechanic. The arrangement only has been altered.

[9] See *Edinburgh Encyclopædia*, vol. ix, p. 180, Art. *Epicycloids*, where the formation of prolate and curtate cycloids is described. The very same is applicable to the epicycloid. See Hachette's *Traité Elementaire des Machines*, p. 296, for a demonstration that this curve is a prolate epicycloid.

Table of Pitches of Wheels in actual Use in Mill-work.

Nature of the Machinery.	Horses' power.	Pitch in inches.	Breadth of teeth in inches.	Wheel.			Pinion.			Breadth proportional to 10 horses' power, and present velocity.	Present velocity per second, in feet.	Breadth in inches proportional to 10 horses' power, at 5 f. p. second, that is, reducing all the examples to the same denom.
				Teeth.	Revols. per minute.	Diameter.	Teeth.	Revols. per minute.	Diameter.			
Horse-mill, ...	1	$2\frac{1}{4}$	4	91	3	$6..0\frac{1}{2}$	22	12.9	$1..5\frac{1}{2}$	40.0	.949	12.65
Horse-mill, ...	1	$2\frac{1}{4}$	$4\frac{1}{2}$	91	3	$6..0\frac{1}{2}$	20	13.13	1..4	45.0	.949	14.23
Water-wheel,[1] ...	$5\frac{1}{2}$	3	4	207		$16..5\frac{1}{4}$	50		$3..11\frac{5}{8}$	7.27	3.	7.27
Water-wheel,[2] ...	10	$4\frac{1}{8}$	$5\frac{1}{2}$									5.5
Water-wheel, ...	15	3	6	204	$4\frac{1}{2}$	16..2	44	20	3..6	4.	3.8	5.06
Water-wheel,[3] ...	30	3	$10\frac{1}{4}$	304	$3\frac{1}{7}$	24	3	318.47		3.41	3.95	4.489
Steam-Engine,[4] ..	4	$2\frac{1}{4}$	$4\frac{3}{4}$	48	32	$2..10\frac{1}{5}$	25	61.11	1..6	11.87	4.8	18.99
Ditto, ditto,[5]	6	$2\frac{1}{4}$	$5\frac{1}{4}$	60	28	3..7	27	62.6	$1..7\frac{3}{8}$	8.75	5.25	15.31
Ditto, ditto,[6]	10	$2\frac{1}{4}$	$5\frac{3}{4}$	77	25	$4..7\frac{1}{8}$	40	48.5	$2..4\frac{5}{8}$	5.75	6.2	11.88
Ditto, ditto,[7]	10	$1\frac{7}{8}$	6	77	25	3..10	40	48.5	$1..11\frac{1}{8}$	6.	5.	10.
Ditto, ditto,......	12	$1\frac{3}{8}$	3	66	44	2..8	48	60.5	1..9	2.5	5.99	4.95
Ditto, ditto,......	12	2	$4\frac{1}{2}$	62		3..6			2..	3.75	8.8	11.
Steam Engine, by B. & W.	14	3	5	64	25	5..1	29	55	2..4	3.57	6.65	7.91
Steam Engine,...	20	$2\frac{1}{2}$	5	90	18	5..11	38	42.63	2..7	2.5	5.57	4.64
Steam Engine, by B. & W.	24	$3\frac{1}{4}$	6	96	19	8..0	42	43.32	3..6	2.5	7.95	6.625
Ditto, ditto,......	32	3	6	116	19	8..10				1.87	8.78	5.47
Ditto, ditto,......	46	3	8	152	$17\frac{1}{2}$		54	50		1.7	11.	6.2

[1] The only defect in this geering, which has been 16 years at work, is the want of breadth in the spur-wheel and pinion: they ought to have been 6 inches or more, as they will not last half so long as the bevel-wheels and pinions connected with them.

[2] Has been 16 years at work. The teeth were much worn.

[3] Has been 16 years at work. This geering was found rather too narrow for the strain, as it is wearing much faster than the rest of the wheels in the same mill.

[4] This wheel has wooden teeth, and has been working for three years.

[5] Ditto.

[6] This is a better pitch for the power than the following.

[7] This pitch has been found to be too fine.

Explanation of the Table of Wheels actually used in Millwork.

The wheels are all reduced to what may be called one denomination,—

1*st*, By proportioning all their breadths to what they should be to have the same strength, if the resistance were equal to the work of a steam-engine of ten horses' power.

2dly, By supposing their pitch-lines all brought to the same velocity of three feet per second, and proportioning their breadths accordingly. This particular velocity of three feet per second has been chosen, because it is the velocity very common for overshot wheels.

Such cases as appear to have worn too rapidly, are marked, which may tend to discover the limit in point of breadth.

The following table of pitches of wheels was drawn up by Mr. John Roberton, engineer, and is constructed in the following manner :—

The thickness of the teeth in each of the lines is varied one tenth of an inch. *The breadth of the teeth* is always four times as much as their thickness. *The strength* of the teeth is ascertained *by multiplying the square of their thickness into their breadth*, taken in inches and tenths, &c. The *pitch* is found *by multiplying the thickness of the teeth by* 2. 1. The number that represents the strength of the teeth, will also represent the number of horses' power, at a velocity of about four feet per second. Thus in the table where the *pitch is* 3.15 *inches, the thickness* of the teeth 1.5 *inches*, and the *breadth* 6. *inches*, the strength is valued at 13½ *horses' power*, with a *velocity of four feet per second* at the pitch line.

A Table of Pitches of Wheels, with the breadth and thickness of the Teeth, and the corresponding number of horses' power, moving at the pitch line at the rate of three feet, of four feet, of six feet, and of eight feet per second.

Pitch in inches.	Thickness of teeth in inches.	Breadth of teeth in inches.	Strength of teeth, or number of horses' power at 4 feet per second.	Horses' power at 3 feet per second.	Horses' power at 6 feet per second.	Horses' power at 8 feet per second.
3.99	1.9	7.6	27.43	20.57	41.14	54.85
3.78	1.8	7.2	23.32	17.49	34.98	46.64
3.57	1.7	6.8	19.65	14.73	29.46	39.28
3.36	1.6	6.4	16.38	12.28	24.56	32.74
3.15	1.5	6.	13.5	10.12	20.24	26.98
2.94	1.4	5.6	10.97	8.22	16.44	21.92
2.73	1.3	5.2	8.78	6.58	13.16	17.34
2.52	1.2	4.8	6.91	5.18	10.36	13.81
2.31	1.1	4.4	5.32	3.99	7.98	10.64
2.1	1.0	4.	4.0	3.0	6.0	8.0
1.89	.9	3.6	2.91	2.18	4.36	5.81
1.68	.8	3.2	2.04	1.53	3.06	3.08
1.47	.7	2.8	1.37	1.027	2.04	2.72
1.26	.6	2.4	.86	.64	1.38	1.84
1.05	.5	2.	.5	.375	.75	1.

CHAP. III.

ON THE NATURE OF FRICTION, AND THE METHOD OF DIMINISHING ITS EFFECTS IN MACHINERY.

The term friction in mechanics, is employed to denote the obstruction to motion which arises from one surface rubbing upon another. If we place a heavy body upon a surface perfectly level, it is not in a state of equilibrium between all the forces which act upon it, otherwise it would move by the application of the smallest force, in a direction parallel to the plane. Its friction upon the level surface is the unbalanced force which occasions this want of perfect equilibrium; and if a new force, of equal magnitude, is applied so as to balance that force in any given direction, the body will obey the least impulse in that direction, and the force thus employed will be an exact measure of the retarding force of friction. "Friction," as Mr. Playfair has justly remarked, "destroys, but never generates motion, and in this is unlike gravity, or any of the forces hitherto considered, which, if they retard motion in one direction, always accelerate it in the opposite." "Though friction destroys motion, and generates none, it is of essential use in mechanics. It is the cause of stability in the structure of machines, and is necessary to the exertion of the force of animals. A nail, or screw, or a bolt, could give no firmness to the parts of a machine, or of any other structure, without friction. Animals could not walk, or exert their force anyhow, without the support which it affords. Nothing could have any stability but in the lowest possible situation; and an arch which could sustain the greatest load, when properly distributed, might be thrown down by the weight of a single ounce, if not placed with mathematical exactness at the very point which it ought to occupy."

The resistance which friction generates in the communicating parts of machinery is so powerful, and the consequent defalcation from the impelling power is so great, that a knowledge of its nature and effects must be of the highest importance to the philosopher and the practical mechanic. The theory of mechanics must continue imperfect till the nature and effects of

friction are thoroughly developed, and their performance must be comparatively small, and the expense of their erection and preservation comparatively great, till some effectual method is discovered for diminishing that retardation of the machine's velocity, and that decay of its materials which arise from the attrition of the connecting parts. The knowledge, however, which has been acquired concerning this abstruse subject has not been commensurate with the labours of philosophers, and the progress of other branches of mechanical science; and those contrivances which ingenious men have discovered for diminishing the resistance of friction, have either been overlooked by practical inquirers, or rejected by those vulgar prejudices which prompt some of the mechanics of the present day to persist in the most palpable errors, and neglect such maxims of construction as are authorized both by theory and experience. It may be proper, therefore, in a work like this, to give a summary view of the opinions of different philosophers upon the nature of friction, and the means which may be adopted for diminishing its effects.

M. Amontons was the first philosopher who favoured us with any thing like correct information upon this subject. He found that the resistance opposed to the motion of a body upon a horizontal surface was exactly proportional to its weight, and was equal to *one third* of it, or more generally to one third of the force with which it was pressed against the surface over which it moved. He discovered also that this resistance did not increase with an increase of the rubbing surfaces, nor with the velocity of its motion.[1]

The experiments of M. Bulfinger authorised conclusions similar to those of Amontons, with this difference only, that the resistance of friction was equal only to *one fourth* of the force with which the rubbing surfaces were pressed together.[2]

This subject was also considered by Parent, who supposed that friction is occasioned by small spherical eminences in one surface being dragged out of corresponding spherical cavities in the other, and proposed to determine its quantity by finding the force which would move a sphere standing upon three equal

[1] *Mem. de l'Acad. Par.* 1699, p. 206. Amonton's experiments were confirmed by Bossut and Belidor. See *Architect. Hydraulique*, vol. i, chap. ii, p. 70.

[2] *Comment. Petropol.* tom. ii, p. 40.

spheres. This force was found to be to the weight of the sphere as 7 to 20, or nearly one third of the sphere's weight.[3] In investigating the phenomena of friction, M. Parent placed the body upon an inclined plane, and augmented or diminished the angle of inclination till the body had a tendency to move; and the angle at which the motion commenced, he called the angle of equilibrium. The weight of the body, therefore, will be to its friction upon the inclined plane, as radius to the sine of the angle of equilibrium, and its weight will be to the friction on a horizontal plane, as radius to the tangent of the angle of equilibrium.[4]

The celebrated Euler seems to have adopted the hypothesis of Bulfinger respecting the ratio of friction to the force of pressure; and in two curious dissertations which he has published upon this subject,[5] has suggested many important observations, which have been of great use to future inquirers. He observes, that when a body is in motion, the effect of friction will be only one half of what it is when the body has begun to move; and he shews that if the angle of an inclined plane be gradually increased, till the body which is placed upon it begins to descend, the friction of the body at the very commencement of its motion will be to its weight or pressure upon the plane, as the sine of the plane's elevation is to its co-sine, or as the tangent of the same angle is to radius, or as the height of the plane is to its length. But when the body is in motion, the friction is diminished, and may be found by the following equation $F = \text{Tan.}\ a \dfrac{m}{15625\, n\, n \cos.\ a,}$ in which F is the quantity of friction, the weight or pressure of the body being $= 1$; a the angle of the plane's inclination, m the length of the plane in 1000th parts of a Rhinland foot, and n the time of the body's descent. Respecting the cause of friction, Euler is nearly of the same opinion with Parent; the only difference is, that instead of regarding the eminences and corresponding depressions as spherical, he supposes them to be angular, and imagines the friction to arise from the bodies ascending a perpetual succession of inclined planes.

[3] *Recherches de Mathematique et Physique*, 1713, tom. ii, p. 462.

[4] *Mem. de l'Acad. Par.* 1704, p. 174.

[5] The first is entitled, *Sur le frottement des Corps solides*, and the other, *Sur la diminution de la resistance du frottement*, published in the *Mem. de l'Acad. Berlin*, ann. 1748, pp. 122–133.

Mr. Ferguson found that the quantity of friction was always proportional to the weight of the rubbing body, and not to the quantity of surface, and that it increased with an increase of velocity, but was not proportional to that increase. He found also that the friction of smooth soft wood, moving upon smooth soft wood, was equal to $\frac{1}{3}$ of the weight; of rough wood upon rough wood $\frac{1}{2}$ of the weight; of soft wood upon hard, or hard upon soft, $\frac{1}{5}$ of the weight; of polished steel upon polished steel or pewter $\frac{1}{4}$ of the weight; of polished steel upon copper $\frac{1}{5}$, and of polished steel upon brass $\frac{1}{6}$, of the weight.[6]

The Abbé Nollet[7] and Bossut[8] have distinguished friction into two kinds; that which arises from one surface being dragged over another, and that which is occasioned by one body rolling upon another. The circumference of a wheel rolling on the ground is an example of the first of these, and the friction of the axle of a wheel in motion is a combination of the two kinds of friction. M. Bossut agrees with Amontons in his opinion, that an increase of surface does not occasion an increase of friction. He took a rectangular parallelopiped of wood, weighing 51 lbs. and having dragged it over a horizontal table, and loaded it with different weights, he found, that though one of its surfaces was *five* times greater than the other, the same force was capable of putting the body in motion, whether it rested on the large or the small base. Muschenbroek, and other writers, maintained that the friction increased with the surface.

Bossut has noticed two very important facts, viz. that the friction is affected by the time in which the surfaces remain in contact, and that it does not follow exactly the ratio of the pressures. He found that when the surfaces had been for some time in contact, their friction increased either in consequence of a greater number of eminences having entered into the corresponding cavities from a continuance of the pressure, or from some physical cause which united the two surfaces more firmly together. Bossut likewise noticed, that in large masses the friction is a less part of the pressure than in small masses; but he does not seem to have observed, that this arose from the

[6] *Tables and Tracts*, edit. 2d, p. 289.
[7] Nollet, *Lecons de Physique*, tom. iii, p. 231, ed. 1770.
[8] *Traité Elementaire de Mecanique*. par Bossut, § 306–7.

greater velocity which the mass derived from its magnitude. "Ship-builders," he observes, "give only a declivity of from 10 to 12 lines in a foot to the inclined planes upon which vessels are launched. But this declivity, which is sufficient for putting large masses in motion in spite of the resistance of friction, is too small for weights of a moderate size." Bossut, however, seems to have suspected, that friction might diminish as the velocity increased, when he says, "that if it happens on one hand, that in proportion as the velocity increased, there are more points to disengage, or more springs to bend; yet it may happen, on the other hand, that this same velocity does not give to the pressure time to permit the points to enter the cavities so deeply as they would be allowed to do if the velocity were less. But a diminution of this depth ought to produce a diminution of friction.[9]

Result of Vince's experiments. The subject of friction has more lately occupied the attention of the learned Mr. Vince of Cambridge. He found that the friction of hard bodies in motion is an uniformly retarding force, and that the quantity of friction, considered as equivalent to a weight drawing the body backwards, is equal to $M \frac{\overline{M \times W \times S},}{r\, t^2}$ where M is the moving force expressed by its weight, W the weight of the body upon the horizontal plane, S the space through which the moving force or weight descends in the time t and $r = 16.087$ feet, the force of gravity. Mr. Vince also found that the quantity of friction increases in a less ratio than the quantity of matter or weight of the body, and that the friction of a body does not continue the same when it has different surfaces applied to the plane on which it moves, but that the smallest surfaces will have the least friction.[1]

Experiments of Coulomb. Notwithstanding these various attempts to unfold the nature and effects of friction, it was reserved for the celebrated Coulomb to surmount the difficulties which are inseparable from such an investigation, and to give an accurate and satisfactory view of this complicated part of me-

[9] *Traité de Mecanique*, Part I, chap. iv, § 1, p. 178, edit. 1802. These observations of Bossut were first published in 1763, and with some slight additions in 1775, before the appearance of Coulomb's *Researches*.

[1] *Philosophical Transactions*, vol. lxxv, p. 167.

chanical philosophy. By employing large bodies and ponderous weights, and conducting his experiments on a large scale, he has corrected several errors which necessarily arose from the limited experiments of preceding writers; he has brought to light many new and striking phenomena, and confirmed others which were hitherto but partially established. As it would be foreign to the nature of this work to follow Coulomb through his numerous and varied experiments, we shall only present the reader with a general abstract of the new and interesting results which they authorise.[2]

The apparatus which M. Coulomb employed in his experiments, consists of a solid table (Plate VI, Fig. 1), sustained by legs of great strength. The plank $c\,c'\,d\,d'$ (Fig. 1,) and $d\,d'$ (Fig. 2), of which the table is formed, is 8 feet long, 2 feet wide, and 3 inches thick. Upon this table are placed two pieces of oak $A\,B$, $A'\,B'$, 12 feet long and 8 inches thick, lying in the direction of the table's length, and three inches distant from one another. In the space between the two extremities B, B' of these pieces is placed a pulley H, of very hard wood, 1 foot in diameter, and weighing 14 lbs. This pulley turns upon an axis of green oak 10 lines in diameter A cord, attached to a sledge S (Fig. 2), which runs upon the table, passes over this pulley and suspends a scale P (Fig. 1), for containing weights, which can descend into a pit O 4 feet deep, cut out beneath the pulley P. At the other end $A\,A'$ of the pieces of wood $A\,B$, $A'\,B'$, is placed a small horizontal wheel and axle, with levers L', L. Another plank $a\,a'\,b\,b'$, 8 feet long, 16 inches wide, and 3 inches thick, is firmly fixed upon the pieces of wood $A\,B$, $A'\,B'$, its upper surface being very carefully planed. Upon this plank, sledges of the form shewn in Fig. 2 No. 1, and Fig. 2 No. 2, are made to slide. $A\,B\,D\,C$ (No. 1) is a plank 18 inches wide, but of variable length. Beneath this plank are nailed two pieces $A\,C\,m\,m'$, $B\,D\,n\,n'$, so that when the sledge is placed upon the fixed plank $a\,a'\,b\,b'$, it may be retained on both sides by these pieces, with a play of 2 or 3 lines, in order that it may follow without any obstruction

[2] A full account of Coulomb's experiments may be seen in the *Journal de Physique* for September and October 1785, vol. xxvii, pp. 206 and 282, &c. An excellent summary of them may also be found in Van Swinden's *Positiones Physicæ*. They were originally published in the *Memoires des Savans Etrangers*, tom. x, p. 163, and obtained the double prize offered by the Academy in 1779 and 1782.

the direction of the plank. When it is required to diminish the touching surfaces, other pieces of different widths are nailed upon the plank $A\,B\,D\,C$, and their ends rounded for receiving the nuts, in order that they may not rub upon the plank. The two hooks h, h', seen in No. 2, as fixed to the two extremities of the sledge, are used, the one for attaching the cord which suspends the pulley P, and the other for fixing the cord which goes round the wheel and axle, which is employed for bringing back the sledge to the side $A\,A'$ of the apparatus.

In some of the experiments a steelyard $a\,b$ (Fig. 2) was employed, and a variable weight obtained by shifting the weight P to or from the fulcrum above a.

1. *Friction of one Wood upon another.*—When the bodies rubbed upon one another *in the direction of the fibres*, after they had remained in contact for *one* or *two* minutes, he obtained the following average results:—

	Friction in parts of the weight.		Friction in parts of the weight.
Oak against oak	$\frac{1}{2.34}$	Fir against fir	$\frac{1}{1.78}$
Oak against fir	$\frac{1}{1.50}$	Elm against elm	$\frac{1}{2.18}$

When oak rubbed upon oak, and when the touching surfaces were reduced to the smallest possible dimensions, the friction was $\frac{1}{2.36}$, $\frac{1}{2.42}$, $\frac{1}{2.40}$.

When the friction was *across the grain, or at right angles to the direction of the fibres*, Oak against oak was $\frac{1}{3.76}$.

The preceding ratios are constant quantities, which do not depend upon the velocities, excepting in the case of elm, when the pressures are very small, for in that case the friction sensibly increases with the velocity.

2. *The Friction increases with the time of Contact.*—When wood was moved upon wood in the direction of the fibres, the friction gradually increased, and reached its *maximum* in 8 or 10 seconds. When the friction was across the grain of the wood, a longer time elapsed before it reached its maximum.

3. *Friction of metals upon metals after a certain time of rest.*—The following experiments were made with two flat rulers of iron 4 feet long and 2 inches wide, attached to the fixed plank of the apparatus. Other four rulers, two of iron and two of brass, 15 inches long, and 18 lines wide were also used. All

the angles of these rulers were rounded off, and the rubbing surfaces of the rulers were 45 square inches.

	Pressure.	Friction in parts of the pressure.		Pressure.	Friction in parts of the pressure.
Iron upon iron	53 lbs.	$\frac{1}{3.5}$	Iron upon brass	52 lbs.	$\frac{1}{4.2}$
	453	$\frac{1}{3.6}$		452	$\frac{1}{4.1}$

In these experiments the friction is nearly the same in each set, although the pressures are to one another nearly as 9 to 1. Hence it follows, that in metals, the friction is independent of the extent of the rubbing surfaces. Coulomb found also, that in this case the friction is independent of the velocities.

The ratio of 4 to 1, between the pressure and friction in the case of iron moving upon brass, must not be considered as exact unless in the case when the surfaces are new and very large. When the surfaces are diminished till they become very small, the ratio varies from 4 to 1 to 6 to 1, but it does not reach this last ratio unless when the friction has been continued for more than an hour, and when the iron and brass have taken the highest polish of which they are susceptible, free of all scratches.

4. *Friction of Oak upon Oak when greased with Tallow, and the Tallow renewed at every experiment.*

	Pressure.	Friction in parts of the pressure.		Pressure.	Friction in parts of the pressure.
Oak against oak	3250 pounds	$\frac{1}{27.6}$	Oak against oak	450 pounds	$\frac{1}{21.5}$
	1650 ...	$\frac{1}{25.8}$		250 ...	$\frac{1}{18.5}$
	850 ...	$\frac{1}{23.6}$		50 ...	$\frac{1}{7.7}$

The very remarkable increase of friction which takes place in the preceding results by a diminution of pressure, is ascribed by Coulomb to the cohesion of the parts of the tallow, and to the extent of the rubbing surfaces. If this cohesion is the principal cause, then it is obvious that the constant resistance which it produces must have had very little effect in Exp. 1*st*; and therefore Coulomb regards the ratio of $\frac{1}{27\cdot6}$, as expressing the real relation between the friction and the pressure in all the rest, but particularly in Exp. 6*th*. Since the friction, therefore, *plus* the cohesion, gives a resistance of 6.5, with a pressure of 50 lbs. or a friction of $\frac{1}{7\cdot7}$, the resistance is composed of a friction $= \frac{50 \text{ lbs.}}{27.6}$ or 1.8 lb. and of a cohesive force = 6.5 lb. —

1.8 = 4.7 lb. Taking the even number 5 lbs. as a measure of this cohesion, we shall have 2.83 for the mean and approximate value of the ratio of pressure to friction, when corrected, for the resistance arising from the cohesion of the tallow.

When the touching surfaces were reduced from their original magnitude of 180 square inches, to *one sixth* or about 30 square inches, the preceding ratio of $\frac{1}{27.6}$ is reduced to $\frac{1}{16}$ or $\frac{1}{17}$. The velocity appeared to have an influence on the results in this kind of friction.

5. *Influence of the time of contact upon the Friction of Oak upon Oak, when greased with tallow renewed at every experiment.* In the following very interesting experiments, the surface of contact was 180 inches.

Pressure.	Time of contact.	Friction in parts of the pressure.
47 pounds,	0 minutes	$\frac{1}{7.7}$
	4 minutes	$\frac{1}{5.87}$
	2 hours	$\frac{1}{5.25}$
1650 pounds,	0 seconds	$\frac{1}{25.8}$
	3 seconds	$\frac{1}{10.3}$
	15 seconds	$\frac{1}{7.9}$
	1 minute	$\frac{1}{5.82}$
	4 minutes	$\frac{1}{5.29}$
	2 hours	$\frac{1}{3.65}$
	6 days	$\frac{1}{2.65}$

Pressure.	Time of contact.	Friction in parts of the pressure.
3250 pounds,	0 seconds	$\frac{1}{27.08}$
	3 seconds	$\frac{1}{10.16}$
	15 seconds	$\frac{1}{9.16}$
	1 minute	$\frac{1}{7.87}$
	4 minutes	$\frac{1}{5.48}$
	1 hour	$\frac{1}{3.7}$
	2 hours	$\frac{1}{3.53}$
	5 days	$\frac{1}{2.66}$
		$\frac{1}{2.09}$

In the preceding experiments the tallow was new, but in the following it had been laid on, and used for some time, and was not renewed during the experiments.

Pressure.	Time of contact.	Friction in parts of the pressure.
2310 pounds,	0 minutes	$\frac{1}{12.3}$
	2 minutes	$\frac{1}{5.9}$
	1 hour	$\frac{1}{5.12}$
	16 hours	$\frac{1}{4.5}$
5810 pounds,	0 minutes	$\frac{1}{11.57}$
	2 minutes	$\frac{1}{7.35}$

Pressure.	Time of contact.	Friction in parts of the pressure.
5810 pounds,	4 minutes	$\frac{1}{6.71}$
	9 minutes	$\frac{1}{6.12}$
	26 minutes	$\frac{1}{5.6}$
	1 hour	$\frac{1}{4.9}$
	16 hours	$\frac{1}{3.8}$

6. *Friction of Iron upon Brass, and Iron upon Iron, when greased.*

BRASS UPON IRON.

Pressure.	Time of contact.	Friction in parts of the pressure.
50 pounds,	0 minutes	$\frac{1}{8.33}$
	4 minutes	$\frac{1}{7.14}$
	30 minutes	$\frac{1}{7.14}$
450 pounds,	0 minutes	$\frac{1}{10.7}$
	4 minutes	$\frac{1}{9.37}$
	2 hours	$\frac{1}{9.37}$

Pressure.	Time of contact.	Friction in parts of the pressure.
1650 pounds,	0 minutes	$\frac{1}{11}$
	3 minutes	$\frac{1}{10.4}$
	4 hours	$\frac{1}{9.8}$
	4 days	$\frac{1}{9.8}$

The excess of the friction under a pressure of 50 lbs. obviously arises from the cohesion of the tallow, which amounted to $1\frac{1}{2}$ pounds; making the friction $\frac{1}{11}$ when the time of contact was 0, and 9.5 when it had reached its maximum.

The following experiments were performed under different circumstances.

In order to prepare the metals for these experiments, which was found absolutely necessary before regular results could be obtained, they were first polished as highly as possible, and after being greased with oil or tallow, they were attached to the sledge, and made to rest upon one another under a great pressure for half an hour, the grease being, from time to time, renewed, till it had penetrated the pores of the metal, and thus given the rulers a degree of polish which they could not have received by any other means. The friction was at first uncertain, but became more regular as the polish increased.

By this operation the grease penetrates the pores of the metal. The size of the rulers and the touching surfaces were the same as in Art. 3, p. 155, the velocities of the sledge being imperceptible, or below an inch in a second. The following were the results:

	Pressure.	Friction in parts of the pressure.
Iron upon iron,	53 pounds	$\frac{1}{6.2}$
	443 ...	$\frac{1}{10.1}$
	1653 ...	$\frac{1}{10.3}$
Iron upon brass,	53 ...	$\frac{1}{8}$
	443 ...	$\frac{1}{10.7}$
	1653 ...	$\frac{1}{11}$

	Pressure.	Friction in parts of the pressure.
Iron upon brass, when the grease was not renewed,	53 pounds	$\frac{1}{8}$
	443 ..	$\frac{1}{8.1}$
	1653 ...	$\frac{1}{7.9}$

The ratio between the pressure and the friction depends, in these experiments, on the nature of the unguent and the velocity of the sledge. By greasing the metallic plates with tallow, the friction diminishes greatly under great pressures, in proportion as the velocity augments. In one of these experiments, when the velocity of the sledge was *one foot in a second*, the friction of the sledge under a pressure of 1652 pounds was more than one-third less than when the velocity was insensible, or even *one inch in a second.*

When the velocities are insensible, and the pressure is great, the ratio of the pressure and the friction is for

Iron against iron, greased with tallow, - -	$\frac{1}{10}$
Iron upon brass, greased with tallow, - -	$\frac{1}{11}$
Iron upon brass, greased with tallow, and afterwards with oil, - - - -	$\frac{1}{8}$.

When the plates are greased with tallow, this ratio is less under a pressure of 52 lbs. than under one much greater. This arises from the cohesion of the tallow, which opposes under all pressures a constant resistance proportional to the area of the surfaces. This constant resistance, which is not perceptible but under small pressures, may be estimated in the same manner as in Art. 4, p. 155. When olive oil is used, the cohesion may be considered as nothing.

When the surfaces are rubbed with hog's lard, the friction was never less than $\frac{1}{9}$ of the pressure. The resistance depends always on the consistence of the grease, and the friction increases as the grease is more soft.

When the surfaces are greased with tallow, and have a large area, the friction alters the nature of the tallow, and it increases sensibly in proportion as the experiments are continued without renewing the grease. It is, however, always less than $\frac{1}{8}$ of the pressure. When the unguent is oil, and the surfaces very small, this effect is less perceptible.

7. *Friction of Iron upon Oak.*—When iron moved upon oak the following were the results :—

Pressure—53 pounds. Time of contact.	Friction in parts of the pressure.	Pressure—1650 pounds. Time of contact.	Friction in parts of the pressure.
½ a second, ...	$\frac{1}{10.6}$	½ a second, ...	$\frac{1}{13.2}$
30 seconds, ...	$\frac{1}{10.1}$	10 seconds, ...	$\frac{1}{12.7}$
60 seconds, ...	$\frac{1}{8.1}$	80 seconds, ...	$\frac{1}{11.28}$
1 hour, ...	$\frac{1}{5.89}$	4 hours, ...	$\frac{1}{8.25}$
4 days, ...	$\frac{1}{5.3}$	16 hours, ...	$\frac{1}{5.9}$
		4 days, ...	$\frac{1}{4.86}$

Hence it appears, that in wood rubbing upon metals, the friction does not reach its maximum till after a long time.

When *brass* rubbed against *oak*, results analagous to the preceding were obtained. The increase of the friction, however, relative to the time, was more slow than in iron ; and when it reached its maximum, the ratio was $\frac{1}{5.5}$.

When the velocity was insensible, and the bodies not greased, the following were the results :—

IRON UPON OAK.

Pressure.	Friction in parts of the pressure.	Pressure.	Friction in parts of the pressure.
53 pounds	$\frac{1}{11.8}$	853 pounds	$\frac{1}{12.7}$
453	$\frac{1}{12.9}$	1653	$\frac{1}{13.2}$

From these results, it appears that in the first degree of velocity the friction of iron upon oak is nearly $\frac{1}{13}$ of the pressure.

When the sledge had the velocity of one foot per second, the following were the results :

Pressure.	Friction in parts of the pressure.	Pressure.	Friction in parts of the pressure.
53 pounds	$\frac{1}{5.9}$	853 pounds	$\frac{1}{5.5}$
453	$\frac{1}{5.8}$	1653	$\frac{1}{6.3}$

From which it follows, that the friction has greatly increased with the velocity, that it is nearly constant for the same velocity whatever be the pressure, and that when the frictions increase in an arithmetical progression, the velocities increase in a geometrical progression. Coulomb also found, that under the same

pressure, and with the same velocities, the friction is nearly the same for large and small surfaces.

7. *Friction of Iron and Copper on Oak when greased with Tallow.*—When the metals rub upon woods greased with fatty bodies, the friction is greatly diminished, and small velocities are produced with less force than in almost any other species of friction; but when the velocities are increased, the friction augments greatly with the velocity, as in the rubbing of dry metals upon wood. The following were some of the results in very small velocities:

	Pressure.	Friction in parts of the pressure.
Oak upon iron ...	1650 pounds	$\frac{1}{35.1}$
Oak upon brass ...	1650	$\frac{1}{47.1}$

In experiments of this kind, care must be taken to remove the grease, otherwise it acquires consistence, and the resistance sensibly increases. As a proof of this, Coulomb caused the sledge, when furnished with plates of brass, to move 15 times over the fixed oaken plank without renewing the tallow. The force of friction was triple that which was employed to give the sledge an insensible velocity when the coat of tallow was new. The velocity of the sledge diminished at every trial, and at the 15th it ceased to move. Hence it follows that the tallow is actually injurious, or increases the friction when the surfaces move a long time without the tallow being renewed.

8. *Friction of Iron upon Oak when greased, and when the Surfaces are very small, and the Friction across the Grain of the Wood.*—When the surfaces were covered with tallow, and were afterwards wiped, in order to be merely greasy, the following were the results:

	Pressure.	Friction in parts of the pressure.
Iron upon oak	47 pounds	$\frac{1}{13.4}$
	447	$\frac{1}{14.9}$
	1647	$\frac{1}{14.3}$

These results were nearly the same, whether the surfaces were well tallowed or merely unctuous; and therefore it follows, that in this kind of friction, which is analogous to that of axles of iron turning upon wood, the ratio of the pressure to

the friction is constant, and is not much influenced by the velocity.

9. *Friction of Oak upon Oak when the Surfaces are in motion.*—In order to examine this kind of friction, Coulomb caused the sledges, when differently loaded, to move over a length of 4 feet, and having observed the ratio of the times employed to describe successively the two halves of this length, he regarded the force of friction as a constant force during the continuance of the experiment, and the motion of the sledge as uniformly accelerated. He supposes also that the friction is constant, and that it does not vary with the velocity.

When oak rubbed upon oak, in the direction of the fibres, and without any grease, the following results were obtained:

OAK UPON OAK SURFACES ONE FOOT SQUARE.

Pressure in pounds upon a square foot.	Friction in parts of the pressure.	Pressure in pounds upon a square foot.	Friction in parts of the pressure.
25 ...	$\frac{1}{5.7}$	25 ...	$\frac{1}{9.4}$
188 ...	$\frac{1}{9.4}$	1788 ...	$\frac{1}{9.2}$
291 ...	$\frac{1}{9.5}$	6588 ...	$\frac{1}{10.4}$

When the touching surfaces were reduced to the smallest possible dimensions, so that the sledge rested only on compressed and rounded angles, the results were,

OAK UPON OAK SURFACES REDUCED ALMOST TO LINES.

Pressure.	Friction in parts of the pressure.
47	$\frac{1}{10.4}$
447	$\frac{1}{12.4}$
847	$\frac{1}{14.6}$

The velocities in both these sets of experiments being varied from nothing to 4 feet in 4 or 5 seconds, the friction was not perceptibly altered.

When the surfaces were very large compared with the pressures, the friction *appeared to augment with the velocities*; but when the touching surfaces were very small, compared with the pressures, the *friction diminished,* ~~or~~ *the velocities in-* and *creased.* In the first experiment of the first of the above tables, the friction became greater than $\frac{1}{5.7}$ when the velocity was increased. In this case Coulomb ascribes the variations to some

cause foreign to friction, and depending on the extent of the surfaces; and he seems to think that the surfaces may be covered with a sort of down, which must be bent during the motion of the surfaces. The resistance produced by this down is independent of the cavities and solid parts which enter into one another, and which occasion a friction proportional to the extent of the surface. If this is the cause which alters the law of friction under small pressures, the friction ought also to be augmented with the velocity, since, under a great velocity, a greater number of the parts which form the down are bent, and this is actually what was found by experiment.

General Results.—From the preceding experiments M. Coulomb has drawn the following general conclusions:

1. The friction of wood upon wood, when ungreased, occasions, after a sufficient period of contact, a resistance proportional to the pressure. This resistance increases sensibly in the first instants of repose, but after some minutes it generally reaches its maximum.

2. When wood, ungreased, moves upon wood, with any velocity whatever, the friction is still proportional to the pressure, but its intensity is much less than that which is experienced in detaching the surfaces after some minutes of rest. The force, for example, which is necessary to detach two surfaces of oak after some minutes of rest, is to that necessary for overcoming the friction when the surfaces have already any degree of velocity whatever, nearly as 9.5 to 2.2.

3. The friction of metals sliding over metals, ungreased, is also proportional to the pressures, but its intensity is the same whether the surfaces detached have been any time at rest, or whether they have received any uniform velocity whatever.

4. Heterogeneous substances, such as those of woods and metals, sliding upon one another without grease, give for their friction very different results from the preceding; for the intensity of their friction relative to the time in which they have continued in contact increases slowly, and does not reach its maximum till after four or five days, and sometimes more, whereas in the metals it reaches its maximum in an instant, and in wood, after a few minutes. This increase is even so slow that the resistance of friction in insensible velocities is nearly the same as that which takes place in moving or detaching the surfaces after three or four seconds of rest. In the

motion of wood rubbing upon wood without grease, and in metals moving upon metals, the velocity has very little influence upon the friction, but in the present case the friction increases very sensibly in proportion as we increase the velocities; so that the friction increases nearly in arithmetical progression as the velocities increase in geometrical progression.

On the Friction of Axles.

The principal object of Coulomb in his experiments on this subject, was to determine the friction of the axles of machines in motion. He employed axes of iron moving in boxes of brass. The iron axis was 19 lines in diameter, and had a play of $1\frac{3}{4}$ lines in the brass box. The pulley in which the axis was fixed was 144 lines in diameter, and its weight 14 lbs. Previous to the experiments, the iron axis was made to work in the brass box till the touching surfaces acquired the highest polish of which they were susceptible. M. Coulomb caused the suspended weights to run through a space of 6 feet, and he measured separately in half-seconds the time employed to run through the first three feet, and the time occupied in running through the last three feet. In this way he obtained the results in the following Table arranged by M. Prony:

Table of Coulomb's Experiments on the Friction of Axles.

Number of experiments.	Kind of cord used.	Kind of greasing.	Weight used to bend the cord over the pulley.	Weight hung on each side of the pulley.	Additional weight to move the pulley.	Motion of the weight suspended on each side of the pulley.	Pressure on the axis.	Friction reduced to surface of axis.	Ratio of friction to the pressure.
1	Very flexible thread, 3 lines circumference	Friction without greasing	0.0	103	6	Slow and irregular.	226	42	0.186
2	Cord No. 1, of 6 threads in a yarn.	Idem.	1.5	200	10.5	Slow and irregular.	424	65	0.153
					13.5	First 3 ft. fallen thro' in 6″; last 3 in 3″.			
3	Idem.	Idem.	3.0	400	21	Slow but continual.	825	130	0.156
					28	1st 3 ft. described in 5″.5; last 3 in 2″.5.			
					39	1st 3 ft. described in 3″, last 3 in 1½″.			
4	Very flexible thread, 2 lines circumference	Tallow.	0.0	100	2.5	Slow but continual.	216.5	17.5	0.081
					6	1st 3 ft. desc. in 3″.5, the last 3 in 1″.5.			
5	Cord No. 1, of 6 threads in a yarn.	Idem.	1.5	200	6.5	Slow but continual.	420	36	0.086
					10.0	1st 3 ft. desc. in 3″.5, the last 3 in 1″.5.			
6	Idem.	Idem.	3.0	400	13	Slow and continual.	827	72	0.087
					18	1st 3 ft. desc. in 5″.5, the last 3 in 2″.			
					24	1st 3 feet in 3″, the last 3 in 2″.			
7	Thread of 2 lines in circumference	Cart grease.	0.0	50	2.5	Slow and continual.	117	17.5	0.15
8	Idem.	Idem.	0.0	100	3.7	Idem.	218	26	0.119
9	Idem.	Idem.	0.0	150	5.7	Idem.	320	40	0.125
10	Cord No. 1, of 6 threads in a yarn.	Idem.	0.7	100	4.3	Slow and uncertain.	218	26	0.119
					9	1st 3 feet in 3″, the last 3 in 1½″.			
11	Idem.	Idem.	1.5	200	8.5	Uncertain.	422	50	0.118
					14	1st 3 feet in 4″, the last 3 in 2″.			
					20	All 6 feet in 3″.5.			
12	Idem.	Idem.	3.0	400	17	Uncertain.	831	101	0.121
					22	1st 3 feet in 6″.5, the last 5 feet in 2″.5.			
					28	1st 3 feet in 4″, the last 3 feet in 1″.			
13	The cart-grease of preceding experiment wiped, the pores of the metal remained unctuous.				...	... ...	200 to 1200		0.127
14	The surface fresh done with oil.				...	... ...	...		0.127 0.133
15	The greasing not renewed for a long time, though the machine had been much used.				...	... ...	...	...	0.133

The weights in column 4, which were employed to bend the cord, were computed from the numbers in column 5, by means of the formulæ, and the results of some previous experiments. The weights in column 4 being subtracted from those in column 6, which put the pulleys in motion, give the weights actually employed in overcoming the friction. As these last weights acted at the end of a lever equal to the radius of the pulley, *plus* the radius of the cord, the friction upon the axis may be regarded as equal to the product of these weights into the ratio of the sum of the radii of the pulley and the cord to the radius of the axis, which is nearly that of 7 to 1 when the weight is suspended by a thin packthread, and nearly 7.2 to 1, when it is suspended by the cord No. 1. By this method the frietion in column 9 was calculated. The pressures on the axis in column 8 are obviously composed, 1*st*, Of the weight of the pulley or cylinder; 2*d*, Of double the corresponding weight in column 5; and 3*d*, Of the weights contained in column 6. The ratio therefore of the friction to the pressure in column 10 is the quotient arising from dividing column 9 by column 8.

It appears from experiments 7th, 8th, 9th, 10th, 11th, and 12th columns, that the friction of axes of iron in boxes, or cheeks of brass, is much less diminished by the cart-grease than by tallow.

M. Coulomb likewise extended his experiments to the friction of axles made of the different kinds of wood used in rotatory machines. He used pulleys of 12 inches in diameter, with axes three inches in diameter. The axes were sometimes moveable, and at other times fixed, though in both cases the friction was the same. The levelling surfaces were carefully smoothed.

Names of the wood used for the axles.	Ratio of friction to pressure.
1. Axis of *holm oak* running in a box of lignum vitæ coated with tallow,	$\frac{1}{26.3}$
2. Do. the coating of tallow wiped off, and the surface remaining greasy,	$\frac{1}{16.7}$
3. Do. after being used several times without renewing the tallow,	$\left\{\begin{array}{l}\frac{1}{16.7}\\ \frac{1}{12.5}\end{array}\right.$
4. Do. running in a box of *elm* coated with tallow, - - -	$\frac{1}{33.3}$
5. Do. both axis and box wiped, and the surfaces remaining greasy,	$\frac{1}{20}$
6. Axis of *boxwood*, and box of *lignum vitæ*, coated with tallow, -	$\frac{1}{23}$

Names of the wood used for axles.	Ratio of friction to pressure.
7. Do. the coating wiped, and the surfaces remaining greasy, -	$\frac{1}{14.3}$
8. Axis of *boxwood*, and box of *elm*, coated with tallow, - -	$\frac{1}{28.6}$
9. Do. the coating wiped off, and the surfaces remaining greasy, -	$\frac{1}{20}$
10. Axis of *iron*, and box of *lignum vitæ*, the coating wiped off, and the pulley turned for some time,	$\frac{1}{20}$

In these experiments the velocity did not appear to influence the friction unless in the first instants of rest; and in all cases the friction was least when the surfaces were merely greasy, and not coated with tallow.

On the Friction and Rigidity of Ropes.

When a rope passes over a cylinder, or over the groove of a pulley, whose axis is horizontal, and has a weight suspended at each end of it, then it is obvious that if the rope were destitute of rigidity, or perfectly flexible, the axis of the ropes on each side would be perfectly parallel and vertical. But as every rope is stiff or rigid, the two branches of it that suspend the weights will deviate from the perpendicular. If one of the ends of the rope is fixed in such a manner that the first branch of the rope is in a vertical direction, and if we suspend, at the other end of the rope, a weight W, which will stretch the second branch of the rope to such a degree as to force it into a vertical direction, and to bring it in contact with a semicircumference of the pulley, the weight W *may be taken as the measure of the rigidity of the rope.*

If the rope, instead of being treated in this manner, has two equal weights, W, W, suspended at each extremity, and if we add to one of the ends an additional weight w, capable of destroying the equilibrium of the equal weights, W, W, this weight may be taken as a measure of the friction of the rope. A small portion indeed of the weight w is employed in overcoming the rigidity of the rope; but it is so small, when compared with w, that it may be safely neglected.

The first experiments that appear to have been made on the rigidity of ropes, were those of Amontons, who contrived an ingenious apparatus for this purpose. He has published, in the *Memoirs of the Academy of Sciences* for 1699, a table of the forces required to bend ropes, founded on the supposition that the difficulty of bending a rope of the same thickness, and

loaded with the same weight, "decreases when the diameter of the roller or pulley increases, but not so much as that diameter increases."

Another series of experiments was afterwards made by Desaguliers, (*See Course of Natural Philosophy*, vol. i, p. 243, 244, 245, &c.) who published a table, "shewing what forces were required to bend ropes of different diameters, stretched by different weights round rollers of different bignesses." The general result of these experiments was, *that the difficulty of bending a rope round a roller, is ceteris paribus* inversely as the diameter of the roller.

The experiments on the rigidity and friction of ropes made by Coulomb, were performed, both with the apparatus used by Amontons, and by another of his own. It consists of two tressels 6 feet high, on which are laid two pieces of square wood. On these two pieces are fixed two rulers of oak, well planed, and polished with fish-skin. With two cylinders of lignum vitæ, one 6 inches in diameter, and the other 2 inches; and, with several cylinders of elm, from 2 to 12 inches in diameter, the apparatus was ready for the experiments.

In order to ascertain the friction of the rollers, they were laid on the planks, and a weight of 50lbs. was suspended on each side of the roller, with very fine and flexible pack-thread; and the weights could be increased in any degree by laying additional weights of 50lbs. by different threads, so as to give any required pressure to the rollers. By adding a counterweight on each side of the roller alternately, till they received a motion barely sensible, Coulomb ascertained the friction of the rollers. The following were the results with rollers of lignum vitæ:

Pressure upon the rollers.	Weights which produced an extremely slow motion.	
	Roller of 6 *in. diam.*	*Roller of* 2 *in. diam.*
100	0.6 lb.	1.6
500	3.0	9.4
1000	6.0	18.0

Hence it follows, *that the friction of cylinders rolling upon horizontal planes are directly as the pressures, and inversely as the diameters of the rollers.*

Coulomb found that greasing the ropes did not diminish the friction sensibly. He found also, that rollers of elm had a friction about two-fifths greater than lignum vitæ; and that under small pressures the friction was rather greater than would result from its being proportional to the pressure.

The rigidity of ropes was measured in the way already described, and the following results were obtained. The rollers varied from 2 to 12 inches in diameter, and the pulley ropes were used.

	Threads in a yarn.	Threads in a strand.	Circumference.	Weight of a foot.
No. 1.	6	2	$12\frac{1}{2}$ lines.	$4\frac{1}{2}$ drs.
No. 2.	15	5	20	$12\frac{1}{2}$
No. 3.	30	10	28	$24\frac{1}{2}$

After making a series of experiments in the case of motions nearly insensible, Coulomb proceeded to ascertain the effect produced by the rigidity of the cords as changed by the velocity. With this view, he took a pulley, and a box of copper, and an axis of iron, coated with tallow. The pulley was 144 lines in diameter, and the axis $20\frac{1}{2}$ lines, and the cord was No. 3, the same as that used in the above experiment. The weights were made to run above a distance of 6 feet, and the times of describing the first three and the last three feet, were measured by a half-seconds pendulum.

The following are the general results of both sets of experiments:—

1. The rigidity of ropes increases, the more that the fibres of which they are composed are twisted.

2. The rigidity of ropes increases in the duplicate ratio of their diameters. According to Amontons and Desaguliers, the rigidity increases in the simple ratio of the diameters of the ropes; but this probably arose from the flexibility of the ropes which they employed.

3. The rigidity of ropes is in the simple and direct ratio of their tension.

4. The rigidity of ropes is in the inverse ratio of the diameters of the cylinders round which they are coiled. This result was obtained also by Desaguliers.

5. In general, the rigidity of ropes is directly as their tensions and the squares of their diameters, and inversely as the diameters of the cylinders round which they are coiled.

6. The rigidity of ropes increases so little with the velocity of the machine, that it need not be taken into the account when computing the effects of machines.

7. The rigidity of small ropes is diminished when penetrated with moisture; but when the ropes are thick, their rigidity is increased.

8. The rigidity of ropes is increased, and their strength di-

minished, when they are covered with pitch; but when ropes of this kind are alternately immersed in the sea and exposed to the air, they last longer than when they are not pitched. This increase of rigidity, however, is not so perceptible in small ropes as in those whichare pretty thick.

9. The rigidity of ropes covered with pitch is a sixth part greater during frost than in the middle of summer, but this increase of rigidity does not follow the ratio of their tensions.

10. The resistance to be overcome in bending a rope over a pulley or cylinder may be represented by a formula composed of two terms. The first term $\frac{a D^n}{r}$ is a constant quantity independent of the tension, a being a constant quantity determined by experiment, D^n a power of the diameter D of the rope, and r the radius of the pulley or cylinder round which the rope is coiled. The second term of the formula is $T \times \frac{b D^n}{r}$, where T is the tension of the rope, b a constant quantity, and $D n$ and r the same as before. Hence the complete formula is $\frac{a D^n}{r} + T \times \frac{b D^n}{r} = \frac{D^n}{r} \times a + T b$. The exponent n of the quantity D diminishes with the flexibility of the rope, but is generally equal to 1.7 or 1.8; or, as in No. 2, the rigidity is nearly in the duplicate ratio of the diameter of the rope. When the cord is much used, its flexibility is increased, and n becomes equal to 1.5 or 1.4.*

On the Friction and Form of Pivots.

The needles of compasses are generally suspended upon a pivot, by means of caps of agate, or other hard substances. The cap has a conical form, terminated above with a small concave summit, whose radius of curvature is very small. The pivots themselves are commonly of tempered steel, but frequently reduced to the state of a spring. The point of the pivot which supports the cap is a small curve surface, whose radius of curvature is smaller than that of the summit of the cap. Coulomb generally found, that even when every care was

* A drawing of the apparatus employed, and tabular views of all the circumstances and results of the experiments, will be found in the *Edinburgh Encyclopædia*, Art. *Mechanics*, vol. xiii, p. 599, 600.

taken by the artist, the curvature of the summit of the cap was very irregular, and that the friction of an agate cap, turning upon the point of a pivot, was often five or six times greater than the momentum of friction of a highly polished agate plane turning on the same pivot.

In his experiments on pivots, Coulomb supported the body by a highly polished plane in place of a cap, and having given it a rotatory motion, he noted the time employed in making the *four* or *five* first turns, from a mean of which he obtained the primitive velocity; and he next counted the number of turns which it made before it stopped. The revolving body is obviously brought to rest by the friction of the point of the pivot, and also by the resistance of the air; but in order to get rid of this last resistance, Coulomb gave the body the form of a glass receiver, and found, that when the velocity was not great, and when the receiver weighed 5 or 6 gros (a gros is the eighth of an ounce), the resistance of the air bore no sensible ratio to that of the friction.

In order to render the results more certain, he made several of the experiments in vacuo by an apparatus protected from currents of air by being placed under a receiver.

With the apparatus, which consisted of a glass receiver 4 inches in diameter, 5 inches high, and weighing 5 ounces, Coulomb obtained the following results :—

	Time of making one revolution, in seconds.	Number of revolutions before it stopped.	Friction.
Experiment 1.	4″	$34\frac{1}{10}$ turns.	$\frac{1}{547}$
Experiment 2.	$6\frac{1}{4}$	$14\frac{1}{10}$	$\frac{1}{550}$
Experiment 3.	11	$4\frac{6}{10}$	$\frac{1}{557}$

By calculating the friction from a formula which he had previously given in the Memoirs of the Academy for 1779, p. 451, Coulomb found that the friction of pivots is independent of the velocity, and is therefore necessarily proportional to some function of the pressure.*

In order to examine the friction of pivots, when they support planes of different materials, M. Coulomb made the following experiments. The angle of the summit of the pivot which supported the planes was about 18 or 20 degrees.

* The reader will find this formula and a drawing of the apparatus in the *Edinburgh Encyclopædia*, Art. *Mechanics*, vol. xiii, p. 601.

	Time of making one revolution.	Number of revolutions before it stopped.	Friction.
Garnet plane highly polished.	12″	7	$\frac{1}{1008}$
	23	2	$\frac{1}{1050}$
Agate plane highly polished.	9	$10\frac{1}{2}$	$\frac{1}{851}$
	15	$3\frac{1}{2}$	$\frac{1}{844}$
Rock crystal plane highly polished.	13	$4\frac{5}{8}$	$\frac{1}{781}$
	$14\frac{1}{2}$	$3\frac{3}{4}$	$\frac{1}{787}$
Glass plane highly polished.	$8\frac{3}{4}$	$7\frac{1}{2}$	$\frac{1}{570}$
	$4\frac{1}{4}$	$2\frac{9}{10}$	$\frac{1}{589}$
Steel plane tempered and polished.	17	$1\frac{3}{4}$	$\frac{1}{510}$
	8	$7\frac{1}{4}$	$\frac{1}{464}$

Hence it follows, that *Garnet* is far superior to the other substances for the caps of pivots.

In order to ascertain the effect produced by giving different forms to the pivot of the needle, Coulomb made the following experiments, the circumstances being the same as in the preceding ones.

Nature of the planes.	Friction. Angle of pivot 45 deg.	Angle of pivot 6 or 7 deg.
Garnet,	$\frac{1}{2500}$	
Agate,	$\frac{1}{2100}$	$\frac{1}{800}$
Glass,	$\frac{1}{1400}$	$\frac{1}{450}$
Steel,	$\frac{1}{2000}$	$\frac{1}{230}$

Hence for a plate of agate we have

	Friction.
For a pivot of 45°	$\frac{1}{2100}$
15°	$\frac{1}{1200}$
6°	$\frac{1}{800}$

From these experiments, it appears that in *garnet*, *agate*, and *glass*, under a pressure of $5\frac{1}{3}$ gros, the friction *increases as the pivots become more acute*, and follows nearly the same ratio. The case, however, is different with steel. In agate and steel, the frictions upon a pivot of 45° are nearly equal, being $\frac{1}{2100}$ for agate, and $\frac{1}{2000}$ for steel; whereas, on a pivot of 6° or 7°,

the friction for agate is $\frac{1}{800}$, and $\frac{1}{230}$ for steel. Coulomb ascribes this difference to the irregular contexture of metals, and particularly steel, which he says is covered with an infinity of small pores, even when it has the finest polish. He conceives, therefore, that the increase of friction in the steel arises from the sharp point of the pivot working in these irregular pores.

When the weight of the needle, or body supported on the pivot, is under 100 grains, very little advantage is gained by giving the pivots a greater angle than 18° or 20°. When the needles are very light, the angle of the pivot should be less than 18°.

As the preceding results are not applicable to all degrees of pressure, Coulomb repeated the experiments by varying the weights. He used also a small plane of highly polished glass, and then a plane of garnet, and made the angle of the pivot about 45°. The fork always weighed $1\frac{1}{3}$ gros.

Load upon the pivot in gros.	Time of performing one revolution.	Number of revolutions before it stopped.	Friction.
	With a Plane of Polished Glass.		
3.33	24	2	$\frac{1}{1152}$
	14	$5\frac{3}{4}$	$\frac{1}{1127}$
	10	$11\frac{3}{4}$	$\frac{1}{1175}$
16.66	9	$10\frac{1}{4}$	$\frac{1}{830}$
	13	$4\frac{3}{4}$	$\frac{1}{802}$
	With a Plane of Garnet.		
5.33	...	...	$\frac{1}{2400}$
16.66	...	...	$\frac{1}{1550}$

It is a remarkable circumstance, that Coulomb found the results much more regular with glass than either with garnet or agate. Hence, as the friction was likewise greater, he preferred it for the preceding experiments.

From these experiments, Coulomb has shewn, that the exponent of the power of the pressure to which the friction is proportional, is 1.333, or $\frac{4}{3}$.

The following is a recapitulation of Coulomb's results :

1*st*, The friction of pivots is independent of the velocities, and has a relation to the pressure.

2*d*, The friction of garnet is less than that of agate, and that

of agate less than that of glass; but the friction of different parts of a plane of polished glass is less irregular.

3d, The angle of the points of pivots has an influence on the friction. When the body weighs five or six gros, the best angle is from 30° to 45°. When the body weighs less, the angle of the pivot may be progressively diminished without the friction experiencing any sensible increase. We may even, without much inconvenience, and when the steel is good, reduce it to 10 and 12 degrees, provided the weight of the body does not exceed a hundred grains.

4th, With a pivot of the best steel, well tempered and brought to the first degree of steel temper, and having an angle of 45 degrees, the momentum of friction varies as the $\frac{4}{3}$d power of the pressure. When the pressure was very considerable, and the pivot shaped to any angle, the friction varied nearly as the pressure.

5th, All the caps which Coulomb procured from the best workmen, appeared to be very irregular in their concavity. The momentum of their friction, under pressures of even more than five or six grains, is always much more considerable, and sometimes triple and quadruple of that of a well-polished plane of the same substance; and in order to support these caps, it is necessary that the points of the pivots be shaped to an angle less than that which is necessary to support planes.

On the Methods of diminishing Friction in Machinery.

The experiments above detailed will furnish the practical mechanic with various rules respecting the nature and form of the materials which should form the supports and the communicating parts of machines, respecting the nature of the unguents which should be applied to them, and the mode of their application.

The most efficacious contrivances for diminishing friction are *friction wheels*, by means of which, that species of friction which arises from one body being dragged over another, is changed into that which arises from one body rolling upon another. Friction wheels are nothing more than two wheels having their planes nearly touching each other, and the distance of their axes a little greater than their radius. The axle whose friction is to be relieved, is then placed in the angle formed by their circumferences.

When the moving force is not exerted in a perpendicular direction, but obliquely as in undershot wheels, the gudgeon will press with greater force on one part of the socket than on any other part. This point will evidently be on the side of the bush opposite to that where the power is applied; and its distance from the lowest point of the socket, which is supposed circular and concentric with the gudgeon, being called x, we shall have tang. $x = \frac{H}{V}$, that is, the tangent of the arch contained between the point of greatest pressure and the lowest point of the bush, is equal to the sum of all the horizontal forces, divided by the sum of all the vertical forces and the weight of the wheel, H representing the former, and V the latter quantities. The point of greatest pressure being thus determined, the gudgeon must be supported at that part by the largest friction wheel, in order to equalize the friction upon their axles.

Friction wheels seem to have been first recommended by Casatus. They were afterwards mentioned by Sturmius and Wolfius, but were not in actual use till Sully applied them to clocks in 1716, and Mondran to cranes in 1725. They remained, however, almost unnoticed till the celebrated Euler explained their nature and advantages in his Memoirs on Friction.

Another method of diminishing friction is to apply the impelling power (when it can be done) in such a way as to act either in opposition to, or obliquely to the force of gravity.

If we suppose, for example, that the weight of a wheel, whose iron gudgeons move in bushes of brass, is 100 pounds, then the friction arising from both its gudgeons will be equivalent to 25 pounds. If we suppose also that a force equal to 40 pounds is employed to impel the wheel, and acts in the direction of gravity, as in the cases of overshot wheels, the pressure of the gudgeons upon their supports will then be 140 pounds, and the friction 35 pounds. But if the force of 40 pounds could be applied in such a manner as to act in direct opposition to the wheel's weight, the pressure of the gudgeons upon their supports would be 100 — 40, or 60 pounds, and the friction only 15 pounds. It is impossible, indeed, to make the moving force act in direct opposition to the gravity of the wheel, in the case of water-mills; and it is often impracticable for the engineer to apply the impelling power otherwise than in a given way; but there are many cases in which the moving force may be so

exerted, as at least not to increase the friction which arises from the wheel's weight.

As it appears from the experiments of Coulomb, that the least friction is generated when polished iron moves upon brass, the gudgeons and pivots of wheels, and the axles of friction rollers, should all be made of polished iron ; and the bushes in which these gudgeons move, and the friction wheels, should be formed of polished brass. In small and delicate machinery, the caps, or planes, which support pivots, or upon which knife-edges rest, as in balances and pendulums, should be made of garnet in preference to any other substance.

When every mechanical contrivance has been adopted for diminishing the obstruction which arises from the attrition of the communicating parts, it may be still farther removed by the judicious application of unguents. The most proper for this purpose are swine's grease and tallow, when the surfaces are made of wood, and oil when they are of metal. When the force with which the surfaces are pressed together is very great, tallow will diminish the friction more than swine's grease. When the wooden surfaces are very small, unguents will lessen their friction a little, but it will be greatly diminished if wood moves upon metal greased with tallow. If the velocities, however, are increased, or the unguent not often enough renewed, in both these cases, but particularly in the last, the unguent will be more injurious than useful. The best mode of applying it, is to cover the rubbing surfaces with as thin a stratum as possible, for the friction will then be a constant quantity, and will not be increased by an augmentation of velocity.

In small works of wood, the nterposition of the powder of black lead has been found very useful in relieving the motion. The ropes of pulleys should be rubbed with tallow, and whenever the screw is used, the square threads should be preferred.

In order to apply unguents to the communicating parts of machines, various contrivances have been adopted. The spindles of trundles have been made hollow, to contain oil, so that when they had a horizontal position, they allowed it to drop from small apertures upon the wheel below, by which they were driven. In the *Edinburgh Encyclopædia*, Art. *Mechanics*, the reader will find a drawing and description of a method of diminishing friction, by dispensing with the axes of rotation, and

a method in which the wheel is sustained by two floating cylinders.

CHAPTER IV.

ON THE ELEMENTS OF MACHINERY, AND THE CONTRIVANCES USED IN THE COMPOSITION OF MACHINES.

As every machine consists of a number of elementary parts, it is necessary that the mechanic should be acquainted with their nature and mode of action before he combines them together so as to compose a machine for producing a particular effect. The most important parts of machines may be arranged as follows :—

I. Contrivances in which the acting parts have no permanent connexion.

1. Cylindrical wheels and pinions.
2. Bevelled or conical wheels.
3. Crown wheels.
4. Rackwork.
5. Ratchet-wheels.
6. Wheels driven by belts, bands, or ropes.
7. Rag-wheels and chains.
8. Axles, gudgeons, and pivots.
9. Apparatus for locking and unlocking machinery.
10. Endless screws.
11. Lever of Lagaroust.
12. Springs.

II. Contrivances in which the communicating parts have a permanent connexion.

1. - Single crank.
   - Double crank.
   - Triple crank.
   - Variable crank.
2. Hooke's universal joint.
3. Sun and planet wheel.
4. Ball and socket.
5. Arched head and chain.

I.—*Account of Contrivances in which the Acting Parts have no Permanent Connexion.*

1. *Cylindrical Wheels and Pinions.*—When a rod or axle is put into a rotatory motion by any power or first mover, the motion of this axle is conveyed to a second axle, parallel to it, by means of cylindrical wheels. If nothing more than motion was to be conveyed to the second axle, it might be done by two wheels, one placed on each axle, and having their cylindrical surfaces in contact. The mere friction of these circumferences

would give motion to the second wheel, and the axle upon which it is placed.[1] As it is necessary, however, to transfer the force as well as the motion of the first mover, the two wheels are generally made to communicate by means of teeth cut out of their respective circumferences, which act upon one another in the way already described.

If the second axle is to move with a different velocity, the wheel placed upon it must have its diameter one half the diameter of the first wheel, if it is to move with twice the velocity; one third if it is to move with thrice the velocity, and so on; or twice the diameter if it is to move twice as slow, and so on. In these cases the smallest wheel is generally called the pinion.

2. *Bevelled or Conical Wheels.*—When the motion of the first axle is to be communicated to a second axle, inclined at any angle to it, bevelled wheels (or bevelled geer) are employed, as already described, and the number of teeth in the one must always be to those in the other inversely as the velocities with which they are to move.

3. *Crown Wheels.*—In very small works, the motion of one axle may be conveyed to another at right angles to it, by means of a cylindrical pinion acting upon teeth at right angles to the radii of the wheel. Such a wheel is called a *crown wheel.*

4. *Rackwork.*—When the motion of an axle is to be used for giving a rectilineal motion backwards and forwards to a rod or beam, the machinery is called rackwork. An example of this is shewn in Vol. I, Plate VI, Fig. 2, 3, Plate XIII, Fig. 4, and in Plate V, Fig. 9 of this volume.

5. *A Ratchet-Wheel*, or detent wheel, is a wheel fixed upon an axis which allows the axis to turn round in one direction, but prevents it from turning in the opposite direction. Ratchet-wheels are generally employed to prevent a weight raised by a machine from descending.

[1] In a saw-mill erected by Mr. Taylor of Southampton, the wheels act upon each other by the contact of the end grain of the wood. This method continued in use for twenty years. The wheels were of course made to bear against each other by some mechanical means. In small works a film of caoutchoc round the circumference of the wheels would enable the one to drive the other with great effect; or the same result might be produced in an inferior degree by rims of buff leather, similar to what Mr. Nicholson saw in the drawing of a spinning-wheel.

In Plate XI, Fig. 1, Vol. I, a ratchet-wheel is shewn at *N*. (See also Plate II, Fig. 5). Its teeth have the shape of the point of a crescent lying in the same direction. When the wheel turns in one direction, a click such as *O* rolls over the convex surfaces of the teeth ; but when the wheel attempts to move in the opposite direction, the click *O* presses itself, sometimes by the help of a spring, into the angular space between the two teeth, and prevents the wheel from moving.

6. *Wheels with Belts and Ropes.*—When the motion of one axle is to be conveyed to another at a distance, a wheel or pulley on the one is connected with a wheel or pulley on the other, by means of a belt or rope passing over each, as shewn at *E F* in Plate I, Fig. 12, Vol. I. When ropes are used, they lie in grooves cut in the circumference of each wheel ; but when a belt is employed, it acts simply upon their cylindrical circumferences. When the belts become loose by being stretched, they lose their power of turning the axles, and in this case they must either be shortened, which is sometimes done by means of buckles, or the friction must be increased by chalk or other means. Sometimes this evil is remedied by having grooves on a pulley of different diameters in one or both of the wheels, and when the belt or rope becomes slack, it is shifted to the next largest circumference. If this, however, is not done on both wheels, the velocity of the machinery will be changed. Belts are often apt to quit the wheels upon which they work, which generally arises from the circumference not being accurately cylindrical. If the circumferences are conical, the belt has always a tendency towards the base of the cone, and hence it is usual to give the circumferences of the pulleys or wheels a barrel shape, or to make their diameter greatest in the middle, by which the belt always keeps its place. When the belt or rope crosses between the wheels, the second will move in a contrary direction to the first; but when they do not cross, the wheels will both move in the same direction.

7. *Rag-wheels with Chains.*—When the machinery is exposed to great strains, or when its movements require to be very regular, chains of various kinds are substituted in place of belts and ropes. These chains generally lay hold of pins or hooks, or enter into notches on the circumference of the wheel, and hence such wheels have been called *Rag-wheels.* Three different

kinds of rag-wheels are shewn in Plate VI, Figs. 3, 4, 5. In Figs. 3 and 4 the chains take hold of pins *p*, *p*, &c. projecting from the wheel, and in Fig. 5 there are sharp angular points on the chain *A B C*, which enter into notches *t t* upon the wheel's circumference. Another example of rag-wheels is shewn in Plate II, Fig. 11, at *A* and *B*.

8. *Axles, Gudgeons, and Pivots.*—Axles are generally fixed permanently to the wheels which they carry; but Mr. Brunell, in the block machinery at Portsmouth, connected the wheels with their axles solely by friction. A hollow conical shoulder in the centre of the wheel was fitted upon a conical part of the axle. By driving the interior upon the exterior cone, with a few blows from a hammer, the friction was sufficiently powerful to prevent the wheel from slipping round, while the machine was performing its ordinary work; but when the force applied to the wheel was increased by any unforeseen cause, this force, in place of crushing the machinery, overcame the friction between the two cones, and the wheel revolved upon its axle without turning it, and consequently without injuring the machinery. The *Gudgeons* are the metallic and cylindrical extremities of axles upon which they rest and turn. They should never be made much larger than the nature of the machinery requires. When the extremities of axles are conical, they are called *Pivots*. The hollows in which the gudgeons rest are called *Bushes*. A vertical gudgeon with its bush is shewn in Fig. 6 of Plate VI, and a horizontal one in Fig. 7. They should be made of hard substances of uniform texture, and plentifully supplied with oil or grease. Sometimes they are made of siliceous stones in mills. In wind-mills the great collet of the axle is sometimes supported in blocks of marble slightly hollowed. M. Borgnis has found that green oak that has been soaked in boiling oil is preferable to copper. When the wheel is subject to horizontal or vertical displacements, it is necessary that the gudgeon should have a cover *m* (Fig. 8), kept down by screws *d d*, the tail *b* diverging, in order to keep it firmer in the wood.

9. *Apparatus for Locking and Unlocking Machinery.*—In many machines it is necessary sometimes to connect one axle with another, and at other times to separate them. This is called

throwing in and *out of geer*, and may be effected in various ways. The most elegant method of doing this is to place upon one of the axes a conical shoulder, which can slide freely endways, but is prevented from revolving by fillets or projections from the axle. This conical shoulder is forced by the power of a lever into a hollow conical shoulder, and is rivetted with it so firmly by friction that they act as if they were firmly joined together. By the action of the lever, they are again separated at pleasure.

Another method of throwing either of two wheels into geer is shewn in Plate VI, Fig. 9, where *A* and *B* are the two axles, and *P*, *O* the wheels fixed upon them. When it is required to throw the wheel *O* into geer, the clutch *q'* is pushed by the arm *f e* into hollows in the face of the wheel, and the same is done in the opposite direction by the clutch *q* when *P* is to be thrown into geer. In the position shewn in the figure, the wheels are both out of geer. This contrivance is part of a self-regulating sluice, which is elevated or depressed to admit more or less water according as the pinion *O* or *P* is thrown into action. See *Edinburgh Encyclopædia*, Art. *Hydrodynamics*, vol. xi, p. 561.

10. *Endless Screws.*—The object of an endless screw is to communicate the motion of an axle to a wheel lying in a plane passing through the axle. It is shewn at *H* in Plate I, Fig. 9, where it works in the teeth of the wheels *C*, *H*. In this construction of the endless screw only a few teeth of the screw are engaged at the same time; but when great strength and solidity are required, the wheel in which the endless screw works has a groove *a b* (Fig. 10) in its circumference, like a female screw. By this means many teeth of the screw and the wheel are engaged, and the construction is remarkably solid. This contrivance is used in the dividing engines of Ramsden and Troughton, and is well represented, both as applied to rectilineal and circular motions, in the *Edinburgh Encyclopædia*, Art. *Graduation*, Plate 281, Figs. 7 and 8.

11. *Lever of Lagaroust.*—This contrivance, which produces a rectilineal from a circular motion, is represented in Plate VI, Fig. 11. The lever *A B* has a motion round an axis *C*, in a fixed beam *M N*, the use of which is to elevate or depress the toothed rack *F G*. This is done by the two secondary levers *D E*,

$D' E'$, fixed to $A B$, at the points D and D'. When the extremity A descends, the hook E slides down over the tooth below it into the next hollow, while the ascent of B, which takes places at the same time, causes the hook E' to pull up the rack $F G$. When B, on the contrary, is depressed, the hook E' slides down over the tooth below it into the succeeding hollow, while the ascent of $A C$ causes the hook E to draw upward the beam $F G$. By means of the same contrivance a circular motion may be given to a wheel having teeth similar to those in the rack $F G$.

12. *Springs.*—Springs are frequently used in machinery, not as a moving power, but as a sort of occasional force for keeping together the parts of a machine that would otherwise separate. An example of this is shewn in Plate I, Fig. 5, Vol. I, where the spring H, by bearing against the index $H K$, keeps it in contact with the point m, and the lever $I F$ in contact with the end n of the expanding bar O. See also Plate VII, Figs. 1 and 8. A spiral spring sometimes forms a permanent connexion between the extremities of a chain, in order to tighten it when it has become slack with use.

II. *On Contrivances in which the Communicating Parts are permanently connected.*

1. *A Crank* is one of the simplest and most durable methods of conveying motion. It is represented at F, G, and H, in Fig. 5, and at A, B, and C in Fig. 6 of Plate V, Vol. I, and it acts in the same manner as if the power were applied to a winch fixed at or near the circumference of a wheel. A double crank is shewn in Plate VI, Fig. 12, where $A B$ is the revolving axis, which, by means of the double crank, communicates a reciprocating motion to the two beams $a b$, $c d$, the one ascending when the other is descending. A triple crank is shewn in Fig. 13, where three beams are made to ascend and descend; but the planes passing through each of the three cranks must be inclined 120° to one another. A changeable crank, in which the radius is variable, is shewn in Fig. 14, where $A B$ is the axis, and $a b$ the beam, which can be set to different distances from $m n$, by drawing up the four parallel bars, c, d, e, f, and fixing them in any position, by pins passing through the small holes.

2. *Hooke's Universal Joint.*—This ingenious contrivance, invented by Dr. Hooke, and represented in Plate VI, Fig. 15, consists of two shafts or axes *A*, *B*, terminating in a semicircle, and connected by means of a cross *C D*, *E F*. As the branches *C D*, *E F* have a motion round their pivots at *C*, *D*, *E*, and *F*, it is obvious that when the shaft *A* is turned round, a similar motion will also be communicated to the shaft *B*. This joint may be used when the inclination of the shaft does not exceed 40°, or rather 140°, its complement. It is particularly useful in giving the vertical and horizontal motions to telescopes, by means of rods which the observer holds in his hand, and which are connected with endless screws by means of the universal joint. When the inclination of the shafts is between 50° and 90°, a *double universal joint* is employed, as shewn in Fig. 16. Here there are two crosses, the extremities of which move on their pivots in the semicircles at the ends of the shafts *A*, *B*. These joints may also be constructed with four pins, fastened at right angles upon the circumference of a hoop or a solid ball.

3. *Sun and Planet Wheel.*—This contrivance was first introduced by Mr. Watt, for the purpose of converting the reciprocating motion of the beams of his steam-engine into a rotatory one. It has been already described in pages 95 and 96.

4. *Ball and Socket.*—Although the ball and socket is not used in general machinery, yet we may regard it as a mechanical contrivance, by which two parts of a machine are permanently connected. The object of it is to give a motion in various directions to one axis, while the other remains fixed. This is generally done in telescopes, by the joint effect of a horizontal and a vertical movement; but when the telescope is not heavy, a ball and socket is the simplest and most commodious contrivance. It is represented in Fig. 17, where *B* is a *Ball* of brass, fixed at the lower extremity of the axis *A B*, on which a telescope or any other body is supported. This ball is of the same diameter as the interior diameter of the *Socket* or cap *C S D*, with which the lower axis *S T* is terminated. This socket consists of two parts *C E F D*, *E F S*, the former of which can be removed, or slackened, or tightened, by turning the milled circumference *E F*. On the bottom of the socket is placed a piece of cork, against which the lower part of the ball *B* presses, so that whenever the ball moves too loosely in

the socket, it may be pressed against the elastic cork, by turning *E F*, which draws the ball downwards, by which means its motion is rendered as stiff as we choose. In the upper edge of the moveable part *C D*, a notch *n* is cut, to allow the axis *A B* to come into a position at right angles to *S T*.

In the preceding observations, we have mentioned only those means which are used in connecting together the different parts of machinery, without any particular reference to the nature of the effect which they individually produce. We shall now proceed to consider the various contrivances which have been adopted for changing the direction of motion.

Account of Mechanical Contrivances for transferring Motion, or changing its Direction.

I. Contrivances for transferring a direct circular motion into a direct circular motion in the same plane.

The simplest way of producing a direct circular motion from a motion of the same kind, is by means of a belt, rope, or chain, which does not cross between the two wheels.

The same effect may be obtained by means of *three* wheels acting upon one another, where, if the right-hand wheel is turned from left to right, it will cause the middle wheel to turn from right to left, and the left-hand wheel from left to right. The same effect is likewise produced by one wheel acting upon the interior circumference of another.

II. Contrivances for changing a direct circular into an inverse circular motion in the same plane.

This effect may be produced by a crossed belt, rope, or chain, or by one toothed wheel acting upon another.

III. Contrivances for changing a circular motion in one plane into a circular motion in any other plane.

A motion in any one plane may be changed into a motion in any other plane, by means of bevelled or conical wheels, as described in p. 135, and the same effect may be produced by Hooke's universal joint. An example of this may be seen in Plate V, Fig. 19, where the motion is changed into a plane inclined to the original one. The same effect may be produced by an endless screw working in the teeth of a wheel. In Vol. I, Plate II, Fig. 20, the motion is represented as changed into a plane at right angles to that of the handle; the wheel,

however, may have any degree of inclination to the horizon; but the common section of its plane, and that of a plane passing through the axis of the screw, must always be parallel to the axis of the screw.

IV. Contrivances for transferring and changing a circular motion into another of the same kind.

A contrivance of this kind, invented by M. Camus, for the purpose of moving sieves, is shewn in Plate VI, Fig. 18, where *A B C D* is a table above which is a plank *E F*, capable of revolving round the pivots *m*, *n*. To this plank, or to one of the pivots *m* prolonged, is fixed a crooked arm *s*, upon which is suspended a pendulum *R S*. By means of the moving power, the pendulum is put into a circular alternating motion, and, of course, the same motion is communicated to the plank *E F*.

Another contrivance for this purpose is shewn in Fig. 19, where *a b c* is a cord fixed at *a* to the spring *B*, and after passing round the cylindrical wheel *A*, it is fixed at *c* to the pedal *D*. The circular alternating motion of the pedal *D* round the centre *C* is thus communicated to the wheel *A*.

Account of Contrivances for changing one Motion into another of a different kind.

In the preceding paragraph we have considered only the methods of changing one motion into another of the same kind, either into a new plane, or into a different plane. We shall now, therefore, describe the different kinds of mechanism by which motions are changed into others of a different kind. These changes may be thus arranged.

1. A rectilineal continuous motion into a circular continuous motion.

2. A rectilineal continuous motion into a circular alternating motion.

3. A rectilineal continuous motion into a rectilineal alternating motion.

4. A circular continuous into a rectilineal alternating motion.

5. A circular continuous into a circular alternating motion.

6. A rectilineal alternating into a circular alternating motion.

I. Contrivances for converting a rectilineal continuous motion into a circular continuous motion.

This effect may be produced by the following means :—

1. By the descent of a weight attached to a cord wrapped round the circumference of a cylindrical wheel or axle. The weight, by its continuous rectilineal descent, gives a circular continuous motion to the wheel or axle. See Plate II, Fig. 5, Vol. I, where the weight P gives this motion to the axle $C\,D$, by the rope acting on the circumference of $A\,B$.

2. By a rack acting upon teeth in the circumference of a wheel, where the continued rectilineal motion of the rack gives a continuous circular motion to the wheel.

3. By a rectilineal belt, rope, or chain, acting in the direction of a tangent to the circumference of a wheel.

4. By a nut, with a female screw advancing along a male screw.

II. Contrivances for converting a rectilineal continuous motion into a circular alternating motion.

This effect may be produced by the following means :—

1. By the teeth of a rack $M\,N$, Plate VI, Fig. 20, acting upon the extremity B of a lever $A\,B$ moving round the centre C. After the head A has been raised by any tooth acting upon B, it again descends by its own weight through the same arch; and as soon as its descent is completed, it is again caught by another tooth, which raises it as before. The same effect may be produced by a chain with teeth, which act upon notches cut in the circumference of a wheel fixed at B. If the rack $M\,N$ is fixed to a chain passing round two rag-wheels, it may be called an *endless rack*, and though moving round two centres, its motion is in reality rectilineal.

2. The same effect may be produced by a double rectilineal rack acting upon a double wheel, as shewn in Fig. 21, where $A\,B\,C\,D$ is the double rack, having sets of teeth on each side, with intervals without teeth, the sets of teeth on the side $C\,D$ being opposite the intervals on the side $A\,B$. Between the arms of this double rack is fixed a double wheel $a\,b$, $c\,d$; the half $a\,b$ has a smaller diameter than the other, and is furnished with a few teeth, which are acted upon by the teeth of the rack $C\,D$, and the other half $c\,d$ is also furnished with teeth, which are acted upon by the teeth of the rack $A\,B$. When the rack $A\,B\,D\,C$ descends, the first tooth upon $C\,D$ acting upon the first tooth $a\,b$, gives the double wheel a rota-

tory motion from left to right, and the remaining teeth of the first set conduct it through a given arch. When this arch is completed, which happens when the last tooth of the first set upon CD acts upon the last tooth upon $a\,b$, the first tooth of the first set upon $A\,B$ begins to act upon the first tooth upon $c\,d$, and therefore causes the double wheel to move backwards through the same arch. In this way the circular alternating motion of the double wheel will continue as long as the rack continues to act upon it. The double wheel may be placed between two racks, carried round by two pair of rag-wheels.

3. The *Lever of Lagaroust*, which we have already described, becomes, when its action is inverted, a contrivance by which the rectilineal continuous descent of the hooks E, E produces an alternating circular motion round the centre C.

III. Contrivances for converting a rectilineal continuous motion into a rectilineal alternating motion.

1. The only contrivance which we think capable of producing this effect is shewn in Fig 22 of Plate VI. A chain $A\,B$ moving in the rectilineal direction $A\,B$ by means of two rag-wheels, and furnished with wipers m, n, o, will obviously raise the arm $C\,D$ through a certain height. As soon as the wiper quits it, it will descend by its own gravity, and will then be elevated by the subsequent wiper m, having an alternating rectilineal motion of ascent and descent.

IV. Contrivances for converting a circular continuous motion into a rectilineal alternating motion.

The contrivances which have been invented for producing this change of motion are very numerous, and some of them highly ingenious. Innumerable pieces of mechanism might be devised for this purpose; but it is our object at present to exclude all those in which the effect is produced by more than two or three parts; as all contrivances of a more complicated nature must be considered as real machines, and not as elementary ones.

1. The simplest methods of producing a rectilineal alternating motion from a circular continuous one, are those which consist in raising a beam or stamper vertically, by wheels formed into an irregular outline, or carrying on their circumferences wipers or spirals, as shewn in Plate V, Figs. 9, 10, 11, &c. The stamper is thus raised to a certain height, and descends through the same height by its own weight, when the succeeding wiper

begins to acts upon it, and raise it a second time. The very same effect may be produced by a rack and pinion, provided the pinion has teeth only on a part of its circumference. The toothed part elevates the rack, which of course descends when the last tooth has ceased to act, and by the time that the rack has performed its work, the toothed part of the pinion has come round again, for the purpose of raising the rack a second time. All these contrivances are so simple and well understood, that they do not require to be more particularly described.

2. Another contrivance for producing this effect is shewn in Fig. 23, where *A* is a wheel turned by the winch *H*, and connected by the bar *a b* with the vertical beam *M N*, which can move up and down between the guides *c d*, *e f*. As the handle *H* is turned, the beam *M N* will move up and down, its greatest range being equal to 2 *A b*. If the beam *M N* is placed horizontally, the same effect will be produced. When the power of a man is to be applied to the handle *H*, the wheel *A* is unnecessary; and we have only to join the extremity *b* to the centre of motion *A*, and convert it into a crank. But when the motion is to be taken from any other moving shaft, it is generally done by a belt or chain passing round the wheel *A*. This apparatus, when horizontal, may be employed for grinding and polishing flat surfaces, and is used in silk mills.

3. In Figs. 24 and 25, we have represented other two contrivances, which are essentially the same. In Fig. 24, a pinion *P*, having teeth on nearly one half of its circumference, is placed within a double rack *A B C D*. After the pinion, turning from right to left, has raised the double rack by acting upon *D B*, it then proceeds, by continuing to turn in the same direction, to act upon the side *A C*, and depress the double rack. If the teeth of the wheel were infinitely small, the half of the wheel should be furnished with the teeth, and the length of the rack be exactly equal to the toothed part.

The contrivance shewn in Fig. 24, consists in reducing the teeth of the rack to one, and in diminishing the number of teeth in the wheel. When the wiper *a*, turning from left to right, has raised the rack by acting upon *m*, the wiper *b* begins to depress the rack by acting upon *n*; and in this way the alternate motion is continued.

4. The rack and pinion of the form shewn in Plate VI, Fig. 26, produces a similar effect. When the teeth of the pinion *P* have acted upon one side of the rack, and begin to enter upon the circular end, a small lateral motion is given to the rack by means of the jointed pieces *a b*, *c d*, which brings the other side of the rack within the action of the pinion, and enables it to work on the opposite side to produce the returning motion.

5. The contrivance shewn in Fig. 27, consists of a plate of metal *A B*, with a number of rectilineal openings *a b*, *c d* cut through and through it. Behind this plate, and very near it, is another *M N*, in which a spiral aperture marked with a dotted line is also cut through and through. If small cylinders *r*, *s* are placed in the opening at the intersection of the spiral and the rectilineal apertures, it is obvious that these cylinders will approach to or recede from the axis, by turning the plate *M N*, as they must always be found at the points of intersection. If two contrivances of this kind are placed together, as shewn in the section, Fig. 27, and if the small cylinders *r*, *s*, in each set of plates, are connected with each other by means of straight rods *n n*, *n n*, these rods will form the circumference of a drum of a greater or less diameter, according as the cylinders *r*, *s* recede from, or approach to, the centre. By this means an *expanding crane* is formed, by which the relation between the power and weight can be varied at pleasure. This is the principle of the expanding crane invented by Mr. Robert Hall, and described in the *Transactions of the Society of Arts*, vol. xii.

6. Another contrivance is shewn in Plate VII, Fig. 1, where *AB* is a wheel turned by a winch or by a belt upon its circumference. Its exterior part is cut into teeth, the form of which may be varied according to the nature of the case. A small rod *a b*, supported between two guides *m*, *n*, has one of its ends *a* bearing upon the teeth of the wheel, while its other extremity *b* buts against a spring *s*. As the wheel *A B* turns round, an alternating rectilineal motion is obviously communicated to the rod *a b*. M. Zureda, has applied this contrivance to a machine for pricking holes in leather for making cards, and it has also been employed in the manufacture of fishing nets. If all the teeth in *A B* were reduced to one, the surface of the wheel would be an

inclined plane, and the oscillations of $a\,b$ would be performed once in every revolution of the wheel.

7. A piece of ingenious mechanism, invented by M. Zureda, is shewn in Fig 2, where $A\,B$ is a cylinder revolving about its axis either by the handle H, or by a belt going round a wheel at one end of it. On its surface are cut two opposite grooves like the threads of a screw, which unite or run into one another at both ends of the cylinder. The lower extremity b of the small rod $a\,b$ exactly fills this groove, and its upper end a has a head attached to it, which runs in a groove made in the frame $C\,D$. By turning the handle H, the rod $a\,b$ follows the spiral direction of the grooves till it reaches the end B; and as this end of the groove communicates with the commencement of the opposite groove, the rod $a\,b$ returns to its original position by the opposite groove. See Lanz and Betancourt's *Essai, &c.* p. 62, Edit. 1819. If the cylinder in this mechanism is converted into a cone, the path of the small rod $a\,b$ may have any inclination to the axis of the circular motion.

8. Two analogous contrivances are shewn in Figs. 3 and 4. A wheel $A\,B$ (Fig. 3), turned by a handle H, or by a belt, has a projecting pin or cylinder C, which is made to move with a little play in the rectilineal groove $D\,E$, cut in a cross piece $D\,E$, attached to the ascending and descending rod $a\,b$, supported vertically by the guide $m\,n$. As the wheel turns, the cylinder C, acting upon the lower side of the groove, depresses $a\,b$, and bringing it to its lowest point, it raises it by acting upon the upper side of the groove, and thus gives the beam $a\,b$ a rectilineal alternating motion. As the space of descent or ascent is always equal to the versed sine of the arch described by the wheel, the motion of the beam $a\,b$ is very slow at the beginning and end of each oscillation, and varies rapidly towards the middle.

This want of uniformity is remedied in the contrivance shewn in Fig. 4, where the groove $D\,E$, instead of being rectilineal, is formed into two similar curves, as shewn in the Figure. In order that the beam $a\,b$ may descend through equal spaces $p\,1$; 1, 2; 2, 3, &c. while the pivot s describes the equal arcs $s\,1$; 1, 2 of the arch $p\,D$, it is obvious that the point p of the beam must be at the points 1, 2, 3 of the radius of the wheel, while the pivot s is at the divisions 1, 2, 3, &c. of the quadrant of the

circle. Hence the distances 1, 1; 2, 2, between the quadrant and the curvilineal groove must be taken equal to *s* 1, *s* 2, *s* 3 of the radius. The other quadrants of the curve are formed in the same manner. But it must be noticed, that if the pivot *s* causes the beam *a b* to move through a space exactly equal to what it moved through when it described the first quadrant, then the curves must cross one another at *s* like the figure ∞. If it is required that the beam *a b* moves through a smaller space in the 2d than in the 1st quadrant, then the form of the curves must be that in the figure, where the two branches are separated at *s*; but if it is required that the beam should describe a greater space, then the curves will cross one another at two points, thus .

9. Another contrivance is shewn in Plate VII, Fig. 5, where *A B* is a double rack with circular ends, driven by a pinion *P*, which can move freely in a groove *m n* cut in the cross piece *C D*; then if, when the pinion *P* comes to the end *B*, the projecting piece *a* meets the spring *s*, the pinion *P* will descend in its groove, and carry back the other side of the rack, so that the alternate rectilineal motion of the rack will thus be kept up continually.

10. Another contrivance is shewn in Fig. 6, where *A B* is a concave toothed wheel, in which another wheel *C* of half its diameter works. If this wheel *C* is driven by a handle, so as to make it revolve within the other, any point of its circumference will describe a straight line. This arises from the property that the interior epicycloid thus described is a straight line. See p.

In all the preceding contrivances, the rectilineal alternating motion is performed in a path which has absolutely the same position in every successive oscillation. It may, however, often be necessary that this path should be different in different oscillations. If, for example, we wish to bruise any thing with a stamper, it may be required that the stamper should not always fall upon the same spot, but should strike different places at different times. The following contrivance for producing this compound rectilineal alternating motion has occurred to us.

11. Let it be required to raise a stamper vertically, so that it shall strike different places at the end of different oscillations. If the rectangles *A*, *B*, *C*, *D* (Fig. 7), represent the curvilineal acting surfaces of four wipers lying in different planes, let

grooves $a\,b$, $a'\,b'$ be cut into two of them, so as to conduct laterally the pin, or projecting point, by which the stamper is raised, and let the other two have similar grooves $c\,d$, $c'\,d'$ cut in an opposite direction. Then when the pin of the stamper is taken up at the point a of the groove upon A, it will be conducted in the groove $a\,b$, and quitted at b, so as to have received a lateral displacement equal to $a\,a'$. It will then be taken up by the point a' of the groove $a'\,b'$ of the wiper B, and in like manner conducted to b', where it falls. The next wiper C will take it up at the point c of its groove, and conduct it back again to d, where it quits it; and here it is taken up by the fourth wiper D at c', and conducted by the groove $c'\,d'$ to the point where it was when it was taken up by the wiper A. The only practical difficulty in this construction consists in guiding the stamper during its lateral displacement.

V. Contrivances for converting a continuous circular motion into an alternating circular motion.

1. This effect may be produced by means of wipers fixed upon a wheel, and raising a forge hammer $A\,B$ moveable round a centre c as in Fig. 9, Plate ~~VII~~, where the teeth or wipers are V
fixed upon a straight line instead of a wheel. This contrivance admits of numerous varieties, but they are all precisely the same in principle.

2. Another contrivance, shewn in Plate VII, Fig. 8, consists of a wheel $A\,B$, with one tooth as it were, or an inclined plane $M\,N$ fixed upon its outer rim. This plane acts against the extremity a of a crooked lever $a\,b\,c$, and gives it an alternating vibrating motion round the centre C, the extremity a being always pressed against the plane in its returning oscillation by the spring s. If the bent arm $C\,b\,a$ of the lever is heavier than the other arm, the spring will be unnecessary, as the lever will return like a pendulum by its own gravity. The plane of the wheel may have any position with regard to the horizon.

3. Another piece of mechanism for this purpose is shewn in Fig. ~~11~~, where A is a wheel partly toothed; and B, C, other 9
two toothed wheels upon the same axis $D\,E$. After the toothed arch A has driven the wheel B through a certain arch, it then quits it, and begins to act upon C in an opposite direction, by which means an alternate circular motion is communicated to the axis $D\,E$.

4. A contrivance invented by M. St. Cyr, for changing an uniform circular motion into an alternating circular motion, the velocity varying according to a given law, is shewn in Plate VII, Fig. 10. The use of it is to construct equation clocks. The annual wheel *A* carries the equation curve *B C D*, in the rim of which is cut a groove, in which the pivot *E* is guided. This pivot is connected with the levers *E F* and *E G*, the latter of which is attached to the cannon *H*, which carries the minute-hand *H I*, so that it follows the vibration of the curve in more than one half of the circumference of the minute dial, and which is sufficient for marking the inequalities arising from the equation.

5. The crank, which we have already described in p. 181, is another contrivance for converting an alternating circular motion into a continuous circular motion.

6. The sun and planet wheel is also a contrivance for the same purpose. See p. 182.

7. The lever of Lagaroust, where the ratchet-wheel is circular, likewise answers the same purpose. See page 180, and 181. This contrivance admits of the modification shewn in Fig. 11, where the lever *A B* turns round the point *C*, above and below which are two rods *E*, *D*, moveable round pins on *A B*, and so formed at their other extremities as to rest upon the spindles or teeth of the wheel *M N*. When the end *B* is pushed towards the wheel, the ends of the rods *E*, *D* push against the spindles, and drive the wheel round; and when *B* moves backwards from the wheel, the rods *D* and *E* fall from the spindles which they formerly acted on, upon the subsequent spindles, which they push forward at the next returning vibration of *B C*.

8. The various *escapements* which are used in time-keepers, &c. may also be ranked in this class of elementary machines.

VI. Contrivances for converting a rectilineal alternating into a circular alternating motion.

The mechanical contrivances which have been invented for this purpose are of very great use in the arts, and deserve to be carefully studied by those who are professionally occupied with the construction of machinery.

1. A very simple piece of mechanism of this kind is shewn in Fig. 12, where *A B* is a lever moving round *C*. The two extremities of a rope, fastened at the points *D*, *F*, pass round the semicircle *D E F*, and after crossing below *E*, pass over two

sheaves *M*, *N*. If we turn the lever round *C*, by depressing *B*, the point *L* of the rope will obviously move towards *N*, and if we raise the extremity *B*, the same point *L* will return to its former place. Hence this point receives a rectilineal alternating motion from the circular alternating motion of the lever.

2. A third contrivance for the same purpose is represented in Plate VII, Fig. 13, which is nothing more than the common bow used by watchmakers for drilling holes. The cord *A B*, passing round the sheave *C*, gives the latter a circular alternating motion when the bow is drawn backwards and forwards.

3. A lever, with two arched heads or ends, working in a vertical rack, or raising a weight by means of a chain passing over the arched ends, the weight descending by its own weight, is also a machine which produces the same effect.

4. In Fig. 14, the wheel *A B* approaches to the point *C*, by turning it round its axis in one direction, and recedes from it by turning it round in the opposite direction. The ropes *A C*, *B C*, being twisted round the axis *C D*, the distance between *A B* and *C* is necessarily shortened, while their untwisting increases that distance. This contrivance is often used for drilling or boring with the extremity *D*.

5. Another contrivance for the same purpose is shewn in Fig. 15, where the rod *a b* rises and descends vertically, between the two guides *m n*, *m n*, by means of the alternate circular motion of the lever *A B* moving round the centre *C*, and connected with *a b* by the ruler *B D*, having an axis of motion at each end.

6. The contrivance called the *Parallel Motion*, invented by Mr. Watt, for giving a circular alternating motion to the beams of his steam-engines, by means of the rectilineal alternating motion of the piston, belongs to this class of machines, and has been described in p. 100 of this volume.

CHAPTER V.

ON THE NATURE AND OPERATION OF FLY-WHEELS, AND OTHER CONTRIVANCES FOR REGULATING THE MOTION OF MACHINES.

Fly-wheels.

A FLY in mechanics is a heavy wheel or cylinder which moves rapidly upon its axis, and is applied to machines for the purpose of rendering uniform a desultory or reciprocating motion, arising either from the nature of the machinery, from an inequality in the resistance to be overcome, or from an irregular application of the impelling power.

Causes of unequal motion in machines.

When the first mover is inanimate, as wind, water, and steam, an inequality of force obviously arises from a variation in the velocity of the wind, from an increase of water occasioned by sudden rains, or from an augmentation or diminution of the steam in the boiler, produced by a variation in the heat of the furnace; and accordingly various methods have been adopted for regulating the action of these variable powers. The same inequality of force obtains when machines are moved by horses or men. Every animal exerts its greatest strength when first set to work. After pulling for some time, its strength will be impaired, and when the resistance is great, it will take frequent, though short relaxations, and then commence its labour with renovated vigour. These intervals of rest and vigorous exertion must always produce a variation in the velocity of the machine, which ought particularly to be avoided, as being detrimental to the communicating parts as well as the performance of the machine, and injurious to the animal which is employed to drive it.

These inequalities remedied by a fly.

But if a fly, consisting either of cross bars, or a massy circular rim, be connected with the machinery, all these inconveniencies will be removed. As every fly wheel must revolve with great rapidity, the momentum of its circumference must be very considerable, and will consequently resist every attempt either to accelerate or retard its motion. When the machine, therefore, has been put in motion, the fly wheel will be whirling with an uniform celerity, and with a force capable of continuing that celerity when there is any relaxation

in the impelling power. After a short rest the animal renews his efforts, but the machine is now moving with its former velocity, and these fresh efforts will have a tendency to increase the velocity: the fly, however, now acts as a resisting power, receives the greatest part of the superfluous motion, and causes the machinery to preserve its original celerity. In this way the fly secures to the engine an uniform motion, whether the animal takes occasional relaxation, or exerts his force with redoubled ardour.

Exemplified in a thrashing machine.

We have already observed, that a desultory or variable motion frequently arises from an inequality in the resistance, or work to be performed. This is particularly manifest in thrashing-mills, on a small scale, which are driven by water. When the corn is laid inequally on the feeding board, so that too much is taken in by the fluted rollers, this increase of resistance instantly affects the machinery, and communicates a desultory or irregular motion even to the water wheel or first mover. This variation in the velocity of the impelling power may be distinctly perceived by the ear in a calm evening, when the machine is at work. The best method of correcting these irregularities is to employ a fly-wheel, which will regulate the motion of the machine, when the resistance is either augmented or diminished. In machines built upon a large scale there is no necessity for the interposition of a fly, as the *inertia* of the machinery supplies its place, and resists every change of motion that may be generated by an inequal admission of the corn.

Irregularities arise from the nature of the machinery.

A variation in the velocity of engines arises also from the nature of the machinery. Let us suppose that a weight of 1000 pounds is to be raised from the bottom of a well 50 feet deep, by means of a bucket attached to an iron chain which winds round a barrel or cylinder; and that every foot in length of this chain weighs 2 pounds: it is evident that the resistance to be overcome in the first moment is 1000 pounds, added to 50 pounds, the weight of the chain; and that this resistance diminishes gradually, as the chain coils round the cylinder, till it becomes only 1000 pounds, when the chain is completely wound up. The resistance therefore decreases from 1050 to 1000 pounds; and, if the impelling power is inanimate, the velocity of the bucket

will gradually increase; but if an animal is employed, it will generally proportion its action to the resisting load, and must therefore pull with a greater or less force, according as the bucket is near the bottom or top of the well. In this case, however, the assistance of a fly may be dispensed with, because the resistance diminishes uniformly, and may be rendered constant, by making the barrel conical, so that the chain may wind upon the part nearest the vertex at the commencement of the motion, the diameter of the barrel gradually increasing as the weight diminishes. In this way the variable resistance will be equalized much better than by the application of a fly-wheel; for the fly, having no power of its own, must necessarily waste the impelling power.

Having thus pointed out the chief causes of a variation in the velocity of machines, and the method of rendering it uniform by the invention of fly-wheels, the utility, and, in some instances, the necessity of this piece of mechanism, may be more obviously illustrated by shewing the propriety of its application in particular cases.

Advantages of fly-wheels exemplified in the pile-engine. Plate IX. Fig. 1. Vol. I.

In the description which has been given of Vauloue's pile-engine (See vol. i, p. 78), the reader must have remarked a striking instance of the utility of fly-wheels. The ram *Q*, is raised between the guides *b b*, by means of horses acting against the levers *S S*; but as soon as the ram is elevated to the top of the guides, and discharged from the follower *G*, the resistance against which the horses have been exerting their force, is suddenly removed, and they would instantaneously tumble down, were it not for the fly *O*. This fly is connected with the drum B, by means of the trundle *X*: and as it is moving with a very great force, it opposes a sufficient resistance to the action of the horses, till the ram is again taken up by the follower.

In the single stroke steam-engine.

When machinery is driven by a single stroke steam-engine, there is such an inequality in the impelling power, that, for two or three seconds, it does not act at all. During this interval of inactivity, the machinery would necessarily stop, were it not impelled by a massy fly-wheel of a great diameter, revolving with rapidity, till the moving power again resumes its energy.

According to Mr. Murray of Leeds, the weight of a fly-wheel in cwts. for steam-engines, may be found by multiplying the number of horses' power of the engine by 200, and dividing the product by the square of the intended velocity of the wheel's circumference. If the steam-engine is one of 20 horses' power, the fly-wheel 18 feet in diameter, and to perform 22 revolutions in a minute, then its weight should be about 90.4 cwt.

In the common winch.

If the moving power is a man acting with a handle or winch, it is subject to great inequalities. The greatest force is exerted when the man pulls the handle upwards from the height of his knee, and he acts with the least force when the handle, being in a vertical position, is thrust from him in a horizontal direction. The force is again increased when the handle is pushed downwards by the man's weight, and it is diminished, when the handle, being at its lowest point, is pulled towards him horizontally. But when a fly is properly connected with the machinery, these irregular exertions are equalized, the velocity becomes uniform, and the load is raised with an equable and steady motion.

In many cases, where the impelling force is alternately augmented or diminished, the performance of the machine may be increased by rendering the resistance unequal, and accommodating it to the inequalities of the moving power. Dr. Robison observes, that "there are some beautiful specimens of this kind of adjustment in the mechanism of animal bodies."

Fly-wheels accummulators of power.

Besides the utility of fly-wheels as regulators of machinery, they have been employed for accumulating or collecting power. If motion is communicated to a fly-wheel by means of a small force, and if this force is continued till the wheel has acquired a great velocity, such a quantity of motion will be accumulated in its circumference as to overcome resistances, and produce effects, which could never have been accomplished by the original force. So great is this accumulation of power, that a force equivalent to 20 pounds, applied for the space of 37 seconds to the circumference of a cylinder, 20 feet diameter, which weighs 4713 pounds, would, at the distance of one foot from the centre, give an impulse to a musket ball equal to what it receives from a full charge of gunpowder. In the space of 6 minutes and 10 seconds, the same effect would be produced, if the cylinde

was driven by a man who constantly exerted a force of 20 pounds at a winch 1 foot long.[1]

Exemplified in the sling. This accumulation of power is finely exemplified in the sling. When the thong which contains the stone is swung round the head of the slinger, the force of the hand is continually accumulating in the revolving stone, till it is discharged with a degree of rapidity which it could never have received from the force of the hand alone. When a stone is projected from the hand itself, there is even then a certain degree of force accumulated, though the stone only moves through the arch of a circle. If we fix the stone in an opening at the extremity of a piece of wood 2 feet long, and discharge it in the usual way, there will be more force accumulated than with the hand alone, for the stone describes a larger arch in the same time, and must therefore be projected with greater force.

When coins or medals are struck, a very considerable accumulation of power is necessary, and this is effected by means of a fly. The force is first accumulated in weights fixed in the end of the fly; this force is communicated to two levers, by which it is farther condensed: and from these levers it is transmitted to a screw by which it suffers a second condensation. The stamp is then impressed on the coin or medal by means of this force, which was first accumulated by the fly, and afterwards augmented by the intervention of two mechanical powers.

Importance of placing the fly-wheels properly. Notwithstanding the great advantages of fly-wheels, both as regulators of machines, and collectors of power, their utility wholly depends upon the position which is assigned them, relative to the impelled and working points of the engine. For this purpose no particular rules can be laid down, as their position depends altogether on the nature of the machinery. We may observe, however, in general, that when fly-wheels are employed to regulate machinery, they should be near the impelling power; and, when used to accumulate force in the working point, they should not be far distant from it. In hand-mills for grinding corn, the fly is for the most part very injudiciously fixed on the axis to which the winch is attached; whereas, it should always

[1] This has been demonstrated by Mr. Atwood. See his *Treatise on Rectilineal and Rotatory Motion.*

be fastened to the upper mill-stone, so as to revolve with the same rapidity. In the first position, indeed, it must equalize the varying efforts of the power which moves the winch; but when it is attached to the turning mill-stone, it not only does this, but contributes very effectually to the grinding of the corn.

Dr. Desaguliers mentions an instance of a blundering engineer, who applied a fly-wheel to the slowest mover of the machine, instead of the swiftest. The machine was driven by four men, and when the fly was taken away, one man was sufficiently able to work it. The error of the workman arose from his conceiving, like many others, that the fly added power to the machine; but we presume that Dr. Desaguliers himself has been accessory to this general misconception of its nature, by denominating it a *mechanical organ* or *power*. By the interposition of a fly, however, as the Doctor well knew, we gain no mechanical force; the impelling power, on the contrary, is wasted, and the fly itself even loses some of the force which it receives, by the resistance of the air.[2]

Having thus explained the nature and position of fly-wheels, we shall conclude this Chapter with the description of two very ingenious contrivances for regulating the motion of machines—namely, Prony's condenser of forces, and Samuel's apparatus for regulating the action of horses when employed to drive machinery.

Description of Prony's Condenser of Forces.

The condenser of forces invented by M. Prony has for its object to transmit the action of the first mover of a machine in such a manner, that the resistance opposed to the first mover may be speedily and easily varied, that it may be rendered constant at pleasure, and that the velocity of the machine may be permanent, notwithstanding any sudden variations in the efforts of the first mover.

In the plan and section of the condenser of forces, given in Plate VII, Figs. 16 and 17, it is represented as applied to a vertical arbor *O O*, to which the sails of a wind-mill are adapted; *e e e e* (Fig. 17) is an assemblage of carpentry, of which one of

[2] An account of the ingenious contrivance by Mr. Woolf, as a substitute for a Fly, will be found in *Nicholson's Journal*, 8vo, No. 23, and in the *Edinburgh Encyclopædia*, Art. *Mechanics*, vol. xiii, p. 590.

the radii *O e* carries a curved piece *b d* of iron or steel; vertical axes of rotation *a a a* being placed round the axis *O O*, also divide the circumference in which they are found into equal parts.

Each of these axes carries a curved metallic piece *a f*, shewn only in one of them, and so situated, that when the wind acts upon the sails, the curve *b d* presses against one of the curves *a f*, and causes the vertical axis to which this last curve is fixed to perform a part of a revolution.

The curves *b d* and *a f* must be so arranged, that when *b d* ceases to press on one of the curves *a f*, it shall at the same instant begin to act upon the succeeding curve. The number of axes which are furnished with these curves must be determined by the particular circumstances of each case; and we may also substitute instead of *b d* a portion of a toothed wheel having its centre in the axis *O O*, and use portions of pinions instead of the curves *a f*, but the mechanism represented in the figure is preferable.

Each of the axes *a a a a* (which are all fitted up alike, though, for the sake of clearness, only one of them has its apparatus represented in the figures) carries upon it a drum or pulley *t t r r* (Fig. 16), on which is wound a cord that passes over a pulley *p*, and serves to support a weight *Q* by means of the lever *F G*, upon which this weight may slide and be fastened at different distances from the centre of motion *G*.

The same axes *a a* pass through the pinions *q q*, to which they are not fixed; but these pinions carry clicks or ratchets which bear against the teeth *r r*, so that when the weight *Q* tends to rise, the ratchet gives way, and no other effect is produced on the pinion *q q* either by the motion of the axis or of the drum *t t r r*, excepting that which causes the ascent of the weight *Q*, but the moment that the curve or tooth *b d* ceases to bear against one of the curves *a f*, after having caused the corresponding weight *Q* to rise, that weight *Q* tends to re-descend, and then the toothed wheel *r r* acts against the ratchet, so that *Q* cannot descend without turning the pinion *q q* along with the drum *t t r r*.

The pinion *q q* works in the wheel *A B*, from the motion of which the useful effect of the machine immediately arises; so that the effect of the descent of one of the weights *Q* is to put the wheel *A B* in motion, or to continue the motion in concur-

rence with all the other weights Q which descend at the same time. The wheel $A B$ carries beneath it oblique or bevelled teeth $C D$, which work in a similar wheel $C E$, and cause the buckets at P to rise by means of the rope and pulley on the axis of the wheel $C E$.

From this description it will be seen that if the machine is supposed to start from a state of rest, the wind will at first raise a number of weights Q sufficient to put the machine into motion, and will continue to raise new weights, while those before raised have fallen, so that the motion once impressed will be continued. It follows that no violent shock can take place in any part of the mechanism; and that the useful effect being proportioned to the number of weights Q, which descend at the same time, this effect will increase in proportion as the wind becomes stronger, and causes the sails to turn with greater velocity.

As the weights Q are moveable along the levers $F G$, they may always be placed in such a manner as to obtain that ratio between the power and the resistance, which will produce a maximum effect. Hence it follows that advantage may be taken of the weakest breezes of wind to obtain a certain product in circumstances under which all other wind-mills are in a state of absolute inactivity. This advantage, says M. Prony, is of great importance, particularly with regard to agriculture; the wind-mills employed for watering land are sometimes inactive for several days, and this inconvenience is more particularly felt in times of drought. A machine capable of moving with the slightest breeze must therefore offer the most valuable advantages. See *Annales des Arts et Manufactures*, tom. xix.

Mr. Samuel's Contrivance for Regulating the Action of Horses when driving Machinery.

This contrivance was invented by Mr. Walter Samuel, smith at Niddrey, Linlithgowshire, and is now in actual use in many thrashing-mills in Scotland. It is represented as applied to a thrashing-machine driven by four horses in Plate VII, Fig. 18, and to a machine driven by six horses in Fig. 19.

In Fig. 18, which is a plan of the perpendicular axle and levers, $A A$ represent the axle or shaft, in which are fixed the levers or arms, B, B, B, B, that carry the great wheel, and by which the machine is turned round when working.

C C are two frames fixed in the axle *A*, and supported by the arms B. Upon these frames are placed the two shifting-blocks 11 and 14, which are easily moved either inward to the axle *A*, or outward from it. In each of these shifting-blocks are placed two running sheaves.

D D is an endless rope or chain, which passes over the two sheaves that are placed in the shifting-blocks at the ends 12 and 13. By this rope the blocks are so connected, that when one is pulled outward from the axle *A*, the other is pulled inward alternately.

E, *E* are two sheaves by which the rope *D D* is kept clear of the axle *A*, when turning round.

E, *F* and *E*, *F* are ropes which pass over the sheaves that are placed in the shifting-blocks at the ends 11 and 14. Upon each end of the ropes *F F* is fastened a small block, in which are placed the running sheaves *G*, *G*, *G*, *G*, and over these sheaves pass the double ropes by which the horses pull when working the machine.

H, *I*, *K*, *L*, *M*, *N*, and *O*, *P* represent the limbers, which are fixed with screw-bolts, near the extremity of the arms B, B. In each of these limbers are placed two running sheaves, which conduct the ropes with their hooks to the line of draught.

Each horse is yoked to two hooks, as 1, 2 ; 3, 4 ; 5, 6 ; 7, 8 ; which are fastened on the ropes that pass over the sheaves *G*, *G*, *G*, *G* ; the sheaves having liberty to turn on their axes. By this means the draught will always press the collars equally upon the horses' shoulders, whether they incline to pull outward from the centre of their walk, or inward to that centre. And therefore, though the horses are walking in a circle, yet the strain of the draught must press fair on their shoulders, without twisting to either side. This advantage cannot be obtained in the ordinary way of yoking the horses in a thrashing-machine, unless the draught-chains on each side of the shoulders be made in exact proportion to the diameter of the circle on which they walk ; or the chain next to the centre of their walk made a degree shorter than the one farthest from it, which is often neglected.

The draught-ropes being thus all connected by the endless rope *D D*, and the shifting-blocks having liberty to move, either inward to the axle *A*, or outward from it, it follows, that if one of the horses relax, the other horse must imme-

diately press the collar to his shoulders, and by this means he is excited to exertion. For instance, if the horse yoked to the hooks at 1, 2, were to relax, then the one yoked at 5, 6, would instantly take up his rope, and pull the collar close to his shoulders, so that the horse at 1, 2, must either exert himself, or be pulled backward. And supposing the horses at 1, 2, and 5, 6, both to relax, then the exertions of the horses at 3, 4, and 7, 8, would pull the shifting-blocks from 11 toward 14, which would tend to drag the horses at 1, 2, and 5, 6, backward, and force them to exertion: So that each horse spurs up his companion in consequence of being all connected by the ropes and shifting-blocks. Thus the exertion of all the horses completely round the circle is united, so as to form one power applied to the machine, instead of as many independent powers as there are horses employed.

As it may sometimes be convenient to employ fewer horses than the whole number of which the machine admits, this is provided for so as still to unite the power of the horses that are employed. Upon the rope *F* is fastened an iron hook, which is hooked into the eye of a bolt fixed in one of the arms as at *R*, and thus the horse at 7, 8, is left out of the circle, whilst the power of the horses at 1, 2; 3, 4; 5, 6, are still united.

Plate VII, Fig. 19.

In Fig. 19, which is a plan of the perpendicular axle and levers of a thrashing-machine driven by six horses, *A* is the axle in which are fixed the arms B, B, B, B, B, B, which carry the great wheel, and upon these arms are fixed with screw-bolts the limbers, in each of which are placed two sheaves, that conduct the draught-ropes or chains to the line of draught.

C, *D*, *E*, are three frames fixed in the axle *A*, and supported by the arms B. Upon each of these frames are placed a shifting-block 1, 4, 6, having freedom to move either inward to the axle *A*, or outward from it. Into each of these shifting-blocks are placed two running sheaves, over three of which passes the endless rope or chain 7, 8, 9. Thus the three blocks with their sheaves are so connected, that if any one is pulled outward from the axle A, the other two blocks must shift inward.

There is also a rope which passes over each of the sheaves that are placed in the shifting-blocks at 1, 4, 6. And upon the

ends of these ropes, at 10, 11, 12, 13, 14, and 15, are fastened a small block with a running sheave in each, and over these sheaves the ropes or chains pass, by which the horses pull or draw, when working the machine.

F, *G*; *H*, *I*; *K*, *L*; *M*, *N*; *O*, *P*; and *Q*, *R*, represent the limbers fixed on the arms *B*, with screwed bolts. Into each of these limbers are placed two running sheaves, which direct the ropes 10, 11, 12, 13, 14, and 15, to the line of draught.

Each horse is yoked to these ropes by the hooks at *S*, *T*, *U*, *V*, *W*, and *X*. Thus, by the endless chain, shifting-blocks, and running sheaves, all the ropes by which the horses pull are connected, and of course the power of all the horses is united, whatever number may be employed. This apparatus will apply to two horses, or to any greater number that may be found necessary.

See Mr. Samuel's pamphlet, entitled "*Account of a New Apparatus for Yoking Horses in Thrashing Machines*," Edin. 1811, from which we have taken the preceding descriptions; also the *Farmer's Magazine*, vol xi, p. 492, and vol. xiii, p. 279.

An account of various other contrivances for regulating machinery will be found in the *Edinburgh Encyclopædia*, art. *Mechanics*, vol. xiii, p. 589, &c.

CHAPTER VI.

ON THE MAXIMUM EFFECT OF MACHINES.

When any power is made to act in opposition to a given resistance, by the intervention either of a simple or a compound machine, an equilibrium will take place when the velocity of the power is to the velocity of the resistance as the weight is to the power. In this state of things, however, the machine must be actually at rest, and therefore incapable of performing any work. If we can increase the power, the machine will move with more and more velocity, and will have its motion gradually accelerated as long as the power exceeds the resistance. But if from any cause the power should begin to diminish, or if the resistance should increase, or if both these changes on the state of the machine should take place at the same time, the acceleration of

the machine will diminish, and it will at last arrive at a state of uniform motion. Now, this increase of resistance may arise in many cases from an increase of friction, which often (though not always) accompanies an augmentation of velocity; or it may arise from the resistance of the air, which must necessarily increase with the velocity; and therefore all machines are found soon to attain a state of uniform motion. When an undershot wheel is driven by the impulse of water, the uniformity of motion to which it arrives, arises principally from the diminution of the power which in this case accompanies an increase of velocity. When the mass of fluid strikes one of the float boards at rest, the impulse is then a maximum. When the float-board is in motion, it withdraws itself, as it were, from the action of the power, and therefore its mechanical effect will diminish as the velocity increases, and if it were possible that the velocity of the wheel should become equal to that of the fluid, the float-board would not be struck at all by the moving water. Hence it follows, that the power itself diminishes by an increase of velocity, and therefore that from this cause alone machines in general would soon acquire a motion sensibly uniform. This effect will be more easily understood, if we suppose an axle to be put in motion by two currents of water, moving with different velocities, and driving two wheels, one of which is placed at each extremity of the axle. When the wheels have begun to move, by the joint action of these falls of water, the motion will at first be slow, and each fall of water will perform its part in giving motion to the axle; but if the greater fall is capable, by the continuance of its action, of giving its wheel a velocity either equal to or greater than the velocity of the smaller fall, then it is manifest that the smaller fall ceases to impel its wheel, and that the whole effect is produced by the action of the greater fall. Hence it is easy to understand from this statement, not only why a diminution of the impelling power accompanies an increase of velocity, but why there is a certain velocity of the machine, which is necessary before we can gain all the useful effect which we wish to have from the powers which we employ.

In order to illustrate this in the case of a real machine, let us suppose that the power of a man is to be employed in raising a load, by means of the machine or walking crane. This machine consists of a large wheel placed upon an axis, round

which is coiled a rope, to the lower extremity of which a weight or resistance is attached; the man walks upon the interior of the wheel, and by his weight gives it a rotatory motion, and therefore coils the rope round the axis, and elevates the weight. Let us suppose that the drum or wheel is so constructed, like the fusee of a watch, that the man can walk to different distances from the axis; and let P be the power or weight of the man, R the weight to be raised, and y the radius of the axle; then since $R : P = y : \frac{P y}{R}$, the distance from the centre of the wheel, at which the man must place himself, in order to be in equilibrio with the resistance R. But as the machine must be moved, and the weight raised, the man must go to a greater distance from the axis than $\frac{P y}{R}$; the motion of the machine will therefore accelerate, and the acceleration would increase if he moved to a greater and greater distance from the centre of the wheel. Hence it is obvious, that as the acceleration increases, the man must walk with greater and greater velocity; but there is an obvious limit to this, for he would soon be fatigued by the rapid walking, and would therefore be rendered unfit to continue his work. He must therefore return to that distance from the axis, where the wheel has such a velocity that he can continue to move with that velocity during the period that his work is to last. There is therefore a particular velocity with which the man must walk, in order to perform the greatest quantity of work; and it would be easy to find this, if we knew the law according to which his force diminished as his velocity increased. We may suppose, however, that his force diminishes in the same ratio as his velocity increases.

Let P represent the force which a man at rest can exert during a given time against a weight or load. This force will obviously be greater than any which he could exert when in motion; for a part of his strength is expended in putting himself in motion, and in continuing it. Let V be the velocity at which he would lose the power of exerting any force; then if he moves with any velocity v less than V, he will obviously lose a part of his maximum force P, and the part lost will be found from the hypothesis, by the analogy $V : v = P : \frac{P v}{V}$. The

real force, therefore, which he exerts against the weight will be $P - \frac{P v}{V}$, or, by separating its two factors, $P\left(1 - \frac{v}{V}\right)$. Now, if x is the length of lever by which this force acts, R the resistance or weight raised, y the radius of the axle, and U the velocity of the weight; then when the machine has obtained a uniform motion, we shall have $P\left(1 - \frac{v}{V}\right) x = R\, y$; but since $x : y = v : U$, we have by substitution $P\left(1 - \frac{v}{V}\right) v = R \times U$, which is the equation to a circle, and this effect will be a maximum when $v = \frac{V}{2}$; and substituting this value of v, we shall have $R \times U = \frac{P V}{4}$; that is, the man will do most work when he moves with half of his greatest velocity, and in that case the greatest effect will be $\frac{P V}{4}$.

It appears, however, from direct experiments, that the force of a man diminishes as the square of his velocity increases; and therefore we must take $V^2 : v^2 = P : \frac{P v^2}{V^2}$, and then we shall have the formula

$$P\left(1 - \frac{v}{V}\right)^2 v = R\, U,$$

an equation to a parabola, which will be a maximum when $v = \frac{V}{3}$, and $R\, U = \frac{4 P}{9}$; that is, the force of a man, or any other animal, will be a maximum when they move with a velocity equal to one-third of their greatest velocity, and when the load is $\frac{4}{9}$ths of the greatest load which they are able to put in motion.

Having thus considered the maximum effect of living agents, we shall proceed to the subject of machines, and shall take the case of a wheel and axle, as almost all other machines may be considered as reduced to this.

It would be inconsistent with the nature of this work to give the investigation of the formula from which the following table is computed. It may be sufficient to state, that Dr.

Robison has shewn that the ratio of the power to the resistance when the effect is a maximum, is represented by the formula $y = x\sqrt{\frac{P}{R} + 1} - 1$, where x represents the radius of the wheel to which the power is applied, and y that of the axle by which the resistance acts. The inertia of the machinery and the friction are supposed to be very small, which is the case in pulleys and simple machines. In the table x is supposed to be 1, and R 10.

Table containing the best Proportions between the Velocities of the Impelled and Working Points of a Machine, or between the Levers by which the Power and Resistance act.

Proportional value of the impelling power, or P.	Value of the velocities of the working point, or y, or of the lever by which the resistance acts, that of x being 1.	Proportional value of the impelling power, or P.	Value of the velocities of the working point, or y, or of the lever by which the resistance acts, that of x being 1.
1	0.048809	20	0.732051
2	0.095445	21	0.760682
3	0.140175	22	0.788854
4	0.183216	23	0.816590
5	0.224745	24	0.843900
6	0.264911	25	0.870800
7	0.303841	26	0.897300
8	0.341641	27	0.923500
9	0.378405	28	0.949400
10	0.414211	29	0.974800
11	0.449138	30	1.000000
12	0.483240	40	1.236200
13	0.516575	50	1.449500
14	0.549193	60	1.645700
15	0.581139	70	1.828400
16	0.612451	80	2.000000
17	0.643168	90	2.162300
18	0.673320	100	2.316600
19	0.702938		

In order to understand the method of using the preceding Table, let us suppose that we wish to raise *two* cubic feet of water in a second, by means of the power of a stream which affords five cubic feet of water in a second, applied to a wheel

and axle, the diameter of the wheel being 7 feet. It is required, therefore, to find the diameter which we must give to the axle, in order to obtain a maximum effect. We have obviously $P = 5$, and $R = 2$ and $P = \frac{5}{2} R$; but, in the above table, $R = 10$: hence $P = \frac{5}{2} 10 = 25$. Now, it appears from the table, that when $P = 25$, the diameter of the axle, or y, is 0.8708 when $x = 1$; but as $x = 7$, the diameter of the axle must be $7 \times 0.8708 = 6.0956$.

When a machine is already constructed, the velocities of its impelled and working points are determined; and therefore, in order to obtain from it its maximum effect, we must seek for the best proportion between the power and the resistance, as these are the only circumstances over which we have any control, without altering the machinery.

The following table is computed from Dr. Robison's formula

$$R = \frac{\sqrt{y + 1} - 1}{y^2}.$$

Table containing the best Proportions between the Power and the Resistance, the inertia of the Impelling Power being the same with its pressure, and the friction and inertia of the Machine being omitted.

Values of y, or the velocity of the working point, x being equal to 1.	Values of R, or the resistance to be overcome, P being = 1.	Ratio of R to the resistance which would balance P.	Values of y, or the velocity of the working point, x being equal to 1.	Values of R, or the resistance to be overcome, P being = 1.	Ratio of R to the resistance which would balance P.
$\frac{1}{4}$	1.8885	0.4724 to 1	7	0.03731	0.26117 to 1
$\frac{1}{3}$	1.3928	0.4639 ——	8	0.03125	0.25000 ——
$\frac{1}{2}$	0.8986	0.4493 ——	9	0.02669	0.24021 ——
1	0.4142	0.4142 ——	10	0.02317	0.23170 ——
2	0.1830	0.3660 ——	11	0.02037	0.22407 ——
3	0.1111	0.3333 ——	12	0.01809	0.21708 ——
4	0.0772	0.3088 ——	13	0.01622	0.21086 ——
5	0.0580	0.2900 ——	14	0.01466	0.20524 ——
6	0.0457	0.2742 ——	15	0.01333	0.19995 ——

In order to understand the method of using this Table, let us suppose that it is required to find the value of the resistance, or the quantity of water which must be put into a bucket to be raised by a wheel and axle, in which the radius of the wheel is 6 feet, and that of the axle 2 feet, and with a power = 8. Since, in the table, $x = 1$, we have $y = \frac{2}{6} = \frac{1}{3}$, which corre-

sponds in the table to 1.3928, the value of R when $P = 1$. But, in the present case, $P = 8$, consequently $1 : 8 = 1.3928 : 11.1424$, the value of R when $P = 8$.

CHAPTER VII.

ON WHEEL-CARRIAGES.

Wheel carriages.

Mr. Ferguson, in his fourth lecture, has treated the subject of wheel-carriages with great perspicuity, and has communicated much practical information of considerable importance. Many of the prejudices, however, which he has there endeavoured to remove, and several others which have escaped his notice, still continue to prevail in this country; and as some of these have been countenanced even by ingenious men, we are laid under a more urgent necessity of attempting to develope the source of their errors, and of regulating the practice of the mechanic by the deductions of theory. The very assistance which theory has, in this case, furnished to the artist, has been rendered not only useless, but injurious, by an erroneous application; and we may safely affirm, that there is no species of machinery where less science is displayed than in the construction and position of carriage-wheels. The few imperfect hints, which we are able to convey upon this subject, regard the formation and position of the wheels, the line of traction, and the method of disposing the load which is to be drawn.

On the Size and Form of Carriage-Wheels.

Wheels act as mechanical powers.

When the wheels of carriages either move upon a level surface, or overcome obstacles which impede their progress, they act as mechanical powers, and may be reduced to levers of the first kind. In order to elucidate this remark, let A be the centre, and $B\,C\,N$ the circumference of a wheel 6 feet in diameter and let the impelling power P, which is attached to the extremity of a rope $A\,D\,P$, passing over the pulley D, act in the horizontal direction $A\,D$. Then, if the wheel is not affected by friction, it will be put in motion upon the level surface $M\,B$, when the power P is infinitely small. For since the whole

Plate VII. Fig. 20.

weight of the wheel rests on the ground at the point *B*, which is the fulcrum of the lever *A B*, the distance of the weight from the centre of motion will be nothing, and therefore the mechanical energy of the smallest power *P*, acting at the point *A*, with a length of lever *A B*, will be infinitely great when compared with the resistance of the weight to be raised ; and this will be the case, however small be the lever *A B*, and however great be the weight of the wheel. But as the wheels of carriages are constantly meeting with impediments, let *C* be an obstacle 6 inches high, which the wheel is to surmount. Then the spoke *A C* will represent the lever, *C* its fulcrum, *A D* the direction of the power ; and if the wheel weighs 100 pounds, we may represent it by a weight *W* fixed to the wheel's centre *A*, or to the extremity of the lever *C A*, and acting in the perpendicular direction *A B*, in opposition to the power *P*. Now, the mechanical energy of the weight *W* to pull the lever round its fulcrum in the direction *A E*, is represented by *C E*, while the mechanical energy of an equal weight *P* to pull it in the opposite direction *A F*, is represented by *C F* ; an equilibrium, therefore, will be produced, if the power *P* is to the weight *W* as *C E* to *C F*, or as the sine is to the cosine of an angle, whose versed sine is equal to the height of the obstacle to be surmounted ; for *E B*, the height of the mound *C*, is the versed sine of the angle *B A C*, and *C E* is the sine, and *C F* the cosine of the same angle. In the present case, where *E B* is 6 inches, and *A B* 3 feet, *E B*, the versed sine, will be 166, &c. when *A B* is 1000 ; and, consequently, the angle *B A C* will will be 33° 33′, and *C E* will be to *E F* as 52 to 83, or as 66 to 100. A weight *P*, therefore, of 66 pounds, acting in a horizontal direction, will balance a wheel 6 feet diameter, and 100 pounds in weight, upon an obstacle 6 inches high ; and a small additional power will enable it to surmount that obstacle. But if the direction *A D* of the power be inclined to the horizon, so that the point *D* may rise towards *H*, the line *F C*, which represents the mechanical energy of *P*, will gradually increase, till *D A* has reached the position *H A*, perpendicular to *A C*, where its mechanical energy, which is now a maximum, is represented by *A C* the radius of the wheel ; and since *E C* is to *C A* as 53 to 100, a little more than 53 pounds will be sufficient for enabling the wheel to overcome the obstacle.

Proceeding in this way, it will be found that the power of wheels to surmount eminences increases with their diameter, and is directly proportional to it, when their weight remains the same, and when the direction of the power is perpendicular to the lever which acts against the obstacle. Hence we see the great advantages which are to be derived from large wheels, and the disadvantages which attend small ones. There are some circumstances, however, which confine us within certain limits in the use of large wheels.

Advantages of large wheels.

When the radius *A B* of the wheel is greater than *D M* the height of the pulley, or of that part of the horse to which the rope or pole *D A* is attached, the direction of the power, or the line of traction *A D*, will be oblique to the horizon, as *A d*, and the mechanical energy of the power will be only *A e*, whereas it was represented by *A E* when the line of traction was in the horizontal line *D A*. Whenever the radius of the wheel, therefore, exceeds *four feet and a half*, the height of that part of the horse to which the traces should be attached,[1] the line of traction *A D* will incline to the horizon, and, by declining from the perpendicular *A H*, its mechanical effort will be diminished; and, since the load rests upon an inclined plane, the trams, or poles of the cart will rub against the flanks of the horse, even in level roads, and still more severely in descending ground. Notwithstanding this diminution of force, however, arising from the unavoidable obliquity of the the impelling power, wheels exceeding four and a half feet radius have still the advantage of smaller ones; but their power to overcome resistances does not increase so fast as before. Hitherto we have supposed the weight of the large and small wheels to be the same; but it is evident, that, when we augment their diameter, we add greatly to their weight; and, by thus increasing the load, we sensibly diminish their power. Another disadvantage of large wheels is to increase the risk of overturning the carriage, by the elevation of the centre of gravity.

Dr. Young[2] has remarked, that if the curvature of the obstacle to be overcome should happen to be intermediate be-

[1] According to M. Couplet, the distance of this part of the horse from the ground is generally *three feet and a half*. (*Mem. de l'Acad. Paris*, 1733, 8vo, p. 75). In horses of a common size, however, it is seldom below *four feet and a half*.

[2] *Lectures on Natural Philosophy*, vol. i, p. 219.

tween those of a larger and a smaller wheel; then in this case the larger wheel will touch a remote part of the obstacle, so that the path of the axle will form an abrupt angle, while the smaller wheel following the curve of the obstacle produces a more equable motion. This, however, Dr. Young adds, is a case of rare occurrence, and an advantage of little importance.

The advantages of large wheels are particularly great, as Dr. Young has remarked, in soft and boggy soils, and in sandy countries, and he finds that the resistance which is in these cases opposed to the motion of a given wheel, may be reduced to one half, either by making the wheel a little less than three times as high, or by making it eight times as broad as the given wheel. Camus in his *Traité des Forces Mouvantes*, has given an account of a great number of experiments on the superiority of large wheels, under a variety of circumstances.

From these remarks, we see the superiority of great wheels over small ones, and the particular circumstances which suggest the propriety of making the wheels of carriages less than $4\frac{1}{2}$ feet radius. Even this size is too great, as we shall afterwards shew, when speaking of the line of traction; and we may safely assert, that they ought never to exceed 6 feet in diameter, and should never be less than $3\frac{1}{2}$ feet. When the nature of the machine will permit, large wheels should always be preferred, and small ones should never be adopted, unless we are compelled to employ them by some unavoidable circumstances in the construction.[3] This maxim, which has been inculcated by every person who has written on the subject, seems to have been strangely neglected by the practical mechanics of this country. The fore-wheels of our carriages are still unaccountably small, and we have seen carts moving upon wheels scarcely *fourteen* inches in diameter. The workman, indeed, will tell us, that, in the one case, the wheels are made small for the conveniency of turning,

The fore-wheels of carriages too small.

[3] For the advantage of those who wish to study this subject with greater attention, and with the view also of recommending the use of large wheels, we shall subjoin the following references to the works of eminent men, who have held the same opinion upon this point: Mersennus' *Geomet.* p. 459; Herigon, *Mecan.* prob. xvi, Schol.; Wallis' *Mecan.* c. vii, prob. 3, Schol. § 15; *Phil. Trans.* vol. xv, p. 856; Camus' *Traité des Forces Mouvantes*, prop. xxviii, xxx; and Deparcieux *Sur le Tirage des Chevaux*, in the *Mem. de l'Acad. Paris*, 1760, p. 263, 4to.

and, in the other, for facilitating the loading of the cart; but how trifling are these advantages when compared with that diminution of the horses' power which necessarily results from the use of small wheels. A convenient place for turning with large fore-wheels, which is not frequently required, may be procured by going to the end of a street; and a few additional turns of a windlass will be sufficient to raise the heaviest load into carts which are mounted upon high wheels. It has been objected against large fore-wheels, that the horses, when going down a declivity, cannot so easily prevent the carriages from running downwards; but this very objection is an acknowledgment that large fore-wheels are advantageous, both in horizontal and inclined planes, otherwise their tendency downwards would not be greater than that of small ones.[4]

On the form of the wheels.

Having thus ascertained the superiority of large wheels, we are now to determine the shape which ought to be given to them. Every person, who is not influenced by preconceived notions, would affirm, without hesitation, that, if the wheels are to consist of solid wood, they should be portions of a cylinder; and if they are to be composed of naves, spokes, and fellies, that the rim of the wheel ought to be cylindrical, and the spokes perpendicular to the naves. But mechanics have renounced this simple shape, and adopted the more complicated form of Fig. 21, where the rim $BsrA$ is conical, and the spokes inclined to the naves.[5] Philosophers, too, have found a reason for this change, and it has been adopted in every country, more from the authority of names, than the force of argument.[6] It is with the greatest diffidence, however, that we presume to contradict a practice which has been defended by the most celebrated mechanics.

[4] From some experiments on wheel-carriages, Mr. Walker conceives that the greatest advantage was obtained when the hind-wheels were 5 feet 6 inches in diameter, and the fore ones 4 feet 8 inches, whereas the large wheels are, in general, only 4 feet 8 inches, and the small ones 3 feet 8 inches. *System of Familiar Philosophy*, vol. i, p. 130.

[5] This inclination is about 1 inch in 11, or A is generally 3 inches when the diameter of the wheel is $5\frac{1}{2}$ feet.

[6] I have seen some carriage-wheels, in which one half of the spokes were inclined about one fourth more than the other half, every alternate spoke being equally inclined to the axis. The reasons for such a construction it is not easy to discover.

Advantages and disadvantages of concave dishing wheels.

The form represented in Fig. 21 is liable to two objections, namely, the inclination of the spokes, and the conical figure of the rim. When the spokes are inclined to the nave, the wheels are said to be *concave*, or *dishing*, and they are recommended by Mr. Ferguson, and every other writer on mechanics, from the numerous advantages which are said to attend them. By extending the base of the carriage, they prevent it from being easily overturned; they hinder the fellies from rubbing against the load or the sides of the cart; and when one wheel falls into a rut, and therefore supports more than one-half of the load, the spokes are brought into a perpendicular position, which renders them more capable of supporting this additional weight.[7] Now, it is evident, that the second of these advantages is very trifling, and may be obtained when it is wanted, by interposing a piece of board between the wheel and the load. The other two advantages exist only in very bad roads; and if they are necessary, which we very much question, in a country like this, where the roads are so excellently made, and so regularly repaired, they can easily be procured by making the axle-tree a few inches longer, and increasing the strength of the spokes. But it is allowed, on all hands, that perpendicular spokes are preferable on level ground. The inclination of the spokes, therefore, which renders concave wheels advantageous in rugged and unequal roads, renders them disadvantageous when the roads are in good order; and where the good roads are more numerous than the bad ones, as they certainly are in this country, the disadvantages of concave wheels must overbalance their advantages. It is true, indeed, that in concave wheels, the spokes are in their strongest position when they are exposed to the severest strains; that is, when one wheel is in a deep rut, and sustains more than one half of the load; but it is equally true, that in level ground, where the spokes are in their weakest position, a less severe strain, by continuing for a much longer time, may be equally, if not more, detrimental to the wheel.[8]

Inclination of the spokes disadvantageous in general.

[7] See Desaguliers' *Natural Philosophy*, vol. i, lect. 4, prop. xxv, or Camus' *Traité des Forces Mouvantes*, chap. iv, lect. 5, which Desaguliers has copied.

[8] Mr. Anstice, in his excellent treatise on wheel-carriages, recommends concave wheels, but candidly allows, that " some disadvantages attend this contrivance;

And more expensive.

It was observed, that concave wheels are more expensive than plain ones. This additional expense arises from the greater quantity of wood and workmanship which the former require; for, in order that dishing wheels may be of the same perpendicular height as plane ones, the spokes of the former must exceed in length those of the latter, as much as the hypothenuse $o\,A$ of the triangle $o\,A\,m$ exceeds the side $o\,m$ (Fig 21); and therefore the weight and the resistance of such wheels must be proportionally great. The inclined spokes, too, cannot be formed or inserted with such facility as perpendicular ones. The extremity of the spoke which is fixed into the nave is inserted at right angles to it, in the direction $o\,p$, and if the rims are cylindrical, the other extremity of the spoke should be inserted in a similar manner, while the intermediate portion has an inclined position. There are, therefore, two flexures or bendings in the spokes of concave wheels, which require them to be formed out of a larger piece of wood than if they had no such flexures, and render them liable to be broken by any sudden strain at the points of flexure.

In the comparison which we have now been stating, between the merits of concave and plane wheels, we have taken for granted what has been uniformly stated by the advocates of the former, that when one of the wheels falls into a rut or surmounts an eminence, the lowest sustains much more than one half of the load. Now, though it be true that the lowest wheel supports more than one half of the load, yet we deny that it bears so much as has generally been supposed,[9] and we may prove the assertion, by pointing out a method of ascertaining the additional weight which is transferred to one wheel by any given elevation of the other.

Proportion of the load borne by each wheel when one of them is surmounting an eminence.

Let $A\,M\,O\,C$ (Fig. 22) represent a cart loaded with coals or lime, or any other material which fills it to the top, and let $A\,B$ be a horizontal line on the surface of a level road. Then, if the wheel A remains fixed, and the wheel C is raised to any height, its lower extremity C will

for the carriage thus takes up more room upon the road, which makes it more unmanageable; and when it moves upon plain ground, the spokes not only do not bear perpendicularly, by which means their strength is lessened, but the friction upon the nave and axle is made unequal, and the more so the more they are dished."

[9] Mr. Ferguson observes (Vol. I, p. 76), that the wheel, which falls into the

describe the arch $B\ O$ round the centre A, while the centre of gravity D of the whole machine and load will move in the arch $N M$ round the same centre. Now, let us suppose that $B\ C$ is an eminence which the wheel C has to surmount, and that it has arrived at the top of it; it is required to find what proportion of the load is sustained by each wheel. Bisect the the horizontal line $A B$ in e, and from e, draw $e\,d$ at right angles to $A\ B$, and meeting the arch $N\ M$ in the point d, join $A\ C$, $A\ d$, $A\ D$, and from the point D let fall the perpendicular $D\ E$. The point d will be the centre of gravity of the load when the points C and B coincide; that is, when the wheels are resting on the horizontal plane $A\ B$. For since, in this case, each wheel bears an equal part of the weight, the line of direction, or a vertical line passing through the centre of gravity, will cut the base $A\ B$, so that $A\ e$ will be to $e\ B$ as the weight upon the wheel A to the weight upon C; and therefore $e\,d$ will be the line of direction, and the point d, where it cuts the circle $N\ M$, in which the centre of gravity moves, will be the centre of gravity of the load in a horizontal position. Now as D is the centre of gravity when the cart is in its inclined position, the perpendicular $D\ E$ will be the line of direction, and the weight sustained by the wheel A will be to that sustained by C as $E\ B$ to $E\ A$, or $E\ e$ will represent the additional weight transferred upon A, when $A\ B$ represents the whole of the load. But $E\ e$ can be easily determined for any value of $B\ C$, the height of the obstacle. For, while the point C moves from B to C, the centre of gravity rises from d to D, so that $D\ d$ and $B\ C$ are similar arches, and $A\ B$, $A\ d$, $B\ C$, are known, $A\ B$ being the distance between the wheels, and $A\ d$ being equal to the square root of the sum of the squares of $A\ E$, the half of that distance, and $d\ e$ the height of the centre of gravity (Eucl. 1, 47), and $B\ C$ being the height of the eminence. But since $d\,e$, the sine of the arch $d\,N$, is known, $d\,N$ is known, and also $D\ N$, the sum of the two arches $D\,d$, $d\,N$. The cosines $A\ E$, $A\ e$, of the arches $D\ N$, $d\ N$, are therefore known, and consequently $E\ e$, their difference may be determined; or, otherwise, $E\ e$ is the difference of the versed sines $E\ N$, $e\ N$, of the same arches. Let us now take a particular value of $B\ C$, or rather of $C\ o$, the perpendicular

rut, bears *much more* of the weight than the other; and, a little afterwards, that it bears *most of* the weight of the load.

height of the eminence, and call it 12 inches; for even in the worst roads there are few eminences which are greater than this. Let *A B*, the distance between the wheels, be 6 feet, and *d e*, the height of the centre of gravity, 4 feet, then *C o* will be $\frac{1}{6}$ of the radius *A B*, and making *A B* = 1000000, *C O* will be 166666, which, being the natural sine of the arch *B C*, gives 9° 35′ for the arch *B C*, and for the similar arch *D d*. Now, since *A e* is 3 feet, and *d e* 4 feet, the sum of their squares will be 25, and its square root 5 will be the length of the hypothenuse *A d*, or the radius of the circle *N D M*. Then, making *A d* radius, or 1000000, *d e*, the sine of the arch *d N*, will be $\frac{4}{5}$ of it, or 800000; and therefore the arch *d N* will be 53° 8′, and the arch *D N* 62° 43′. But *A E*, the cosine of the arch *D N*, is = 458391 = or $\frac{46}{100}$, nearly, of *A D* = 5 feet, and is therefore equal to 2 feet 3 inches and 6 tenths; consequently *E e* = *A e* — *A E* will be 8 inches and 4 tenths, which is nearly $\frac{1}{9}$ of *A B*. We may therefore conclude, that the additional weight sustained by the wheel *A*, while the other wheel is rising over an obstacle 12 inches in perpendicular height, is $\frac{1}{9}$ only of the whole load, or that $\frac{1}{9}$ of the pressure upon the wheel *C* is transferred to the wheel *A*, while surmounting an eminence 12 inches high. If one of the wheels falls into a rut 12 inches deep, the same conclusion will result; and we may affirm, that as the ruts and eminences which are generally to be met with, even in bad roads, are for the most part much less than 12 inches in depth or height, such a small proportion of the load will be transferred to the lowest wheel, that there is no necessity for inclining the spokes in order to sustain the additional weight. When the cart is loaded with stones, or any heavy substance, the centre of gravity will be lower than *d*, so that a less proportion of the weight will be transferred to one wheel by the elevation of the other; and when it is loaded with hay, or any light material, the lowest wheel will sustain a greater proportion of the load.

Concave wheels easily injured.

We shall now dismiss the subject of concave wheels with one observation more, and we beg the reader's attention to it, because it appears to be decisive of the question. The obstacles which carriages have to encounter are almost never spherical protuberances, which permit the elevated wheel to resume by degrees its horizontal position. They are generally of such a nature, that the wheel is instan-

taneously precipitated from their top to the level ground. Now the momentum with which the wheel strikes the ground is very great, arising from a successive accumulation of force. The velocity of the wheel *C* is considerable when it reaches the top of the eminence, and while it is tumbling into the horizontal line *A B* (Fig. 22), the centre of gravity is falling through the arch *D d*, and the wheel *C* is receiving gradually that proportion of the load which was transferred to *A*, till, having recovered the whole, it impinges against the ground with great velocity and force. But in concave wheels, the spoke which then strikes the ground is in its weakest position, and therefore much more liable to be broken by the impetus of the fall, than the spokes of the lowest wheel by the mere transference of additional weight. Whereas if the spokes be perpendicular to the nave, they receive this sudden shock in their strongest position, and are in no danger of giving way to the strain.

Conical rims disadvantageous.

In the preceding observations, we have supposed the rims of the wheels to be cylindrical, as *A C*, *B D* (Fig. 21). In concave wheels, however, the rims are uniformly made of a conical form, as *A r*, *s B*, which not only increases the disadvantages that we have ascribed to them, but adds many more to the number. Mr. Cumming, in a late treatise on wheel-carriages, solely devoted to the consideration of this single point, has shewn, with great ability, the disadvantages of conical rims, and the propriety of making them cylindrical. The defects of conical rims are so numerous and palpable, that it is wonderful how they should have been so long overlooked. Every cone that is put in motion upon a plane surface will revolve round its vertex, and if force is employed to confine it to a straight line, the smaller parts of the cone will be dragged along the ground, and the friction greatly increased. Now when a cart moves upon conical wheels, one part of the cone rolls while the other is dragged along, and though confined to a rectilineal direction by external force, their natural tendency to revolve round their vertex occasions a great and continued friction upon the linchpin, the shoulder of the axle-tree, and the sides of deep ruts.

The shape of the wheels being thus determined, we must now attend to some particular parts of their construction. The iron plates of which the rims are composed should never be less than 3 inches in breadth, as narrower rims sink deep into

the ground, and therefore injure the roads and fatigue the horses. Mr. Walker, indeed, attempts to throw ridicule upon the act of parliament which enjoined the use of broad wheels, but he does not assign any sufficient reason for his opinion, and ought to have known, that several excellent and well devised experiments were instituted by Boulard and Margueron,[1] which evinced, in the most satisfactory manner, the great utility of broad wheels. When any load is supported upon two points, each point supports one half of the weight; if the points are increased to four, each will sustain one fourth of the load, and so on, the pressure upon each point of support diminishing as the number of points increases. If a weight, therefore, is supported by a broad surface, the points of support are infinite in number, and each of them will bear an infinitely small portion of the load; and, in the same way, every finite portion of this surface will sustain a part of the weight inversely proportional to the number of similar portions which the surface contains. Let us now suppose that a cart, carrying a load of 16 hundred weight, is supported upon wheels whose rims are 4 inches in breadth, and that one of the wheels passes over 4 stones, each of them an inch broad, and equally high, and capable of being pulverized only by a pressure of 400 weight. Then, as each wheel sustains one half of the load, and as the wheel which passes over the stones has 4 points of support, each stone will bear a weight of 200 weight, and therefore will not be broken. But if the same cart, with rims only 2 inches in breadth, should pass the same way, it will cover only 2 of the stones; and the wheel having now only two points of support, each stone will be pressed with a weight of 400 weight, and will therefore be reduced to powder. Hence we may infer, that narrow wheels are, in another point of view, injurious to the roads, by pulverizing the materials of which they are composed.

Utility of broad wheels.

Practical Remarks.

As the rims of wheels wear soonest at their edges, they should be made thinner in the middle, and ought to be fastened to the fellies with nails of such a kind, that their heads may not rise above the surface of the rim. In some military waggons, we have seen the heads of these nails rising an inch above the rims, which not only destroys the pave-

[1] The memoir which contains an account of these experiments, was presented to the Academy of Lyons, and is published in the *Journal de Physique*, tom. xix, p. 424.

ments of streets, but opposes a continual resistance to the motion of the wheel. If these nails were eight in number, the wheel would experience the same resistance as if it had to surmount 8 obstacles, 1 inch high, during every revolution. The fellies on which the rims are fixed, should, in carriages, be $3\frac{1}{4}$ inches deep, and in waggons 4 inches. The naves should be thickest at the place where the spokes are inserted, and the holes in which the spokes are placed should not be bored quite through, as the grease upon the axle-tree would insinuate itself between the spoke and the nave, and prevent that close adhesion which is necessary to the strength of the wheel. A very great improvement in the nave has lately been made by Mr. Morton of Edinburgh, who constructs them all of cast iron, with openings ready to receive the wooden spokes.

On the Position of Carriage-Wheels.

Position of the wheels.

It must naturally occur to every person reflecting upon this subject, that the axle-trees should be straight, and the wheels perfectly parallel, so that they may not be wider at their highest than at their lowest point, whether they are of a conical or a cylindrical form. In this country, however, the wheels are always made concave, and the ends of the axle-trees are *universally* bent downwards, in order to make them spread at the top and approach nearer below. In some carriages which we have examined, where the wheels were only 4 feet 6 inches in diameter, the distance of the wheels at the top was fully 6 feet, and their distance below only 4 feet 8 inches. By this foolish practice, the very advantages which may be derived from the concavity of the wheels are completely taken away, while many of the disadvantages remain; more room is taken up in the coach-house, and the carriage is more liable to be overturned by the contraction of its base.

Disadvantages of bent axle-trees.

With some mechanics it is a practice to bend the ends of the axle-trees forwards, and thus make the wheels wider behind than before. This blunder has been strenuously defended by Mr. Henry Beighton, who maintains that wheels in this position are more favourable for turning, since, when the wheels are parallel, the outermost would press against the linch-pin, and the innermost would rub against the shoulder of the axle-tree. In rectilineal motions, however, these converging wheels engender a great deal of friction, both on the axle and on the

ground, and must therefore be more disadvantageous than parallel ones. This, indeed, is allowed by Mr. Beighton; but he seems to found his opinion upon this principle, that *as the roads are seldom straight lines*, the wheels should be more adapted for curvilineal than for rectilineal motion.

On the Line of Traction, the Length of Traces, and the Method by which Horses exert their Strength.

Line of Traction.

M. Camus, a gentleman of Lorrain, was the first person who treated of the line of traction.[2] He attempted to shew that it should be a horizontal line, or rather that it should always be parallel to the ground on which the carriage is moving, both because the horse can exert his greatest strength in this direction, and because the line of draught being perpendicular to the vertical spoke of the wheel, acts with the largest possible lever. M. Couplet,[3] however, considering that the roads are never perfectly level, and that the wheels are constantly surmounting small eminences, even in the best roads, recommends the line of traction to be oblique to the horizon. By this means the line of draught $H A$, Plate VII, Fig. 20, which is by far too much inclined in the figure, will in general be perpendicular to the lever $A C$ which mounts the eminence, and will therefore act with the longest lever when there is the greatest necessity for it. We ought to consider also, that when a horse pulls hard against any load, he always brings his breast nearer the ground, and therefore it follows, that if a horizontal line of traction is preferable to all others, the direction of the traces should be inclined to the horizon when the horse is at rest, in order that it may be horizontal when he lowers his breast and exerts his utmost force. If a carriage is drawn by several horses, as shewn in Fig. 23, the length of the traces has a considerable influence on the draught when the declivity changes. The following method of ascertaining and computing this effect has been given by M. Prony:—

Disadvantages of long traces.

Plate VII. Fig. 23.

From the point E, where the traces are fastened to the horse next the load, draw $E R$ to the same point in the second horse R, and let R' be another position of the second horse; it is required to find the difference of effect that will be produced by

[2] *Traité des Forces Mouvantes*, p. 387.

[3] *Reflexions sur le Tirage des Charettes*, in the *Mem. de l'Acad. Paris*, 1733, 8vo, pp. 75, 86.

placing the second horse at R or R', or the comparative advantages of short and long traces. From R', the point where the traces are fixed, draw $R'\,F'\,E$; and from E draw $E\,m\,n$ parallel to the declivity $D\,A$. Take $E\,F = E\,F'$ to represent the power of the horse in the direction of the traces, which will be the same whether he is yoked at R or at R'; draw $E\,A$ perpendicular to $D\,A$; $F\,n$, $F'\,m$ parallel to $E\,A$, and $F\,\varphi$, $F'\,f$ parallel to $E\,n$. Then since the second horse when at R pulls with a force represented by $F\,E$, in the direction $F\,E$, we may resolve this force into the two forces $E\,n$, $E\,\varphi$, one of which $E\,n$ is solely employed in dragging the cart up the inclined plane $D A$, while the other $E\,\varphi$ is solely employed in pressing the first horse E to the ground. Let the horse be now removed from R to R', the direction of the traces becomes $R\,F'\,E$ and $F'\,E = F\,E$ is the power exerted by the horse at R' and the direction in which it is exerted. But this force is equivalent to the forces $E\,m$, $E\,f$, the first of which acts directly against the load, while the other presses the horse against the ground. Hence we see the disadvantages of long traces, for the force which draws the load when the horse is at R' is to the force when the horse is at R, as $E\,m$ to $E\,n$, and the forces which press the horse upon the ground as $E\,f$ to $E\,\varphi$, or as $F'\,m$ to $F\,n$. Now $E\,\varphi = F\,n = F\,E \times \sin.\ n\,E\,F$; hence $E\,\varphi = F\,E \times \sin.\ \overline{(n\,E\,g' - F\,E\,g')}$ ($g'\,E$ being parallel to $A\,B'$), and $E\,n = E\,F \times \cos.\ (n\,E\,g' - F\,E\,g')$. In like manner we have $E f = F\,E \times \sin.\ (n\,E\,g' - F\,E\,g')$, and $E\,m = E\,F \times \cos.\ (n\,E\,g' - F\,E\,g')$. Now $\sin.\ F\,E\,g' = \sin.\ F\,E\,g = \frac{R\,g}{E R}$, and $\sin.\ F\,E\,g' = \frac{R'\,g'}{E\,R'} = \frac{R\,g}{E\,R'}$; but $R\,g = R'\,g' = B\,R - E\,Q = B\,R - B\,R \times \cos.\ n\,E\,g' = B\,R \times (1 - \cos.\ n\,E\,g)$. By substituting this value in the equations which contain the values of $E\,\varphi$, $E\,n$, $E f$, $E\,m$, and considering that the angles $F\,E\,g'$, $F'\,E\,g'$ are always so small that their arcs differ very little from their sines, we have

$$F\,E\,g = \frac{B\,R \times \overline{1 - \cos.\ n\,E\,g}}{E\,R}, \text{ and}$$

$$F'\,E\,g' = \frac{B\,R \times \overline{1 - \cos.\ n\,E\,g}}{E\,R'}.$$

By substituting these values in the preceding equations, we have

$$E\varphi = EF \times \sin.\ (nEg - \frac{\overline{BR \times 1 - \cos.\ nEg}}{ER},$$

$$Efn = EF \times \sin.\ (nEg - \frac{\overline{BR \times 1 - \cos.\ nEg}}{ER'},$$

$$En = EF \times \cos.\ (nEg - \frac{\overline{BR \times 1 - \cos.\ nEg}}{ER},$$

$$Em = EF \times \cos.\ (nEg - \frac{\overline{BR \times 1 - \cos.\ nEg}}{ER'}.$$

If AB is horizontal, and the declivity $AD = \frac{1}{6}$, we shall have $nEg = 9^\circ\ 28'$, or in parts of the radius $= 0.16522$, and cos. $nEg = 0.98638$. Then, if $EF = 200$ pounds, $BR = 3\frac{1}{2}$ feet, $ER = 8$ feet, $ER' = 12$ feet, then we shall have from the preceding formulæ, $E\varphi = 31.716$ pounds, $Ef = 32.350$ pounds, $En = 197.470$ pounds, and $Em = 197.404$. Hence an additional length of four feet to traces eight feet long, presses the horse E to the ground with an additional force of $32.250 - 31.716 = 0.534$ pounds, and diminishes the effect of the other horse by 0.066 pounds.

When horses are yoked abreast, they are commonly attached to the opposite ends of a lever or bar, unequally divided by the fulcrum in the ratio of the strength of the horses, the weaker horse pulling by the longer arm, and the stronger horse by the shorter arm. Dr. Young has remarked, that even without this inequality a compensation takes place, as shewn in Fig. 24, for the centre E on which the bar moves is always considerably behind the points of attachment A, B. Hence when one of them falls back a little so that the arm AB assumes the position CD, the foremost horse has the disadvantage of acting by a lever equal only to EF, while the other horse acts by a lever equal to EC.

In yoking horses to carriages, a very considerable advantage, both in reference to the horses and to the passengers, is gained by attaching them to the lever AB, by strong springs at A and B.

How horses exert their strength.

The particular manner in which living agents exert their strength against great loads, seems to have been unknown both to Camus and Couplet,

and to many succeeding writers upon this subject. It is to M. Deparcieux, an ingenious mechanic, that we are indebted for the only accurate information with which we are furnished; and we are sorry to see, that philosophers who flourished after him have overlooked his important instructions. In his Memoir on the draught of horses,[4] he has shewn, in the most satisfactory manner, that animals draw by their weight, and not by the force of their muscles. In four-footed animals, the hinder feet is the fulcrum of the lever by which their weight acts against the load, and when the animal pulls hard, it depresses its chest, and thus increases the lever of its weight, and diminishes the lever by which the load resists its efforts. Thus, let P (Fig. 20) be the load, $D A$ the line of traction, and let us suppose $F C$ to be the hinder leg of the horse, $A F$ part of its body, A its chest or centre of gravity, and $C E$ the level road. Then $A F C$ will represent the crooked lever by which the horse acts, which is equivalent to the straight one $A C$. But when the horse's weight acts downwards at A,[5] round C as a centre, so as to drag forward the rope $A D$, and raise the load P, $C E$ will represent the power of the lever in this position, or the lever of the horse's weight, and $C F$ the lever by which it is resisted by the load, or the lever of resistance. Now, if the horse lowers its centre of gravity A, which it always does when it pulls hard, it is evident that $C E$, the lever of its weight, will be increased, while $C F$, the lever of its resistance, will be diminished, for the line of traction $A D$ will approach nearer to $C E$. Hence we see the great benefit which may be derived from large horses, for the lever $A C$ necessarily increases with their size, and their power is always proportioned to the length of this lever, their weight remaining the same. Large horses, therefore, and other animals, will draw more than small ones, even though they have less muscular force, and are unable to carry such a heavy burden. The force of the muscles tends only to make the horse carry continually forward his centre of gravity; or, in other words, the weight of the animal produces the draught, and the play or force of its muscles serve to continue it.[6]

[4] *Sur le Tirage des Chevaux*, published in the *Mem. de l'Acad. Par.* 1760, 4to, p. 263; 8vo, p. 275.

[5] It may be imagined that the fore feet of the horse prevent it from acting in this manner; but Deparcieux has shewn by experiment that the fore feet bear a much less part of the horse's weight when he draws than when he is at rest.

[6] When I first compared Deparcieux's theory with the manner in which horses

Position of the line of traction.

From these remarks, then, we may deduce the proper position of the line of traction. When the line of traction is horizontal, as $A\ D$, the lever of resistance is $C\ F$; but if this line is oblique to the horizon, as $A\ d$, the lever of resistance is diminished to $C f$, while the lever of the horse's weight remains the same. Hence it appears that inclined traces are much more advantageous than horizontal ones, as they uniformly diminish the resistance to be overcome. Deparcieux, however, has investigated experimentally the most favourable angle of inclination, and found, that when the angle $D\ A\ F$, made by the trace $A\ d$ and a horizontal line, is 14 or 15 degrees, the horses pulled with the greatest facility and force. This value of the angle of draught will require the height of the spring-tree bar, to which the traces are attached in four-wheeled carriages, to be *one-half* of the height of that part of the horse's breast to which the fore end of the traces is connected.[7]

Notwithstanding the great utility of inclined traces, it will not be easy to derive complete advantage from them in two-wheeled carriages without diminishing the size of the wheels. In all four-wheeled carriages, however, they may be easily employed; and in many other cases where wheels are not concerned, great advantage may be derived from the discovery of Deparcieux.

On the Position of the Centre of Gravity, and the manner of Disposing the Load.

Position of the centre of gravity.

From Mr. Ferguson's observations on the centre of gravity, (Vol. i, p. 11,) it must be evident, that if the axle-tree of a two-wheeled carriage passes through the centre of gravity of the load, the carriage will be in equilibrio in every position in which it can be placed with respect to the axle-tree, and in going up and down hill, the whole load will be sustained by the wheels, and will have no tendency either to press the horse to the ground or to raise him from it. But if the centre of gravity is far above the axle-tree, as it must necessarily be according to the present construction of

appear to exert their strength, I was inclined to suspect its accuracy; but a circumstance occurred which removed every doubt from my mind. I observed a horse making continual efforts to raise a heavy load over an eminence. After many fruitless attempts, it raised its fore feet completely from the ground, pressed down its head and chest, and instantly surmounted the obstacle.

[7] This height is about 4 feet 6 inches, and therefore the height of the spring-tree-bar should be only 2 feet 3 inches, whereas it is generally 3 feet.

wheel-carriages, a great part of the load will be thrown on the back of the horses from the wheels, when going down a steep road, and thus tend to accelerate the motion of the carriage, which the animal is striving to prevent; while in ascending steep roads a part of the load will be thrown behind the wheels, and tend to raise the horse from the ground, when there is the greatest necessity for some weight on his back, to enable him to fix his feet on the earth, and overcome the great resistance which is occasioned by the steepness of the road. On the contrary, if the centre of gravity is below the axle, the horse will be pressed to the ground in going up hill, and lifted from it when going down. In all these cases, therefore, where the centre of gravity is either in the axle-tree, or directly above or below it, the horse will bear no part of the load in level ground: In some situations the animal will be lifted from the ground when there is the greatest necessity for his being pressed to it, and he will sometimes bear a great proportion of the load when he should rather be relieved from it.

The only way of remedying these evils is to assign such a position to the centre of gravity, that the horse may bear some portion of the load when he must exert great force against it; that is, in level ground, and when he is ascending steep roads; for no animal can pull with its greatest effort, unless it is pressed to the ground. Now, this may, in some measure, be effected in the following manner. Let $B\,C\,N$ (Plate VII, Fig. 20) be the wheel of a cart, $A\,D$ one of the shafts, D that part of it where the cart is suspended on the back of the horse, and A the axle-tree; then, if the centre of gravity of the load is placed at m, a point equidistant from the two wheels, but below the line $D\,A$, and before the axle-tree, the horse will bear a certain weight on level ground, a greater weight when he is going up hill and has more occasion for it, and a less weight when he is going down hill and does not require to be pressed to the ground. All this will be evident from the figure, when we recollect, that if the shaft $D\,A$ is horizontal, the centre of gravity will press more upon the point of suspension D the nearer it comes to it; or the pressure upon D, or the horse's back, will be proportional to the distance of the centre of gravity from A. If m, therefore, be the centre of gravity, $b\,A$ will represent its pressure upon D, when the shaft $D\,A$ is horizontal. When the cart is ascending a steep road, $A\,H$ will

be the position of the shaft, the centre of gravity will be raised to *a*, and *A a* will be the pressure upon *D*. But if the cart is going down hill, *A C* will be the position of the shaft, the centre of gravity will be depressed to *n*, and *c A* will represent the pressure upon the horse's back. The weight sustained by the horse, therefore, is properly regulated, by placing the centre of gravity at *m*. We have still, however, to determine the proper length of *b a* and *b m*, the distance of the centre of gravity from the axle, and from the horizontal line *D A*; but as these depend upon the nature and inclination of the roads, upon the length of the shaft *D A*, which varies with the size of the horse, on the magnitude of the load, and on other variable circumstances, it would be impossible to fix their value. If the load along with the cart weighs 400 pounds, if the distance *D A* be 8 feet, and if the horse should bear 50 pounds of the weight, then *b A* ought to be 1 foot, which being $\frac{1}{8}$ of *D A*, will make the pressure upon *D* exactly 50 pounds. If the road slopes 4 inches in one foot, *b m* must be 4 inches, or the angle *b A m* should be equal to the inclination of the road, for then the point *m* will rise to *a* when ascending such a road, and will press with its greatest force on the back of the horse.

Method of disposing the load.

When carts are not constructed in this manner, we may, in some degree, obtain the same end, by judiciously disposing the load. Let us suppose that the centre of gravity is at *O*, when the cart is loaded with homogeneous materials, such as sand, lime, &c.; then if the load is to consist of heterogeneous substances, or bodies of different weights, we should place the heaviest at the bottom and nearest the front, which will not only lower the point *O*, but will bring it forward, and nearer the proper position *m*. Part of the load, too, might be suspended below the fore part of the carriage in dry weather, and the centre of gravity would approach still nearer the point *m*. When the point *m* is thus depressed, the weight on the horse is not only judiciously regulated, but the cart will be prevented from overturning, and in rugged roads the weight sustained by each wheel will be in a great degree equalized.

In loading four-wheeled carriages, great care should be taken not to throw much of the load upon the fore-wheels, as they would otherwise be forced deep into the ground, and require

great force to pull them forward.[8] In some modern carriages this is very little attended to. The coachman's seat is sometimes enlarged so as to hold two persons, and all the baggage is generally placed in the front, directly above the fore-wheels. By this means, the greatest part of the load is upon the small wheels, and the draught becomes doubly severe for the poor animals, who must thus unnecessarily suffer for the ignorance and folly of man.

CHAPTER VIII.

ON THE FORCES OF ELASTICITY AND TORSION.

Elasticity is that property of bodies, in virtue of which their particles return to their original state, when their equilibrium has been disturbed by any external force. When we bend a long stripe of glass, it will resume exactly the same shape which it had before it was bent; but if we subject a plate of steel to the same force, it will not resume its original form. In the first case, the *elasticity* is said to be *perfect*, and in the second *imperfect*.

It would be unsuitable to the character of this work, to enter into those theoretical discussions into which such a subject tends to lead us. It will be sufficient to describe the apparatus which has been employed for determining the elasticity of bodies, and to give a popular abstract of the most prominent results which philosophers have obtained on this subject.

The vast importance of these experiments, in a practical point of view, and the extensive utility of the force of torsion in physical inquiries, will justify us in devoting a few pages to their illustration.

The earliest and most correct experiments on the elasticities of bodies were made by S'Gravesende, with an apparatus shewn in Plate VIII, Fig. 1. It consists of a frame $A\,B\,C\,D$, which carries the pincers or vices M, N, between which is fastened the wire or rod of steel, or any other substance $M\,N$, whose elasticity is to be tried. A scale S, carrying weights, is fixed to

[8] Dr. Young remarks, that the centre of gravity in four-wheeled carriages should divide the distance between the fore and hind wheels in the ratio of the cubes of the diameters. If the diameters of the wheels are as 4 to 5, the centre of gravity should be twice as near the hind as the fore wheels. *Lectures*, Vol. II, p. 201.

a piece of metal $P\,Q$, perforated at Q, so as to permit the wire or rod to pass through it. To the upper end of this piece is fixed a silk thread $Q\,R$, passing over the pulley R, and suspending a weight W, exactly equal to the weight of the piece $P\,Q$. The axis of this pulley carries the index V, which points to the divisions of a dial-plate. If we now put different weights into the scale S, the elastic body $M\,N$ will be bent by the weight, the middle point Q will descend, drawing after it the thread $Q\,R$, and consequently turning the index $T\,V$ from right to left. In order to know the exact space through which the point Q has descended, or the degree of flexure of the body $M\,N$, we must determine by direct experiment the relation between the divisions on the dial-plate and the descent of any point of the thread $Q\,R$. By this apparatus, S'Gravesende determined the degree of flexure, or the descent of the middle points of wires and plates produced by various weights placed in the scale S. The wire which he used was metallic, and such as are employed in musical instruments. Its length was 34 inches, and its weight 24 grains. Having given it a certain degree of tension by means of one of the screws, he measured the degree of flexion, or the descent of the middle point, produced by two different weights. He then gave the wire a second, and even a third degree of tension, and ascertained the flexion as before. The results which he thus obtained are given in the following table:—

Degrees of tension.	Weights in the Scale in grains.	Descent of the middle point in parts of an inch.
First tension, - -	3	0.04
	36	0.40
Second tension, - - -	8	0.05
	70	0.40
Third tension, -	8	0.04
	86	0.40

From these experiments S'Gravesende concludes—

1. That when the string almost returns to its primitive condition, the elongations which it receives from different weights or forces are proportional to these weights or forces.

2. That as the elongations of similar strings of different lengths must be as their lengths, the elongations of similar strings of the same thickness must be in the compound ratio of the lengths of the strings, and the weights or forces which elongate them.

3. That the weights or forces which elongate strings of the

same length are not as the quantities of matter in them, the ratio being sometimes greater and sometimes less.

4. That a greater or less degree of elasticity in bodies of the same kind depend, *cæteris paribus*, on a certain peculiar disposition of their parts.

S'Gravesende extended his experiments to plates of metal, and as these might be considered as a congeries of fibres or strings, he found, as might have been expected, that their elasticities followed the same laws. The following were the results which he obtained with a plate of steel made of clock-spring. This plate was 34.5 inches long, and weighed 67 grains.

Degrees of tension.	Weights in the scale in drachms.	Descent of the middle point in parts of an inch.
First tension, -	20	0.10
	144	0.40
Second tension, -	32	0.10
	192	0.40
Third tension, -	64	0.07
	430	0.40
Fourth tension, -	64	0.06
	492	0.40

As the apparatus from which the preceding results were obtained is not susceptible of very minute accuracy, we shall describe an instrument of very general application, not only for measuring the elasticities of wires and plates, but for studying all the general phenomena of elasticity.

This instrument may be called a *Teinometer*, and is constructed in two forms, which may be distinguished by the names of the *Mechanical* and the *Chromatic* Teinometer, from the different ways in which the tensions of the elastic bodies are measured.

The general instrument is shewn in Plate VIII, Fig. 2, where $E\,F$ is a strong frame of iron or brass, with an open groove $A\,B$ along its centre. A and B are two fulcra which can be fixed by means of the screws $E, F,$ or taken out so as to admit in their place the screw pincers shewn in Fig. 3. A strong piece of brass or steel $G\,H,$ having a horizontal shoulder for receiving the micrometer screw $S\,S'$ may be fixed by means of a screw opposite to the centre of the frame $E\,F$, or may be placed at different distances from the centre. The head of the micrometer screw $S\,S'$ is divided as usual into 100 parts, which are read off by the index I.

In order to perform with this instrument the same experiments which were made with that of S'Gravesende, the screw pincers are put on, the wire or the plate of steel $A\ B$ is fixed with a certain tension by the screws of the pincers, and by means of a scale suspended from the centre S' of $A\ B$, different degrees of elongation may be given to the wire or the plate. The point S of the screw having been nicely adjusted before the application of the weights, so as to touch the upper surface of the wire or the plate, it is now screwed down till it is again brought into contact with it, and the number of turns, and parts of a turn of the screw $S'\ S$ will afford the most accurate measure of the degree of elongation. By shifting the stand $G\ H$ to different parts of the frame $E\ F$, the exact degree of flexure at any point of the wire or plate may be ascertained. When the steel plate rests freely on two fulcra A, B, the micrometer screw will enable us to compare its form with that which it assumes when its extremities are fixed.

The *Chromatic Teinometer* depends upon an optical principle, which will be farther illustrated in the chapter on the Polarisation of Light. When a plate of glass is bent, one side of it is *concave*, and in a state of *compression*, while the other side is *convex*, and in a state of *dilatation*, and there must therefore be some neutral line where there is neither compression nor dilatation. Let $A\ B$, Plate VIII, Fig. 4, be the plate of glass seen edgewise, and $M\ N$ the neutral line. If we expose it in this state to polarised light, and afterwards analyse the transmitted light, we shall perceive the neutral line or axis $m\ n$ marked out by a black line, while the convex and concave sides which it separates are covered with beautiful coloured fringes, the tints being *negative* on the *convex*, and *positive* on the *concave* side. These fringes increase in number as the plate is more bent, and therefore afford a precise measure of the degree of flexure to which it is subjected.

By placing therefore one of these plates in the Teinometer, we may not only determine the degree of flexure from the tints; but we may study the mechanical condition of every part of the glass as modified by flexure; and we may compare the effects of elongation combined with those of flexure, with the effects of flexure alone, according as we use the screw-pincers or the fulcra. In comparing the flexure with the tints in different points of the glass plate, the first Teinometer may be used;

but in comparing the elasticities of glass plates of different dimensions, and of different kinds of glass, the Teinometer represented in Fig. 5 should be employed. In this instrument *A B* is the standard plate of well annealed glass, having its edges parallel, and also flatly ground and well polished. Along this plate there are moved two brass pieces *S a b c*, *S′ a′ b′ c′*, which can be fixed in any position by means of the screws, *S*, *S′*. The plate *C D*, whose elasticity is to be measured, rests with its lower edge upon the projection *b c* (Fig. 6), and with one of its faces against *a b*. It is then pressed into contact with the plate *A B*, and kept in this position by the wooden hold-fast *H*, the brass pieces *S a b c*, *S′ a′ b′ c′*, having been previously placed at such a distance from each other, that the two plates will meet at *H* without breaking, or without any permanent change of form. The apparatus *E R*, for observing the tint, is shewn separately in Fig. 7. It consists of an eye-piece *E*, to which is attached a reflector *R*, made of several plates of the thinnest glass, about 1½ inch long and 1 inch broad, and placed close to each other. The eye-piece *E* consists of two tubes, one of which is moveable within the other. The moveable tube contains a plate of tourmaline, or an achromatic prism *m n* of calcareous spar, with a convex lens *o p* about an inch in focal length, placed either above or below it. When this apparatus is set upon the edge of *A B*, by means of the forked arms *e f*, the reflector *R* is turned round till the plane of reflection is cut at an angle of 45° by the plane of the plate *A B*, and is placed at such an angle that the light which it reflects through the edge of the plate *A B*, and up the tube, is completely polarised. The moveable tube is then turned round till the tints appear upon the edge of one of the images of the glass plate. In order to avoid the confusion arising from two images, the achromatic prism may be constructed in such a manner that only one of the images is visible.[1]

There is another form of the Teinometer shewn in Fig. 8, in which it affords peculiar facilities for the comparison of elasticities. Let the middle plate *E F* be a standard plate of glass, and *A B*, *C D* two plates of steel or other bodies to be compared. When they are placed on the fulcra, as in the figure, and brought in contact at their middle points, the standard

[1] See the *Edinburgh Transactions*, vol. viii, p. 369–372.

plate of glass $E F$ will preserve its rectilineal form, and exhibit no tints if the plates $A B$, $C D$ have equal elasticities; because the standard plate is urged by equal and opposite forces. If one of the plates, however, is more elastic than the other, the intermediate plate will be bent, and its concave side will be towards the plate which has the least elasticity. If the degree of curvature is not perceptible to the eye, it will be immediately ascertained by the tints, and the positive or negative side will shew which side is concave, and which convex; while the maximum tint will be a measure of the difference of the elasticities of the two plates.

On Torsion.

When a metallic wire, or any other body is twisted, the force with which it untwists itself is called the *Force of Torsion.* Let $a b$, for example, Fig. 9, be a metallic wire held firmly by the pincers c, by means of the screw s, and let a cylindrical body P be suspended vertically by the wire $a b$, firmly fixed to it, so that the wire, if prolonged, would pass through the axis of the cylinder. Let an index $P o$ be fixed to the lower end of the cylinder, for the purpose of pointing out the degrees on a graduated circle n concentric with the circumference of the cylinder. If, when the whole is at rest, we turn the cylinder P round its axis, so that the index $P o$ may describe the arch $o n$, the wire $a b$ will be twisted by this operation, and if left to itself, the tendency of the particles to recover their former situation will cause it to untwist; the index $P o$ will return, and will oscillate backwards and forwards like a pendulum, till it is brought to rest by the resistance of the air, and the imperfect elasticity of the metallic wire. The angle $o P n$, through which the index had moved, or through which the wire had been twisted, is called the *Angle of Torsion*, and the force by which the oscillations are produced, the *Force of Torsion.*

If the wire possessed perfect elasticity, and was twisted in a perfect vacuum, the cylinder would oscillate for ever, and the oscillations would all be of the same length. By observing, therefore, the diminution of the amplitude of the oscillations, and abstracting the effect produced by the resistance of the air, we may determine the laws according to which the elastic force of torsion is altered.

The experiments made by Coulomb for the purpose of de-

termining the laws of the force of torsion, were performed with three harpsichord wires of iron, and three brass wires corresponding to the iron wires in their number, and very nearly in their size. The following table will shew the nature of the wires employed:—

Nature of the wires.	Number of the wires.	Number of grains contained in a length of six feet.	Weight requisite to break the wires.	
			lbs.	oz.
Iron wires	N°. 12	5	3	0
	7	14	10	0
	1	56	33	0
Brass wires	12	5	2	3
	7	18.5	14	0
	1	66	22	0

These wires were fixed successively in the torsion balance shewn in Fig. 9, and suspended cylinders of different weights, all of which had a diameter of 19 lines. The various circumstances and results of the experiments are shewn in the following table:—

Nature of the wires.	Number of the wires.	Length of the wires in inches.	Weight of the suspended cylinder in pounds.	Limit of the centre of torsion for vibrations perfectly isochronous.	Time in which twenty isochronous oscillations were performed.
Iron	N°. 12	9	0.5	180	120″
	12	9	2.0	180	242
	7	9	0.5	180	42
	7	9	2.0	180	85
	1	9	2.0	45	23
Brass	12	9	0.5	360	220
	12	9	2.0	360	442
	7	9	0.5	360	57
	7	9	2.0	360	110
	7	36	2.0	1080	222
	1	9	2.0	50	32

In the preceding experiments, the angle of torsion is always so small, that the particles of the wire return to their primitive

state. If the angle of torsion, however, exceeds that in the table, the centre of the re-action of torsion will be displaced.

As in all the preceding experiments the 20 oscillations were sensibly isochronous, we may regard it as a fundamental law, that in all metallic wires, when the angles of torsion are not very great, the force of the re-action of torsion is sensibly proportional to the angle of torsion.

Since by the[2] formula $T = \sqrt{\frac{M a^2}{2\,n}}\,180°$, and since in all the preceding experiments the cylinders of half a pound and of two pounds weight had the same diameter, it follows, that n ought to be always proportional to $\frac{M}{T^2}$.

If we compare the results of the 1st and 2d experiments, of the 3d and 4th, of the 7th and 8th, and of the 9th and 10th, it will be seen, that with the same wire, the weight of two pounds performs its oscillations in double the time employed by a weight of half a pound. Thus 242″, 85″, 442″, and 110″, are nearly double of 120″, 43″, 220″, and 57″. And since $\sqrt{\frac{1}{2}} : \sqrt{2} = .7071 : 1.4142 = 1 : 2$, it follows, that T is proportional to $\sqrt{M}$, or *that the durations of the oscillations are as the square roots of the weights.* Hence it appears, that a greater or a less degree of tension in the wire has no sensible effect upon the re-action of the force of torsion. Coulomb however found, from many experiments made with tensions, very great in proportion to the force of the wire, that these tensions diminished a little the force of torsion. The wire, indeed, is obviously elongated as the tension increases, and as its diameter is thus diminished, the direction of its oscillations ought also to be diminished.

In considering how the force of torsion should be influenced by the length of the wire, it is obvious, that in proportion as we increase the length of the wire, we may cause it to allow, in the same proportion, a greater number of revolutions to be given to the cylinder without changing the degree of torsion. Hence, for the same number of revolutions, the force of the re-action of torsion ought to be inversely proportional to the length of the

[2] In this formula, the investigation of which is inadmissible in a popular work, T is the time of an entire oscillation, M the mass of the cylinder, a its radius, and n a constant co-efficient depending upon the nature, the length, and the thickness of the metallic wire.

wire. Since $T = \sqrt{\frac{M a^2}{2 n}}\, 180°$, we shall have for the same weight T proportional to $\frac{1}{\sqrt{n}}$, so that if n is in the inverse ratio of the lengths, as the theory announces, T will be as the square roots of the lengths of the wires. By comparing the 10th and 13th experiments, it appears that the lengths of the wires are as 1 to 4 when the times of the oscillations are as $1:2$; hence it follows, *that the times of the same numbers of oscillations are, when the same wires are stretched by the same weights, as the square root of the lengths of these wires*, as indicated by the theory.

If we suppose the lengths and the tensions of the wires to be equal, while the size of the wires varies, it will be found, *that the times of the oscillations are reciprocally proportional to the weights of the wires.* Thus, in the three experiments on the iron wires, with a tension of two pounds, we have the times of the oscillations 242″, 185″, and 23″, when the weights are 5 grains, 14 grains, and 56 grains, but $242 \times 5 = 1210$; $85 \times 14 = 1190$, and $23 \times 56 = 1288$; consequently, since these products are nearly equal, the times are nearly in the reciprocal ratio of the weights.

When the lengths of the wires and their tensions remain the same, *the force of torsion is proportional to the fourth power of the diameter of the wires*, or for the same length, *to the square of their weight.* Calling T and T' the times of a certain number of oscillations for a wire whose diameters are D, D', and the weights for the same lengths φ, φ', and ν the exponent of the power of D, to which the force of torsion is proportional, then we may suppose $T : T' = D^{\nu} : D'^{\nu} = \varphi^{\frac{\nu}{2}} : \varphi'^{\frac{\nu'}{2}}$, from which we obtain $\nu = \frac{2\,(\text{Log. } T - \text{Log. } T')}{\text{Log. } \varphi - \text{Log. } \varphi'}$.

Now, if we compare, by means of this formula, the 2d and 4th experiments, the 2d and 6th, the 8th and 10th, and the 8th and 12th, we shall obtain

$$\nu = -\,182 \qquad \nu = -\,2.04$$

$$\nu = -\,195 \qquad \nu = -\,2.02, \text{ whence}$$

$$T : T' = \frac{1}{D^2} : \frac{1}{D'^2} = \frac{1}{\varphi} : \frac{1}{\varphi'}.$$

But we have already seen that n is proportional to $\frac{M}{T^2}$, or when the weights are equal, to $\frac{1}{T^2}$. Hence it follows, that the force of

torsion for wires of the same nature and of the same length, but of different thicknesses, are as the fourth power of the diameter, a result which the theory also gives.

From all these observations it follows in general, *that the momentum of the force of torsion, for wires of the same metal, is directly in the compound ratio of the angle of torsion, and the fourth power of the diameter, and inversely as the length of the wires.* If l, therefore, is the length of the wire, D its diameter, and B the angle of torsion, the force of torsion will be $\frac{s\,B\,D^4}{l}$, in which s is a constant co-efficient, depending on the elasticity of each metal, but invariable in wires of the same metal.

In order to determine the effective value of n, we must apply the formula $n = \frac{P\,a^2}{2\,\lambda}$ to the 2d experiment; where $P = 2$ lbs. $a = 9\frac{1}{2}$ lines, and $\lambda = 440\frac{1}{2}$ lines, and where 20 oscillations were performed in 242″. The length of a pendulum isochronous with the oscillations of the cylinder, will be $440\frac{1}{2}\left(\frac{242}{20}\right)^2$, consequently

$$n = \frac{2 \text{ lbs. } (9\frac{1}{2})^2}{2 \times 440\frac{1}{2}\left(\frac{242}{20}\right)^2} = \frac{1 \text{ lb.}}{715};$$

that is, the momentum $n\,B$ of the iron wire No. 12, having a length of 9 inches, is equal to $\frac{1}{715}$ lb. multiplied by the angle of torsion B acting at the extremity of a lever a line in length; or what is the same thing, if we suppose the angle of torsion to be a line long, measured on a circle whose radius is a line, the force of torsion will be equal to $\frac{1}{715}$ of a pound, acting perpendicularly at the extremity of the arm of a lever a line long.

By the same mode of calculation applied to the brass wire No. 12, we shall find $n = \frac{1}{2384{\cdot}2}$, so that the strength of the iron is to that of the brass wire as $\frac{1}{715} : \frac{1}{2384{\cdot}2}$, or as 3.34 to 1.

In the preceding investigations, we have considered only that degree of torsion when the twisted wire returns exactly to its primitive state, by a series of decreasing oscillations. In order to determine if the resistance of the air had any share in diminishing the amplitude of the oscillations, Coloumb made the following experiment. The weights used for stretching the wires were 26 lines high and 19 lines in diameter. He then formed out of very thin paper a cylindrical surface of the same diameter as these weights, but with a height of 70 lines. The leaden

weight was then made to enter partly into this cylinder of paper, and the oscillations were executed as before without any sensible change. With another paper cylinder, 78 lines high, or three times the height of the leaden cylinder, with which the effects of the resistance of the air ought to have been tripled, no retardation whatever was observed, and therefore he concluded that the diminution of the amplitude of the oscillations arose from the imperfect elasticity of the metal. Coulomb then proceeds to observe the effects which take place when the wires are twisted to a greater degree than in the first experiment, or to such a degree that the particles do not return to their primitive state, but have experienced a permanent change in their position. If the angle of torsion, for example, is 180°, and if the index, instead of returning after the oscillations are completed to the same point, or through the same arc of 180°, has returned only through 170°, then the centre of torsion is said to be displaced, or to have advanced 10°.

In order to observe from the diminution of the oscillations how the elastic force of torsion is altered in the oscillatory motion, when the centre of re-action is not displaced, he used the iron wire No. 1, with a length of six inches and six lines, and supporting a weight of two pounds; then, when the angle of torsion was

Angle of Torsion.	Degrees lost.	
90°	10° were lost in	3½ oscillations.
45	10	10½
22½	10	23
11¼	10	46

When the angle of torsion exceeded 90°, the centre of torsion was displaced, as shewn in the following table :—

Angles of torsion given to the wire.	Successive displacement of the centre of torsion.	Total displacement of the centre of torsion.	Angle through which the wire untwists itself.
½ circle	8	8	1728
1	50	58	310
2	310	1 circle + 8	410
3	1 circle + 300	2 + 308	420
4	2 + 290	5 + 238	430
5	3 + 280	9 + 158	440
6	4 + 260	14 + 58	460
10	8 + 240	22 + 298	480
14	wire split into two in the direction of its length.		

In the preceding table, the first column contains the angle of torsion through which the index has been turned. The second contains the arch or angle which it wanted of returning to its primitive state, or the displacement of the centre of torsion produced at each successive experiment. The third contains the total displacement, and is found by adding any one displacement in column 2d to all those that precede it. The fourth column contains the angle through which the index has returned, or the effect produced by the re-action of the torsion of the wire, and is found by subtracting the second column from the first.

From the first of these tables it appears, that below 45 degrees the alterations are nearly proportional to the amplitudes of the angles of torsion, and that above 45° the alterations augment in a much greater ratio. It appears from the second table, that the centre of the re-action of torsion does not begin to be displaced till the angle of torsion is nearly 180°;—that this displacement increases in proportion as the thread is twisted;—that it is irregular till the angle of torsion is 310°,—and that beyond this angle the re-action of torsion remains nearly the same for all angles of torsion. In the fourth experiment, for example, where the angle of torsion is three circles, the displacement is 1 circle + 300°, so that the re-action of torsion has brought back the cylinder only 1 circle + 60° or 420°; whereas in the 7th experiment, the cylinder was brought back only 1 circle + 100° or 460°, after having undergone a total displacement of 14 circles + 58°.

The following experiments were made with the iron wire No. 7, whose length was 6 inches 6 lines.

			Oscillations.
When the angle of torsion was	180°	10° were lost in	$3\frac{1}{2}$
	90	10	12
	45	10	27
	$22\frac{1}{2}$	10	54

The following table shews the successive displacement of the centre of torsion, and also the total displacement in the centre of torsion:—

Angles of torsion given to the wire.	Successive displacement of the centre of torsion.	Total displacement.	Angle through which the wire untwists itself.
3	300	300	2 circle + 60°
4	1 circle + 180	2 circle + 120	2 + 180
6	3 + 90	5 + 210	2 + 270
8	5 + 90	10 + 300	2 + 270
12	9 + 40	19 + 340	2 + 320
20	16 + 310	36 + 290	3 + 50
30	26 + 180	63 + 110	3 + 180
50	46 + 20	110 + 130	3 + 340

From all these experiments, M. Coulomb has deduced the following ingenious theory of the elasticity and cohesion of metals.

Coulomb's theory of elasticity and cohesion.

The integrant particles of all metallic wires have an elasticity which may be considered as perfect; that is, the forces necessary for compressing or dilating these particles are proportional to the dilatation or the compression which they experience. These particles, however, are united together by cohesion, a quantity that is constant and absolutely different from elasticity. In the first degrees of torsion the integrant parts change their form, and are elongated or compressed without any change of place in the points by which they adhere, because the force necessary for producing these first degrees of torsion is less than the force of cohesion. But when the angle of torsion becomes such that the force with which the parts are compressed or dilated is equal to the cohesion which unites these integrant particles, then they ought to separate or slide upon one another. This sliding of the particles takes place in all ductile bodies, but if, by this sliding of the particles upon one another, the body is compressed, the extent of the points of contact, and the extent of the field of elasticity, become greater. As these integrant particles have a determinate figure, the extent of the points of contact cannot augment but to a certain degree, beyond which the body breaks. This view of the difference between the cause of elasticity and cohesion was confirmed by an experiment in which Coulomb could vary the cohesion at pleasure, without altering the elasticity. He brought a wire of copper to the temper of a white heat; and by this means its cohesion

was so much reduced that it could scarcely support 12 or 14 pounds, while, in its original state, it carried 22 pounds at the instant of rupture. But though the cohesion was thus diminished nearly one-half, and though the amplitude of elasticity was diminished nearly in the same proportion, yet, in the extent of elastic re-action which remained, the elasticity was the same at the same angle of torsion as in the untempered wire, as both of them, when stretched by the same weight, performed the same number of oscillations in the same time.

In order to confirm these views, M. Coulomb subjected to experiment plates of steel, by means of the apparatus shewn in Plate VIII, Fig. 10. A plate of steel *A B*, had one of its extremities *A* fixed between two plates of iron *E*, *F*, by means of the screw *V*, and holdfast *C D*. The plate was 11 lines wide, half a line thick; and its length from *a* to *B*, where the weight was suspended, was seven inches. The descent of the extremity *B*, by the action of the weight *P*, was measured upon a vertical scale *M N*.

When the plate of steel was brought to a white heat, and tempered very hard, it was loaded with different weights, as in the following table:

Value of the weight *P*.	Descent of the extremity *B*.
½ pound	8 lines
1	15½
1½	23 +

The same plate was then heated till it received a violet colour, and returned to the state of an excellent spring, and when placed in the apparatus with the very same weight, its extremity descended the very same quantities.

It was at last brought to a white heat, and allowed to cool very slowly, and the same effects were produced by the same weights as before.

In these three experiments, the first degrees of the elastic forces suffered no change. The same weights produced the same degree of flexure, and by removing them the plate returned to its horizontal position.

In order to find the strength of the plates in these different states, Coulomb formed three plates out of a piece of English steel exactly similar to those above mentioned, and having tempered one of them at a white heat, another when it had a violet

colour, while a third was allowed to cool slowly from a red heat, he placed these plates successively in the apparatus, and applied weights at the point d, $2\frac{1}{2}$ inches from a. The *first* plate broke with *six pounds;* but to whatever angle it was bent below that at which it broke, it always resumed its primitive horizontal state; the *second* broke with *eighteen pounds;* it bent to the point of rupture into an angle nearly proportional to the force applied to it. The third plate bent with a force of *five or six pounds* proportionally to this force, and into an angle exactly equal, under the same force, to that into which it was bent when in its two former states. But in afterwards applying the force perpendicularly to the direction of the plate, in order to preserve the same length of lever, a force of seven pounds was sufficient to bend it to all angles, and when the weight was removed, it returned only by a quantity through which it had been originally bent by a force of six pounds, so that the angle of the reaction of flexion was changed from any angle into which it was bent, with a force greater than seven pounds.

These last experiments lead us to the same result as the preceding ones. It is obvious, that in order to have an idea of what happens in the flexion of metals, we must distinguish the elastic force of the integrant particles from the force of cohesion, which unites these particles together. The elastic force depends, as we have seen, on the compression and dilatation which these integrant parts experience, and is always proportional to the compressing and dilating forces. These integrant particles are not altered either by tempering or annealing, since in these different states the elasticity is the same under the same degrees of flexion. But the same particles are united only by a certain degree of cohesion, which depends probably on their figure, and on the respective portions of different fluids with which their pores are filled, which varies according to the state of temper and annealing. In very hard-tempered steel, and in good springs, the particles cannot either slide upon one another, or experience the least displacement, without the body breaking; but in ductile bodies, or annealed metals, these particles may slide upon one another, and be displaced, without the cohesion being sensibly altered.

What has now been said respecting metals may be applied to all bodies. Their integrant particles have always a perfect

elasticity, but the bodies are hard, soft, or fluid, according to the cohesion of these particles. If in hard bodies they can slide upon one another, without their distance being sensibly altered, the body will be ductile or malleable; but if they cannot slide upon one another without their distance being sensibly altered, the body will break, when the force with which it is compressed or dilated is equal to the cohesion.

CHAPTER IX.

ON THE STRENGTH OF MATERIALS.

THE mathematical investigation of the strength of materials, when exposed under different forms to a variety of strains, would lead us into a field too extensive for the limits of this work. Fortunately for the practical mechanic, theory is on this subject a fallacious guide, and it is only by the results of direct experiment that he is entitled to guide himself in actual practice.

Where a piece of timber, or any other material, is used, it is subjected to a diversity of strains.

1. It may be pulled or drawn in the direction of its length, as in the case of a King-post, a Queen-post, and the upright rod or spear of a pump. In this case the force is called the *Resistance to Tension.*

2. It may be compressed in the direction of its length, as in the case of columns, posts, pillars, and struts. In this case the force is called the *Resistance to Compression.*

3. It may be exposed to a lateral or cross strain, as in the case of beams, rafters, tenons, &c.

4. It may be twisted, as in the axles of wheels, the rudders of ships, and all kinds of screws.

I. *On the Strength of Materials when Pulled in the Direction of their Length.*

When metals, glass, stone, and other bodies of uniform texture, are pulled asunder in the direction of their length, the resistance to extension is the force of cohesion between their particles; and hence the extension will be invariably proportional to the area of the section of the rod employed. As the

quantity of extension must be proportional to the number of parts extended, it must also be directly proportional to the length of the rod.[1]

In the first volume of this work, p. 57, note, we have already given a table of the best experiments which have been made on the cohesive force of woods and metals. When the strength of a beam or rod to resist a longitudinal strain is computed from the data in that table, the beam should be made to bear only *one-fourth* of the load which would tear it asunder.

A number of very interesting experiments on this branch of the subject have been made by M. Duleau on malleable iron.[2] The general result of his experiments is, that the force necessary to break a bar of malleable iron drawn in the direction of its length varies from 35 to 60 kilogrammes for every square millimetre of the transverse section of a bar, according to the quality of the iron.

He found that a bar a metre long (39.37 inches) was lengthened the 10th part of a millimetre (0.00394 of an inch), when he increased by 4 kilogrammes the mean weight carried by a square millimetre of the transverse section. M. Duleau likewise concluded from his experiments, that the degree of compression or dilatation which deprived iron of its elasticity varied from $\frac{1}{4}$ to 1 millimetre upon every metre of length. The weight which produced this dilatation was between $\frac{1}{3}$ and $\frac{2}{3}$ds of the weight capable of breaking the bar.

The following results have been recently obtained by Mr. George Rennie. All the bars were 6 inches long, and $\frac{1}{4}$ of an inch square.

	Pounds Avoird.
Iron bar cut horizontally, - -	1166
Ditto ditto vertically, - -	1218
Cast steel, previously tilted, - -	8391
Blister steel, reduced by the hammer, -	8322
Shear steel, do. do. -	7977
Swedish iron, do. do. - -	4504
English iron, do. do. -	3492

[1] In some of Mr. Barlow's experiments on iron, the extension appears to vary as the square of the length. *Essay on the Strength of Timber*, p. 228–230.

[2] M. Duleau's Memoir is entitled *Essai Theorique et Experimental sur la Resistance du Fer Forgé.* The experiments were made in 1812. An abstract of them is given in the *Ann. de Chim.* tom. xii, p. 133.

	Pounds avoird.
Hard gun-metal, - - - - -	2273
Wrought copper, reduced by the hammer,	2112
Cast copper, - - - - -	1192
Fine yellow brass, - - -	1123
Cast tin, - - - - -	296
Cast lead, - - - - -	114

In these experiments, the stretching of all the wrought bars indicated heat; and the fracture of the cast bars was accompanied with a diminution of section scarcely sensible.[3]

II. *On the Strength of Materials when compressed in the Direction of their Length.*

When a column of timber is compressed in the direction of its length, it resists more powerfully than when employed in any other way.

According to Muschenbroek, the relative strength of the following kinds of wood, when in bars 4 feet long, and $\frac{7}{10}$ square, were

	Pounds.		Pounds.
Fir, -	226	Beech, -	146
Linden, -	206	Oak, -	86

The following proportions are given by Peronnet from experiments on short specimens:

Oak, -	$12\frac{1}{5}$	Poplar, -	$7\frac{2}{5}$
Willow, -	$9\frac{3}{5}$	Ash, -	$7\frac{1}{5}$
Fir, -	$9\frac{2}{15}$	Elm, -	7

Oak has sometimes sustained a pressure of 4000 lbs. on the square inch. In the experiments of Girard, *six* of his specimens of oak broke with a pressure of 2710 lbs. on the square inch, though *fifteen* other specimens sustained a much greater load.

The following tables contain some of the results obtained by M. Lamands on *Seasoned Oak.*[4]

1. *Length,* 2.125 *feet—Breadth,* 2.126—*Thickness,* 2.12.

Deflexion in inches.	Weight that produced it in pounds.	Weight that broke it in pounds.	Duration in hours.
.079	7.86	15.63	4
.039	13.52	21.30	16
.118	14.12	19.99	18
.039	11.75	21.06	8

[3] *Philosophical Transactions,* 1818, p. 126–127.

[4] See Gauthey's *Construction des Ponts,* tom. ii, p. 48, and Tredgold's *Principles of Carpentry,* from the last of which this table is abridged.

2. *Length*, 4.25 *feet*—*Breadth*, 2.126—*Thickness*, 2.126.

Deflexion in inches.	Weight that produced it in pounds.	Weight that broke it in pounds.	Duration in hours.
.079	6.3	11.84	21
.157	6.3	12.22	27
.157	6.3	13.56	
.157	6.3	12.46	6

3. *Length*, 6.37—*Breadth*, 2.126—*Thickness*, 2.126.

.157	3.28	7.24	6
.157	2.86	7.48	
.236	2.75	8.47	5
.157	2.75	7 88	

4. *Length*, 2.125—*Breadth*, 3.18—*Thickness*, 3.18.

.079	34.60	50.96	27
.039	45.17	50.96	24

5. *Length*, 4.25—*Breadth*, 3.18—*Thickness*, 3.18.

.157	20.32	43.64	29
.157	18.65	36.86	5
.197	20.58	36.20	9
.276	21.82	28.18	17

6. *Length*, 6.375—*Breadth*, 3.18—*Thickness*, 3.18.

.157	9.12	26.94	7
.197	9.71	28.99	19
.079	11.00	23.93	4
.236	10.14	33.05	18
.157	12.75	36.90	6

7. *Length*, 2.125—*Breadth*, 4.25—*Thickness*, 4.25.

.079	61.88	95.26	11
.039	56.69	66.11	8
.039	56.69	105.83	23
.079	67.47	94.47	28
.039	57.78	88.44	30

8. *Length*, 4.25—*Breadth*, 4.25—*Thickness*, 4.25.

.039	63.07	100.75	8
.079	29.70	86.00	5
.079	50.52	73.24	19
.039	45.20	96.37	19

9. *Length*, 6.375—*Breadth*, 4.25—*Thickness*, 4.25

.157	21.59	64.09	7
.236	17.33	59.37	5
.157	18.52	54.06	22
.236	27.60	65.61	22

M. Girard, in his *Analytical Treatise on the Resistance of Solids*, has given a copious table of experiments on oak, from which the following results aretaken.

1. When the specimens were broken.

Specific gravity.	Deflexion in inches.	Weight producing the deflexion,—lbs.	Weight that broke it,—lbs.	Duration of experiment.
Length, 7.46 *feet*—*Breadth*, 6.13 *inches*—*Thickness*, 4.09 *in.*				
.972	0.244	38.104	72.865	$12^h.1$
Length, 7.46 *feet*—*Breadth*, 6.22 *in.*—*Thickness*, 4.00 *in.*				
.925	0.267	38.106	62.977	$12^h.1$
Length, 8.52—*Breadth*, 5.15—*Thickness*, 4.17.				
.923	0.665	26.392	50.448	$6^h.7$

2. When the specimens retained a slight flexure.

Length, 8.52—*Breadth*, 6.22—*Thickness*, 4.00.				
1.01	0.09	26.381		$0^h.8$
Length, 8.52—*Breadth*, 5.24—*Thickness*, 3.9.				
1.00	0.445	26.384		$6^h.7$
Length, 7.46—*Breadth*, 4.97—*Thickness*, 4.00				
1.038	0.312	26.397		$10^h.0$
Length, 6.39—*Breadth*, 6.13—*Thickness*, 5.24.				
1.102	0.177	38.107		$7^h.1$

3. When the specimen recovered its original form.

Length, 8.52—*Breadth*, 6.22—*Thickness*, 5.06.				
1.038	0.268	38.105		$0^h.83$
Length, 6.39—*Breadth*, 6.22—*Thickness*, 4.00.				
.987	0.177	38.106		$2^h.1$
Length, 6.39—*Breadth*, 5.24—*Thickness*, 4.17.				
1.032	0.22	38.048		$10^h.$
Length, 7.46—*Breadth*, 6.22—*Thickness*, 4.25.				
9.20	0.114	26.396		$10^h.$

In the experiments where the specimens broke, it is remarkable, as Mr. Tredgold has observed, that the weight which broke them is always nearly twice that which produced the first deflexion. Almost all the specimens bent in the direction of the diagonal, and were therefore crushed both in the direction of the breadth and thickness.

When the length of a square beam is above 7 or 8 times its thickness, as in all the preceding experiments, it first bends and then breaks near the middle of its length; but when the length of the beam is less than 7 or 8 times its thickness, it swells out in the middle, splits in various places, and is crushed.

The experiments of Rondelet give 5000 or 6000 lbs. as the force per square inch which is necessary to crush *oak*, its length being reduced by this pressure more than *one-third*. In order to crush *fir*, 6000 or 7000 lbs. were requisite, and the piece was reduced to *one-half* its length.

In considering the manner in which a body is compressed in the direction of its length, M. Duleau was led to the following theoretical law:—

"The weight capable of bending a bar, when pressed in the direction of its length, is proportional to the product of the length of the bar, or to the resistance which it presents when loaded perpendicularly in the direction of its fibres."

In order to determine the relation between the longitudinal and the lateral strength of the fibres, as given in the above law, M. Duleau submitted to experiment one-third of the number of bars whose strength he had determined when exposed to a lateral strain. The result of all these observations was, that a bar 2 metres long (6.56 feet), by 0.20*m.* (7.87 inches), and 0.02*m.* (0.787 inches), did not bend till the load amounted to 7600 kilogrammes. The theoretical results were to the experimental results as 1 to 1.19. It was the opinion of the commission appointed to examine M. Duleau's memoir, that the theoretical strength was too small, because in the transverse section the number of fibres compressed were probably greater than the number stretched when the bar was exposed to a lateral force.

The following interesting results on the force necessary to crush *one cubical* inch were made by Mr. George Rennie:—

	Pounds Avoird.
Elm, - - - -	1284
American pine, - - - -	1606
White deal, - - - -	1928
English oak, mean of 2 experiments, -	3860
Ditto 5 inches long, slipped with - -	2572
Ditto 4 inches long, - - -	5147
Prism of Portland stone, 2 inches long, -	805
Prism of statuary marble, - -	3216
Stone from Craigleith quarry, Edinburgh, -	8688

The following results with different stones were obtained by Mr. G. Rennie, and will be of great use to builders :—

	Spec. Grav.	Pounds Avoird.
Aberdeen granite, blue kind,	2.625	24556
White Italian veined marble,	2.726	21783
Very hard freestone, -	2.528	21254
Black Brabant marble, -	2.697	20742
Purbeck stone, - -	2.599	20610
Black compact limestone, Limerick,	2.598	19924
Peterhead granite, hard close grained,		18636
Compact limestone, -	2.584	17354
Devonshire red marble, variegated,		16712
Craigleith stone, with the strata,	2.452	15560
Portland stone, 2 inch cube -	2.423	14918
Dundee sandstone, 2 kinds, -	2.530	14919
Cornish granite, - -	2.662	14302
Bramley Fell sandstone, -	2.506	13632
White statuary marble, not veined,	2.760	13632
Yorkshire paving stone, -	2.507	12856
Craigleith white freestone, -	2.452	12346
Portland stone, - - -	2.428	10284
Killaly white freestone, -	2.423	10264
Derby grit, - - -	2.428	9776
Stourbridge fire brick, -		3864
Red brick, - - -	2.168	1817
Roestone Gloucestershire, -		1449
Brick of a pale red, - -	2.085	1265
Chalk, - - -		1127

It appears from the experiments of Reynolds, that a $\frac{1}{4}$ of an inch cube of cast iron required to crush it a force of 448,000 lbs. avoirdupois, or 280 tons, whereas Mr. G. Rennie, from an average of 13 experiments made on cubes of the same size, found that they could be crushed with 10,392 lbs. or less than 5 tons. According to Mr. Rennie, cubes of $\frac{1}{4}$ of an inch of cast iron, of the specific gravity of 7·033, and taken from the centre of a large block, required 1440 lbs. to crush them. When their lengths were increased so as to vary from $\frac{3}{8}$ to 1 inch, the average force which they sustained was about 1758.

Mr. Rennie found that the vertical cube castings were stronger than the horizontal ones, but the difference was by no means striking.

III. *On the Strength of Materials when exposed to a lateral or transverse strain.*

When we place a weight on the middle of a beam of wood

supported at both ends, as on two fulcra, but not fixed, it will be bent into a curve of more or less concavity, in proportion to the weight employed; and by increasing the weight, the beam will be broken. If the degree of concavity or the deflexion is very small, the beam is said to be *stiff*, and if it is considerable, it is said to be *flexible*.

Many experiments were made upon the strength of materials exposed to this strain, by Belidor and Duhamel, and a very fine series upon long pieces by Buffon. From these experiments, Mr. Nimmo, in the article *Carpentry*, in the *Edinburgh Encyclopædia*, has deduced the following rule for oak:—

Divide the number 651 by the length in feet, substract 10 from the quotient, multiply the remainder by the product of the breadth, and the square of the depth, both expressed in inches. The result will be the greatest load in pounds that such an oaken beam will bear.

The load thus found will break the beam in a few minutes. One half of it may be safely laid on the beam, but will give it a set from which it will not recover. One third of it may be laid on it for any length of time without injury.

The weight required to break a beam of fir may be found by the same rule, by taking two-thirds of the results obtained for oak, fir having only two-thirds of the strength of oak, according to the average of the experiments of Buffon, Emerson, and Parent.

The great importance of this branch of the subject has induced several modern authors to make a variety of experiments upon it. The principal experimentalists are Aubrey, Girard, Ebells, Beaufoy, Rennie, and Tredgold, and some of the leading results which they obtained are given in the following table:—

Experiments on the Stiffness of Beams supported at both ends.

OAK.

1. Mr. Barlow's experiments on the stiffness of oak.

Length, 7 feet—Breadth, 2 inches—Depth, 2 inches.

	Specific gravity.	Deflexion in inches.	Weight producing the deflexion.
English oak, -	.960	1.275	200
Canadian oak, -	.867	1.07	225
Dantzic oak, - -	.787	1.26	200
Adriatic oak, - -	.948	1.55	150

2. Mr. Tredgold's experiments on the stiffness of oak.[5]

Length, 25 feet—Breadth, 1 inch—Depth, 1 inch.

	Specific gravity.	Deflexion in inches.	Weight producing the deflexion.
Old ship timber, -	.872	0.5	127
Oak from Beaulieu, Hants,	.616	0.5	78
Ditto, another specimen,	.736	0.5	65
Dantzic oak, seasoned, -	.755	0.5	148

Length, 2 feet—Breadth, 1 inch—Depth, 1 inch.

	Specific gravity.	Deflexion in inches.	Weight producing the deflexion.
Oak from a young tree, Herts,	.863	0.5	237
Oak from old tree, -	.625	0.5	103
Oak from Riga, - -	.688	0.5	233
Oak (*quercus sessilifolia*) -		0.35	149
Oak (*quercus robur*) -		0.35	167

3. M. Buffon found that a beam of green oak 6.87 feet by 5.3 feet square required 7587 lbs. to produce a deflexion of 0.433 inches, and a beam of the same wood and dimensions but 23.58 feet long required 706 lbs. to produce a deflexion of 2.7 inches. M. Girard found that a beam of oak 8.52 feet by 5.06 inches broad and 6.22 inches deep, required 4146 lbs. to produce a deflexion of 0.71 inches; and a beam of oak the whole size of the tree and 16.86 feet by 10.66 inches broad and 11.73 inches deep, required 4559 lbs. to produce a deflexion of 0.67 inches.

FIR.

1. Mr. Barlow's experiments on the stiffness of fir.

Length, 7 feet—Breadth, 2 feet—Depth, 2 feet.

	Specific gravity.	Deflexion in inches.	Weight producing the deflexion.
Pitch pine, - -	.712	1.33	150
New England fir, -	.560	0.97	150
Riga fir, - - -	.765	0.912	150
Scotch fir, Mar forest, -	.715	1.56	125

[5] M. Ebbels found that beams of English oak of the same size as those used by Mr. Tredgold, suffered the same deflexion with 137 lbs. and green English oak with 96 lbs.

2. Mr. Tredgold's experiments on the stiffness of fir.

Length, 2.5 inches—Breadth, 1 inch—Depth, 1 inch—Deflexion, 0.5.

	Specific gravity.	Weight producing the deflexion.		Specific gravity.	Weight producing the deflexion.
Riga fir, yellow,	.480	123	Larch frm Blair, dry	.622	93
	.464 Ebells	116	Do. seasoned,	.644	101
			medium,	.554 Ebells	112
Memel do. medium	.553	143	Do. very young wood,	.396	45
	.544	145	Scotch fir, -	.529	89

3. M. Girard's experiments on the strength of fir, the whole size of the tree.

Length in feet.	Breadth in inches.	Depth in inches.	Deflexion in inches.	Weight producing it.
10.65	10.48	10.48	0.2245	4122
21.3	10.48	10.48	1.02	4389

Mr. Tredgold found that a beam of yellow Riga fir 18 feet long, 2 inches broad, and 7 inches deep, was deflected 0.25 of an inch by 103 lbs.; and that a beam of white spruce from Christiana, 2 feet long and 21 inches square, was deflected 0.5 of an inch by 261 lbs.

Experiments on the Stiffness of different woods.

1. Mr. Barlow's experiments.

Length, 7 feet—Breadth and Depth, 2 inches.

	Specific gravity.	Deflection.	Weight.
Ash, - - -	.760	1.27	225
Beech, - -	.688	1.025	150
Teak, - - -	.744	1.276	300
Elm, - -	.540	1.42	125

2. Mr. Tredgold's experiments.

Length, 2.5 feet—Breadth and Depth, 1 inch—Deflexion, 0.5

	Specific gravity.	Weight producing the deflexion.		Specific gravity.	Weight producing the deflexion.
Ash, young, -	.811	141	Plane, dry, -	.648	99½
Ditto, old, -	.753	113	Alder, ditto,	.555	80½
Ditto, medium,	.690 Ebbels	78½	Birch, ditto,	.720	90½
Cedar, Lebanon,	.486	36	Beech, ditto,	.690	97½
Maple, common,	.625	65	Wych elm, green,	.763	92
Abele, -	.511	84	Lombardy poplar, green,	.374	56½
Willow, -	.405	41			
Horse chesnut,	.684	79	Honduras mahogany,	.560	118
Lime tree, -	.483	84			
Walnut, -	.920	62	Spanish ditto,	.853	93
Spanish chesnut, green,	.895	68½	Sycamore, -	.590	76
			Pear tree, green,	.792	59½
Acacia, green,	.820	125	Cherry tree, green,	.690	92½

When the weight is uniformly distributed over a beam, in place of acting at the middle part, the deflexion produced is less, and the ratio between them is as 5 to 8.

Colonel Beaufoy's Experiments[6] *on the Stiffness of Beams supported at one end, as reduced by Mr. Tredgold.*

Length, 4 feet—Breadth and Depth, 2 inches.

	Specific gravity.	Deflexion.	Weight producing it.
Dantzic oak, -	.854	2.5	112 lbs.
English oak, - -	.922	1.176	
Ditto, another piece, -	-	1.5	
Riga fir, - - -	.537	1.34	
Pitch pine, - -	-	1.12	

Having thus considered the stiffness of beams exposed to transverse strains, as measured by the degree of deflexion produced by given weights, we shall proceed to detail the experiments that have been made on the absolute strength of materials.

Experiments on the Strength of Timber supported at both ends.

1. Barlow's experiments.

Length, 7 feet—Breadth and Depth, 2 inches.

	Specific gravity.	Deflexion in inches.	Weight which broke it,—lbs.
Teak, - -	.744	4.00	820
Mar forest fir, -	.715	5.5	360

2. Ebbels' experiments.

Length, 2.5 feet—Breadth and Depth, 1 inch.

	Specific gravity.	Weight that broke it.		Specific gravity.	Weight that broke it.
Oak, medium, -	.748	284	Elm, Wych, green,	.763	192
Ditto, green, -	.763	219	Acacia, green,	.820	249
Beech, medium,	.690	271	Walnut, green,	.920	195
Alder, - -	.555	212	Poplar, Lombardy,	.374	131
Plane tree, -	.648	243	Birch, - -	.720	207
Sycamore, -	.590	214	Scotch fir, English growth,	.460	157
Chesnut, green,	.875	180	Spruce fir, British growth,	.555	186
Ash, medium, -	.690	254			
Elm, common, -	.544	216			

3. Mr. Tredgold's experiments.

Length 2 feet—Breadth and Depth, 1 inch.

	Spec. grav.	Weight that broke it.		Spec. grav.	Weight that broke it.
Oak, English, young tree, -	.863	482 lbs.	American white spruce,	.465	285 lbs.
Oak, Riga, -	.688	357	American pine, Weymouth, - -	.460	329
Do. old tree, -	.625	218			

[6] In the *Annals of Philosophy*, vol. ix, pp. 274 to 306, the reader will find an immense body of valuable experiments on the strength of materials, by Colonel Beaufoy.

Length, 23.5 feet—Breadth and Depth, 1 inch.

	Spec. grav.	Weight that broke it.		Spec. grav.	Weight that broke it.
Oak, old ship timber,	.872	264 lbs.	Fir Memel, -	.554	218 lbs.
Ash, - -	.753	314	Do. Norway, Longsound, -	.639	396
Mahogany, Spanish, seasoned, -	.852	170	Larch, choice spec.	.640	253
Do. Honduras do. -	.560	255	Do. medium, -	.622	223
Willow, - -	.405	146	Do. very young, -	.396	127
Fir, Riga, -	.480	212			

Experiments on the Strength of Wood supported at both ends.

1. Barlow's experiments.

Length 3 feet—Breadth and Depth 2 inches.

Beech, -	.700	401	Ash, - -	.658	436

Length, 2 feet—Breadth, 1 inch—Depth, 2 inches.

Beech, - -	.740	352	Ash, - -	.730	321

2. Beaufoy's experiments.

Length, 4 feet—Breadth and Depth, 2 inches.

Oak, English, -	.922	266	Fir, Riga, -	.537	210
Do. Dantzic, -	.854	210	Pitch pine, - -		270

3. Peake and Barrelier's experiments.

Length, 5 feet—Breadth and Depth, 2 inches.

Ash, green, -	.858	239	Canadian white pine,	.618	122
Teak, old and dry,	.606	257	Larch, - -	.526	162
Virginia yellow pine,	.522	147			

Mr. G. Rennie's Experiments on the Strength of Bars of Cast Iron.

In the following experiments the bars were loose at the ends, and they all contained the same area, though differently distributed as to their form.

	Weight of bars.		Distance of bearings.		Weight that broke them.
	lbs.	oz.	ft.	in.	lbs.
1. Bar 1 inch square, -	10	6	3	0	897
2. Do. Do. -	9	8	2	8	1086
3. Half the above bar, -			1	4	2320
4. Bar 1 inch square through the diagonal, -	2	8	2	8	851
5. Half the above bar, -			1	4	1587
6. Bar 2 inches deep, and $\frac{1}{2}$ inch thick, -	9	5	2	8	2185
7. Half of the same bar,			1	4	4508
8. Bar 3 in. deep by $\frac{1}{3}$ in. thick,	9	1	2	8	3588
9. Half the above bar, -			1	4	6854
10. Bar 4 inches by $\frac{1}{4}$ in. thick,	9	7	2	8	3979

Equilateral Triangles with the Angle up and down.

	Weight of bars. lbs.	oz.	Distance of bearings. ft.	in.	Weight that broke them. lbs.
11. Edge or angle up, -	9	11	2	8	1437
12. Do. do. down, -	9	7	2	8	840
13. Half of the first bar, -			1	4	3059
14. Half of the second bar, -			1	4	1656

A Feather-edged or ⊥ Bar was cast of the following dimensions.

15. Two in. deep by 2 wide,	10	0 edge up	2	8	3105

The following experiments were made by Mr. G. Rennie on the bar 4 inches deep by $\frac{1}{4}$ thick, when different forms were given it, and when its bearings were 2 feet 8 inches distant.

	Weight that broke it.
16. Bar formed into a semi-ellipse, 7 lbs. -	4000 lbs.
17. Do. parabolic, on its lower edge, -	3860
18. Do. when 4 inches deep by $\frac{1}{4}$ inch thick,	3979

The following experiments were made by Mr. G. Rennie when one end of the bars was made fast, and the weight suspended at the other end, at the distance of 2 feet 8 inches from the bearing.

19. A bar 1 inch square bore - - -	280 lbs.
20. A bar 2 inches deep by $\frac{1}{2}$ an inch thick, -	539
21. An inch bar, the ends made fast, bore	1173

Mr. Rennie tried the paradoxical experiment mentioned by Emerson in p. 114 of his *Mechanics*, viz. that a bar which has the form of an equilateral triangle is made stronger by cutting off a portion of it.

Such a triangle, when tried in its complete state, as in Experiment 12, bore only -	840 lbs.
But with the piece cut off it sustained -	1129

Account of M. Duleau's Experiments on the Strength of Malleable Iron exposed to Transverse Strains.

In the Memoir of M. Duleau already mentioned (See p. 245), he has given the following theoretical[7] rules for horizontal bars of malleable iron.

1. In rectangular horizontal bars of different dimensions, and loaded in the middle, the deflexion, when the weight is the

[7] It may be necessary to warn the reader, that Dr. Thomson, in his *History of Science* for 1819, has given these theoretical rules as the results of M. Duleau's experiments, and has altogether omitted the experimental results.

same, is inversely as the cubes of the lengths, and directly as the breadth and the cubes of the thicknesses.

2. The resistance of a square bar, whether placed on its face or on its angle, is the same.

3. The resistance of a cylindrical is to that of a rectangular bar, whose side is equal to the diameter of the first, as three-fourths of the circumference of a circle is to the section of the circumscribed square. Hence the resistance of a cylinder is to that of a square bar as 20 to 21.

4. When a bar is under the influence of its own weight merely, its deflexion is to that which the same weight would produce if collected in its middle point, as 5 is to 8.

5. When a bar is under the influence of its own weight, but supported by its middle, the deflexion is to that which the same weight would produce if half of the weight was placed at each end, as 3 to 8.

In reference to these theoretical results, M. Duleau made more than 40 experiments on malleable iron bars, which varied in length from 39.37 inches, to to 16.4 feet, and from one-fifth of an inch thickness, to about 4 inches.

In 36 experiments relative to rectangular bars, the experimental result was to the theoretical result as 0.968 to 1. In a very small number of experiments, the difference was remarkable, the ratio being in one as 1.25 to 1, and in another as 0.77 to 1.

The experimental result of five trials with round bars was to the theoretical one as 1.005 to 1.

M. Duleau found that a bar of 7.87 inches in breadth, and 0.78 of an inch thick placed between two bearings 6.56 feet distant, had a deflexion of 0.394 of an inch under a weight of 160 kilogrammes.

From these experiments M. Duleau has concluded that the quality of iron has very little influence on its resistance at the beginning of the deflexion. Iron forged in England is an exception, as it exhibits some of the irregularities of untempered steel.

M. Duleau has also investigated the strength of bars with an interval between them, but bound together in such a way that they can neither separate nor rub upon one another. The principle upon which he computes the strength of any system of bars thus united, is to find its strength if the whole were one

solid mass, and then to subtract from it the strength of the solid made up of the intervals. Hence he finds,

That if two rectangular bars of equal dimensions are placed the one above the other at a distance a, and united so that they can neither separate nor rub upon one another, the resistance of the system of these two pieces is proportional to $(e + a)^3 - a^3$, that is, to the difference of the cubes of the whole thickness of the system, and the thickness of the interval, e being the sum of their thicknesses.

Hence it is easy to see how the resistance of the system increases with the distance a of the beams. If we call $e = 1$, and $a = 14.50$, the resistance of the system will be 180 times greater than that of a piece of the same breadth, and having a thickness equal to 1. These numbers are derived from an experiment which gives the above result within 1-7th. After having verified this law by several experiments, he bound together the bars of 6 metres long (19.68 feet) and $0^m.011$ thick, kept at the distance of 14.5 times this thickness by a St. Andrew's cross, and a tie often employed in iron bridges. The theoretical strength of this system is 1443 times that of one of the two pieces; but experiment made it only 1260 times, a difference which M. Duleau attributes to the flexion of the St. Andrew's cross.

In applying the same considerations to a hollow tube, M. Duleau finds that its resistance to twisting is proportional to the difference between the 4th powers of its exterior and interior diameter; but he has not verified this theoretical result by experiment. Hence it would follow that a hollow tube, whose thickness is 1-20th of its diameter, will have a resistance $9\frac{1}{2}$ times greater than a cylinder of the same weight.

M. Duleau's results for curved bars are new and interesting.

When a curved rib is loaded in its middle, the descent of the loaded point is, with small weights, equal to 2-3ds of the deflexion, which the same weight would produce in a rectilineal bar of the same width and thickness as the curved rib, but of a length three times less, resting by its extremities on two bearings.

The curved rib is quite bent by a weight equal to three times that which would produce a deflexion equal to that of its own curvature when the rib is straightened and placed horizontally on two bearings. The curved rib will bend under two weights equal to the preceding, if they are placed in each at the sixth part of its length from each extremity.

The most unfavourable point at which a curved rib can be loaded is at the 4th part of its length from either extremity. It then bends under two times the weight capable of producing the primitive deflexion, when it is straightened and placed horizontally in two bearings.

The curved ribs by which the above theoretical propositions were confirmed by repeated observations, had 6.4 metres for the length of the chord, with a thickness of $0^m.06$ and $0^m.02$, and a deflexion of $0^m.70$.

IV. *On the Strength of Materials when Twisted.*

The strength of axles and other parts of machines which resist the force of being twisted, was supposed to be proportional to the cube of their diameter. M. Duleau, however, has arrived at a different conclusion. By considering that the angle of tension is the same for all the particles situated in each section, and that this angle varies from one section to another, and in any given section is proportional to its distance from the twisted extremity of the bar, he concludes that the resistance which a piece of round iron opposes to tension is inversely as its length, and directly as the 4th power of its diameter. Twelve experiments made on pieces, whose diameters varied from $0^m.005$ to $0^m.036$ confirmed this theory, and the general result of them was, that a round bar one millimetre long, and $0^m.01$ in diameter, was twisted 1° by the action of a weight of 22 kilogrammes acting on the circumference of the bar.

A very valuable set of experiments on the twisting of $\frac{1}{4}$ inch cast iron bars has been made by Mr. G. Rennie. The apparatus which he used consisted of a wrought iron lever, two feet long, having an arched head about 1-6th of a circle of 2 feet radius, of which the lever represented the radius. The centre about which it moved had a square hole made in it to receive the end of the bar to be twisted. The other end of the bar was fixed in a square hole in a piece of iron, and this again in a vice.

1. *On Twists of $\frac{1}{4}$ inch Bars, close to the bearings.*

Weights with which they were twisted.

	Horizontal cast.			Vertical cast.	
	10 lb.	4 oz.		10 lb.	8 oz.
Bad casting,	8	4		10	13
	10	11		10	11
Average,	9	17	Average,	10	10

Different Metals.[3]

Cast steel, -	17 lb.	9 oz.	Hard gun metal, -	5 lb.	0 oz.
Sheer steel, - -	17	1	Fine yellow brass, -	4	11
Blister steel, -	16	11	Copper, cast, - -	4	5
English iron, wrought,	10	2	Tin, - - -	1	7
Swedish iron, wrought,	9	8	Lead, - -	1	0

2. *On Twists of $\frac{1}{4}$ inch Bars of different lengths.*

	Cast horizontal.		Cast vertical.	
$\frac{1}{4}$ by $\frac{1}{2}$	7 lb.	3 oz.	10 lb.	1 oz.
$\frac{1}{4}$ by $\frac{3}{4}$	8	1	8	9
$\frac{1}{4}$ by 1 in.	8	8	8	5

Twists of bars cast horizontal at 6 inches from the bearing.

$\frac{1}{4}$ Inch square by 6 inches,	10 lb.	9 oz.
$\frac{1}{4}$ Ditto by ditto,	9	4
$\frac{1}{4}$ Ditto by ditto.	9	4

Twists of $\frac{1}{2}$ inch square bars cast horizontally.

	Qrs.	lb.	oz.	
$\frac{1}{2}$ Inch close to the bearing,	3	9	12	End of bar hard.
$\frac{1}{2}$ Inch ditto,	2	18	0	Middle of bar.
$\frac{1}{2}$ Ditto at 10 inches for bearing, Lever in the middle,	1	24	0	

From these experiments Mr. Rennie concludes that the vertical casts are stronger than the horizontal ones; but if we throw out of the experiments the badly cast specimens, the difference between the horizontal and vertical casts is very trifling.

When the averages of the two kinds of casts are taken conjointly, and compared with a similar cast of half inch bars, the strength of the bars appears to be nearly as the cubes of their diameters.

In the horizontal castings of different lengths, the resistance is greater when the lengths are increased; but with the vertical castings the reverse is the case. In the horizontal castings, at 6 inches from the bearing, there is a perceptible increase in the resistance; but it is not so great as when the twist was made close to the bearing.

For an account of the principles of carpentry, the reader is referred to the *Edinburgh Encyclopædia*, Art. *Carpentry*, and to Mr. Tredgold's valuable work already quoted.

[3] Mr. Rennie has, by mistake, given this table *twice* in the same paper. In the repeated table he makes the resistance of cast steel 19 lbs. 9 oz., and in the first 17 lbs. 9 oz. The latter is probably the correct result.

CHAPTER X.

DESCRIPTION OF DIFFERENT MECHANICAL ENGINES.

1. *Description of a Simple and Powerful Capstane.*

This capstane is represented in Plate VIII, Fig. 11, where *A D* is a compound barrel, consisting of two cylinders *C*, *D* of different radii. The rope *D E C* is fixed at the extremity of the cylinder *D*, and after passing over the pulley *E*, which is attached to the load by means of the hook *F*, it is coiled round the other cylinder *C*, and fastened at its upper end. *A B* is the bar by which the compound barrel *C D* is urged about its axis, so that the rope may coil round the cylinder *D*, while it unwinds itself from the cylinder *C*. Let us now suppose that the diameter of the part *D* of the barrel is 21 inches, while the diameter of the part *C* is only 20, and let the pulley *E* be 20 inches in diameter. It is evident that when the barrel *A D* is urged round by a pressure exerted at the point *B*, 63 inches of rope will be gathered upon the cylinder *D*, and 60 inches will be uncoiled from the cylinder *C* by one revolution of the bar *A B*, these numbers representing the circumference of each cylinder. The quantity of wound rope, therefore, exceeds the quantity that is unwound by 63 — 60, or 3 inches, the difference of their respective perimeters; and the half of this quantity, or $1\frac{1}{2}$ inches, will be the space through which the load, or the pulley *E* moves by one turn of the bar. But if a simple capstane of the same dimensions had been employed, the length of rope coiled round the barrel by one revolution of the bar would have been 60 inches, and the space described by the pulley or load to be overcome would have been 30 inches. Now, it is a maxim in mechanics,[1] that the power of any engine is universally equal to the velocity of the impelled point divided by the velocity of the working point, or to the velocity of the power divided by the velocity of the weight, that is, to the velocity of the point *B* divided by the velocity of the pulley *E*; consequently if the lever in both capstanes is the same, and the diameter of their barrels equal, the power of the common will be to the power of the improved capstane as $1\frac{1}{2}$ to 30, that is,

Method of computing its power.

[1] See Vol. I, pp. 38, 39.

inversely as the velocity of their weights, and the power of the latter will be $\frac{30}{1\frac{1}{2}} = 20$, or in other words, will be equivalent to a 20 fold tackle of pulleys.[2] If it is wished to double the power of the machine, we have only to cover the cylinder C with lathes a quarter of an inch thick, so that the difference between the radii of each cylinder may be half as little as before; for the power of the capstane increases as the difference between the radii of the cylinders is diminished. As we increase the power, therefore, we increase the strength of our machine, while all other engines are proportionably enfeebled by an augmentation of power. Were we, for example, to increase the power of the common capstane, we must diminish the barrel in the same proportion, supposing the bar or handspike not to admit of being lengthened, and we not only weaken its strength, but destroy much of its power by a greater flexure or bending of the ropes.

Convertible into a crane.

The reader will perceive that this capstane may be converted into a crane or windlass for raising weights, merely by giving the compound barrel $A\,B$ a horizontal position, and substituting a winch instead of the bar $A\,B$. The superiority of such a crane above the common one is obvious from what has been said; but it has this additional advantage, that it allows the weight to stop at any part of its progress, without the aid of a ratchet wheel and catch, from the two parts of the rope pulling on contrary sides of the barrel. The rope, indeed, which coils round the larger part of the barrel acts with a larger lever, and consequently with greater force than the other; but as this excess of force is not sufficient to overcome the friction of the gudgeons, the weight remains stationary in any part of its path.

A crane of this kind was erected in 1797 at Bordenton in New Jersey, by Mr. M'Kean, for the purpose of raising logs of wood to the frame of a saw-mill, which was 10 feet distant from the ground. The diameter of the largest cylinder was 2 feet, and its length 3 feet; the other cylinder was 1 foot in diameter, and of the same length with the largest. The difference of their circumferences, therefore, was 3 feet, and the log would move through a space of 18 inches with 1 turn of the

[2] In practice it will be found equivalent to a 26 fold tackle of pulleys, as about *one third* of the power of a system of pulleys is destroyed by friction and the bending of the ropes.

handspike; and through the required height with only 8 turns. The length of the bar or handspike was 6 feet, which, at the point where the power was applied, described a circle of about 30 feet, so that the power of the crane was as 1 to 20. The length of the rope was only 55 feet, whereas if the weight had been raised through the same height with a similar power by means of a tackle of pulleys, 270 feet of rope must have been employed. In the latter case, however, the rope sustains only $\frac{1}{20}$ of the weight, but in the former it supports one half of the load.

In describing a capstane of this kind, Dr. Robison remarks, that when the diameters of the cylinders which compose the double barrel are as 16 to 17, and their circumferences as 48 to 51, the pulley is brought nearer to the capstane by about 3 inches for each revolution of the bar. This, however, is an oversight, as the pulley is brought only $1\frac{1}{2}$ inches nearer the axis. In order to understand this, let us conceive a quantity of rope equal to the circumference of the larger cylinder to be wound up all at once, and a quantity equal to the circumference of the lesser one to be unwound all at once. In the present case, 51 inches of rope will be coiled round the larger part of the barrel by one revolution of the capstane bar, and consequently the load would be raised $25\frac{1}{2}$ feet, the rope being doubled. Let 48 inches of rope be now unwinded from the lesser cylinder, and the load will sink 24 feet; therefore $25\frac{1}{2} - 24 = 1\frac{1}{2}$ feet is the whole height or distance through which the weight has been moved.

This capstane appears to have been the invention of George Eckhardt, and likewise of Mr. Robert M'Kean of Philadelphia, son to the present governor of Pennsylvania. Dr. Gregory observes, that he has seen a figure of a similar capstane among some Chinese drawings nearly a century old.

The principle on which the preceding capstane is constructed might be applied with advantage when two separate axles $A\,C$, $B\,D$ (Plate VIII, Fig. 12), are driven by means of the winch H and the wheels B and A. It is evident that when the winch is turned round in one direction, the rope R is uncoiled from the axle $B\,D$; the wheel B drives the wheel A, so that the axle $A\,C$ moves in a direction opposite to that of $B\,D$, and the rope is coiled round the axle $A\,C$. If the wheels A, B are of the same diameter and the same number of teeth, the weight W

will be stationary, as the rope coiled about one axle will be always equal to what is uncoiled from the other.

If it should be required to have the axles of the same diameter, the wheel *A* might be made to revolve either slower or faster than *B* by varying either their size or their number of teeth; or the two wheels might be placed upon one another as in the Odometer shewn in Plate I, Fig. 9 (See page 38, *note.*) The one wheel might have 100 and the other 101 teeth; the axles might extend beyond the outer surfaces of each wheel to receive the ropes; and if the wheels were driven by a winch and endless screw, the two axles would revolve with different velocities.

2. *On the Principle and Construction of Lever Presses.*

During the last twenty years very great improvements have been made in presses both for printing and other purposes. These improvements have consisted chiefly in adapting a kind of mechanical power, which has been generally called a combination of levers. Let *A B*, *B C* (Plate VIII, Fig. 13), be two levers joined together at *B*, let *B C* be moveable round the fixed point *C*, and let the extremity *A* be confined to move in the straight line *A C*. If the force is now applied to *B* in the direction *B D*, the power of thrusting possessed by the extremity *A* will be very great, and will increase as the point *B* approaches to *D*. In this case the power is to the resistance as Tang. BAC + Tang. $BCA : 1$, or when $AB = BC$ as 2 Tang. $BAC : 1$.

If the power is applied in a direction at right angles to *B C*, then the power will be to the resistance as Sin. ABC : cos. BAC; or when $AB = BC$, the power will be to the resistance as 2 Cos. $\frac{1}{2} BAC : 1$; or as $2 BD : AB$ or BC.[3] Upon this principle the presses of Mr. Ruthven, and that of Mr. Clymer (called the Columbian press), have been constructed, and it is capable of an immense variety of beautiful and useful applications. Mr. Wells of Hartford has still more recently introduced it into a new press, which has been pronounced by competent judges to be of a very superior kind, and which we shall therefore describe, in Mr. Fisher's words, as it is yet in a great measure unknown in this country.

[3] See Professor Fisher's paper on printing presses in the *American Journal of Science*, vol. iii, p. 320.

" A perspective view of this elegant piece of mechanism is given in Plate VIII, Fig. 14. The frame is of iron, cast (with the exception of the feet) in a single piece ; and is of such form and dimensions as to be incapable of springing while the press is in operation. The platen (4) is of cast iron, and is of the dimensions of an entire form. The circular projection in the middle, with six radiating pieces, gives it an ample degree of firmness. The platen is immediately acted on by bringing nearly into a straight line the two main levers (6) and (17). These levers, in presses of the medium size, are fifteen inches each in length, and in the position represented in the figure, which is that of the greatest obliquity, they want two and a quarter inches at their point of contact of being straight. The lower end of each lever is four inches broad, and is rounded off into a portion of a cylindrical surface of half an inch radius. A piece of steel fixed within the circular projection in the middle of the platen has a hollow bush or bed of corresponding figure : in this the lower end of the lever (17) is set. The upper end of this lever is hollowed out in the same manner to receive the lower end of (6), and the upper end of (6) to receive a projection from the under side of the top of the frame. At (5) there is a provision for raising or lowering this projection by slips of sheet iron or tin, and thus adjusting the position of the levers to the best working state. The ends of the levers and the beds in which they rest are overlaid with steel, and the beds are so contrived as permanently to retain a small quantity of oil. (9) is a spindle of wrought iron, fastened at the upper end by a screw and nut to the shorter arm of the balance lever (7), and branching below into three parts, each of which is attached by an adjusting screw to the platen. This answers the double purpose of keeping the platen steady, and enabling the weight (18) attached to the longer arm of the lever (7) to lift the platen and carry back the bar immediately after each pull. The platen is still farther guided by lateral projections which run in grooves connected with the cheeks of the press.

" The mode in which the movement of the working bar (12) is transmitted to the main levers will be best understood from Fig. 15, which is a representation of the parts 11, 12, 13, and 15, as they would appear to an eye looking down upon the press from above. The bar *B A* (the lever worked with the

hand) is inserted into a strong cast iron roller (13) which turns in sockets secured to the right cheek of the press. From this roller, about 6 inches above the bar, proceeds an arm *A C*, three inches in length, and to the extremity of this is connected by a joint the driving lever *C D*, $21\frac{1}{2}$ inches long. The extremity *D* is connected in a similar way with the iron rod *E F*; one end slides in a pewter guide (represented by 10 in Fig. 14), while the other end is fastened by a hook and eye to the upper main lever (6) at the distance of an inch from the bottom. (16) is a bar check, which limits the revolution of the bar to a precise arc. The *carriage* part of the press, which stands in front of the upright iron frame, presents nothing materially different from the Columbian press, and will not require a particular description.

" The operation of the mechanism will now, it is believed, be sufficiently apparent. When the bar *B A* is brought round, the roller *A* and the arm *A C* are made to turn with it: this drives forward the lever *C D*, and this in its turn gives motion to *E F*, which by means of the elbow at *F* brings the two main levers (6) and (17) towards the position of a straight line. As the movement of the bar is continued, the mechanical advantage not only increases from the gradual approach of the two main levers to a vertical position, but from the approach of *A D* and *A C* towards a straight line. The combination is therefore one which is eminently adapted to effect that rapid increase of power near the end of the pull, which has been already mentioned as the great desideratum in the construction of this part of the printing press.

" To determine the actual gain of power at the beginning and at the end of the pull, measures have been taken from an individual press of the lines necessary for the computation. When the bar was thrown back, the angle *A C D* (of the triangle *A D C* formed by joining the three centres of motion with straight lines) was found to be $= 113° 52'$, $C D A = 7° 12'$, and the distance of the centre of motion of the two adjacent ends of the main levers from the straight line joining their outer extremities $= 2\frac{1}{4}$ inches. The length of *A C* was $3\frac{1}{6}$, and the distance from *A* to the part of the handle where the hand was generally applied was 24 inches. Hence, as will appear from the theorems given above, the gain of power will be found by compounding the four following ratios: 24 to $3\frac{1}{6}$; cos. $70° 12'$

to sin. 113° 52′; 15 to $2 \times 2\frac{1}{4}$; and 14 to 15; which gives a total of 20 to 1.

"At the end of the pull the angle $ACD = 172°$, the angle $CDA = 1° 3'$, and the distance of the vertical levers from a straight line, according to the specification of the inventor, which was found nearly exact, = half an inch. Hence the gain of power will be found by compounding the following ratios: 24 to $3\frac{1}{6}$; cos. 1° 3′ to sin. 172°; 15 to $2 \times \frac{1}{2}$; and 14 to 15; which gives a result of 763 to 1. It thus appears that the power gained is about thirty-eight times greater at the end than at the beginning of the pull."

3. *Description of the Thrashing Machine.*

Utility of machines for abridging labour.

In a country like this, where agriculture has arrived at such a high state of perfection, the utility of thrashing machines cannot be called in question. The universal prevalence of these engines is a strong proof that they are advantageous to the farmer; and, however much some men may inveigh against the adoption of every kind of machinery that has for its object the abridgment of manual labour, yet we are convinced that no evil consequences can possibly accrue from their introduction; and that such insinuations have a tendency to inflame the minds of the vulgar, and retard the progress of science. As a proof of this, we might mention the fate of the celebrated Arkwright, the inventor of the fly-shuttle, whom the fury of an English rabble banished from his native country.

History of the thrashing machine.

The thrashing machine was invented in Scotland, in 1758, after five years labour, by Mr. Michael Stirling, a farmer in Perthshire. The honour of this invention has been claimed by Mr. Andrew Meikle, an ingenious mill-wright in East Lothian, who obtained a patent for one of these machines about the year 1785; and in this country his claims have been generally admitted. Mr. Meikle, however, was merely an improver of the thrashing machine, and I am assured by a gentleman of the most unquestionable authority, who, from his local situation, had access to the best information, that Mr. Meikle had seen Mr. Stirling's thrashing machine before he erected any of his own, and that he merely altered and improved it. About 26 years prior to the date of Mr. Stirling's invention, a thrashing machine was constructed in Edinburgh, by Mr. Michael Menzies, which operated by

the elevation and depression of a number of flails, by means of the motion of a crank; and in 1767, the model of a thrashing mill, invented by Mr. Evers of Yorkshire, was laid before the Society of Arts in London, who rewarded the inventor with a premium of £60. This machine, which was driven by wind, consisted of a number of stampers, that beat out the grain when laid upon a moveable thrashing floor, and was actually used on a large scale in Yorkshire, where it received the approbation of several intelligent gentlemen of the county.[4] All these machines, however, and others of a similar kind, are completely defective in principle, and are greatly inferior to the worst of those now in use, which operate by the revolution of a thrashing scutch furnished with beaters,—the exclusive invention of our countryman Mr. Stirling.

Wherever a sufficient quantity of water can be procured, it should always be employed as the impelling power of thrashing machines. There are many situations, however, in which it cannot be obtained; and as the erection of steam-engines and wind-mills would be too expensive for the generality of farmers, they are under the necessity of having recourse to animal power. In Plate IX, Fig. 1, is represented a thrashing machine, which may be driven by four or six horses. To the vertical axis *M* six strong bars are fixed, called the horse poles, four of which *P*, *R*, *S*, *L*, are visible in the figure, and to the extremity of each of these poles two pieces of wood, like *o p*, are attached, to which the horses are yoked when the machine is to be used. Upon the top of the six poles is placed the large bevelled wheel *A B*, containing 270 teeth, which drives the pinion *B C* of 40 teeth; on the axle *N* is also fixed the wheel *D D*, which carries 84 teeth, and drives the pinion *b* of 24 teeth, placed upon the axle *b k*. Upon the same axis the wheel *E E* revolves, carrying 66 teeth, which drive the pinion *c* of 15 teeth, and consequently the thrashing-drum *x x*, which is fixed upon the same axle. The feeding rollers are driven by the intervention of the four bevelled wheels *i*, *h*, *e*, *d*, the latter of which is fastened on the axis of the upper feeding roller. The wheel *i*, upon the gudgeon *i b*, contains 25 teeth, the wheel *h* 24 teeth, *e* 22 teeth, and *d* 21 teeth; but when the fluted rollers require a greater velocity, *e* is taken from its iron axle, and a greater or less wheel substituted in its room. The short

[4] Bailey's Drawings of Machines laid before the Society of Arts, vol. i, pp. 54-59.

axle $b\ k$ is furnished with a pulley p, which, by means of the leathern belt $p\,p$, gives motion to the fanners placed below the thrashing scutch and straw-shaker.

Fig. 2 represents a plan of the wheels, thrashing-drum, and straw-shaker, where the corresponding parts, in Fig. 1, are marked with similar letters. The small wheels g and k, however, which convey motion to the straw-shaker, are not seen in Fig. 1. The largest one g is fixed on the axis N, and carries 38 teeth. It drives k, which contains 14 teeth, and is placed upon the axis of the straw-shaker $K\,K$.

An elevation of the working parts of the machine is delineated in Fig. 3, where the corresponding parts in the plan and section have the same letters affixed to them. The sheaves of corn are spread on the feeding-board O, drawn in by the rollers i, i, and thrashed by the beaters o, o, which strike downward. Part of the corn falls through the rack $i\,r$, and some part of it is carried along with the straw into the larger rack $r\,p$, where it falls into the hopper below, while the straw is thrown out at the opening $n\,p$. The drum and straw-shaker are surrounded with a covering of wood $i\,m\,n$. The following Table exhibits, at one view, the number of teeth in the wheels, and the different velocities with which they move.

Names of the wheels.	Number of teeth in each wheel.	Number of turns for one of the wheel.
Plate IX, Figs. 1, 2, 3.	Teeth.	Turns. Dec.
$A\,B$	270	1.000
$B\,C$	40	6.750
$D\,D$	84	6.750
b	24	23.625
$E\,E$	66	23.625
c	15	103.950
Thrashing-scutch	0	103.950
g	38	6.750
k	14	18.293
Straw-shaker	0	18.293
i	25	23.625
h	24	24.617
e	22	26.857
d	21	28.199
Feeding rollers	0	28.199

Driven by horses and water.

In situations where there is an occasional supply of water, thrashing machines are sometimes constructed so as to be driven either by horses or water. In this case, the water-wheel has the position $L H$ (Fig. 1), and is furnished with a large wheel $G H$, consisting of segments of cast iron firmly fixed to the arms of the water-wheel. The wheel $G H$ fixed on the horizontal shaft N, drives $F G$, and thus communicates motion to it and the rest of the machinery. When there is no water for impelling the mill, the water-wheel $L H$ is either lowered in its frame, or one of the segments is taken from the wheel $G H$, in order to keep it clear of the wheel $F G$; and when there is a sufficient discharge of water, $C B$ is either raised above $A B$, or $A B$ is deprived of a few teeth, which can be screwed and unscrewed at pleasure. Sometimes, when there is a small supply of water, its energy may be combined with the exertion of one or two horses.

Driven by wind or steam.

If the thrashing machine is to be driven by wind, the motion is conveyed to the axle N by the small wheel $m C$, fixed at the bottom of the vertical axis n, which is moved by the wheel upon the windshaft. If the mill is to be moved by steam, which is considered by many farmers as advantageous on a large farm, the fly must be fixed on the axis N, parallel to the horizon.

Power of thrashing machines.

The quantity of corn which a machine will thrash in a given time depends so much upon the judicious formation and position of its parts, that one machine will often perform double the work of another, though constructed upon the same principles, and driven by the same impelling power. Misled by this circumstance, those who have given an account of the power of their thrashing mills have published merely the number of bolls which they can thrash in a given time, without mentioning the quantity of impelling power, or the number of horses employed to drive them.

Mr. Fenwick, whose labours in practical mechanics we have already mentioned with commendation, has furnished us with some important information upon this point. He found, from a variety of experiments, that a power capable of raising a weight of 1000 pounds, with a velocity of 15 feet per minute, will thrash two bolls of wheat in an hour; and that a power sufficient to raise the same weight, with a velocity of 22 feet

per minute, will thrash three bolls of the same grain in an hour. From these facts, Mr. Fenwick has computed the following Table, which is applicable to machines that are driven either by water or horses.

Table of the Power of Thrashing Machines.

Gallons of water per minute, ale measure, discharged on an overshot wheel 10 feet in diameter.	Gallons of water per minute, ale measure, discharged on an overshot wheel 15 feet in diameter.	Gallons of water per minute, ale measure, discharged on an overshot wheel 20 feet in diameter.	Number of horses working $9\frac{1}{2}$ hours each day.	Bolls of wheat thrashed in an hour.	Bolls thrashed in $9\frac{1}{2}$ hours' actual working, or in a day.
230	160	130	1	2	19
390	296	205	2	3	$28\frac{1}{2}$
528	380	272	3	5	$47\frac{1}{2}$
660	470	340	4	7	$66\frac{1}{2}$
790	565	400	5	9	$85\frac{1}{2}$
970	680	500	6	10	95
1	2	3	4	5	6

The four first columns of the preceding table contain different quantities of impelling power, and the two last exhibit the number of bolls of wheat in Winchester measure, which such powers are capable of thrashing in an hour, or in a day. Six horses, for example, are capable of thrashing 10 bolls of wheat in an hour, or 95 in the space of $9\frac{1}{2}$ hours, or a working day; and 680 gallons of water discharged during a minute into the buckets of an overshot water-wheel 15 feet in diameter, will thrash the same quantity of grain.

CHAPTER XI.

ACCOUNT OF THE APPARATUS FOR MAKING AND BURNING GAS.

The first person who appears to have obtained gas from coal, and to have converted it into flame, was the Rev. John Clayton, in the year 1739. It was not however till 1792 that Mr. Murdoch of Soho actually employed gas for useful purposes, and succeeded in lighting up his house and offices at Redruth in Cornwall. He afterwards repeated his experiments at Old

Cumnock in Ayrshire, in 1796; and in 1798, when he went to Soho, he constructed a gas apparatus. Many years elapsed before the public saw the advantages of this important discovery, and though numerous trials were made on a small scale, yet it was not till the year 1805 that it began to excite general notice, after the cotton mills of Messrs. Philips and Lee of Manchester had been lighted up. Within the last ten years great improvements have been made on the apparatus, and all the principal towns in the kingdom are now lighted up with gas obtained either from coal or oil.

In the manufacture of coal gas, cannel coal is generally preferred, and one cwt. of it will yield about 520 cubic feet of gas. The coal is placed in an iron retort, which is subjected to a strong heat. The gas is thus driven off mixed with the vapour of tar, oil, and water, and in this state is conducted by pipes into a refrigeratory or condensing apparatus, surrounded with cold water, where the vapours of the tar, oil, and water, are condensed and fall down, while the gaseous product is conveyed along, containing several impure gases, such as sulphuretted hydrogen and carbonic acid.

In order to separate the carburetted hydrogen, and the olefiant gases from these impurities, various contrivances have been adopted, and it is in this part of the manufacture that a considerable improvement still remains to be effected.

The usual method of purifying the coal gas is to make it pass through a mixture of lime and water called *Milk of Lime*, or *Lime Cream*,[1] which absorbs the contaminating gases. For this purpose a considerable number of purifiers are erected, and the lime and water are kept night and day in a state of constant agitation, either by a steam-engine, or by one or two men. By this means the gas issues in a state of tolerable purity, and is then conveyed by a pipe to the *gasometer*.

The *Gasometer* is a large vessel, made of malleable iron, and either of a cylindrical or a rectangular form, and is suspended over a reservoir of water of a little larger size, by means of counter-weights. The gas is introduced by pipes ascending from the bottom of the reservoir, and rising a little above the sur-

[1] Dr. Hope found that subcarbonate of potash, or common potashes, or subcarbonate of soda, absorbed the contaminating gases with much more avidity than lime water; and he proposed to add 4 or 5 per cent. of it to the lime, and to use the latter in the slaked state.

face of the water. As the gasometer is filling with gas, it gradually rises out of the water, and when it is filled no more gas is admitted, and its contents are ready to be distributed through the pipes by which it is to be conveyed to the manufactory, or to the streets. As the gas is forced out by the weight of the gasometer, and is burned, the gasometer descends gradually in the water till the whole of its contents are expelled, when it is again filled by a similar process.

The gas being thus ready for use, it must be carried off by pipes whose diameter is proportional to the degree of light required. It has been found that a pipe 1 inch in diameter will, under a pressure of a column of water from $\frac{5}{8}$ths to $\frac{3}{4}$ths of an inch, supply gas equal to 100 candles; and if there was no friction, the number of candles would be found for other diameters of pipe by multiplying the square of the diameter of the pipe in inches by 100. The friction, however, diminishes so rapidly with the diameter of the pipe, that the number of candles is always greater than this rule gives. Thus a pipe 3 inches in diameter will supply light equal to 1000 candles—a pipe 4 inches, 2000—a pipe 6 inches, 5000—and a pipe 10 inches, about 14,000.

When the gas is to be burned, it is allowed to escape through small circular apertures of from $\frac{1}{40}$th to $\frac{1}{60}$th of an inch in diameter, which may be advantageously arranged in a circle, like an argand burner, with a current of air running between them.

Description of the Oil Gas Apparatus.

The public attention was first called to the gas from oil by the experiments of Dr. Henry in 1805; but it was not till 1815 that Mr. John Taylor introduced an excellent apparatus for decomposing oil and other animal substances.

This apparatus in its most improved state is shewn in Plate VIII, Fig. 16, where *A A* are cast-iron stoves lined with brickwork for holding the retorts, *a a* the doors for cleaning the retorts, *b* the fire-door, and *c* the ash-pit; *B B* is the frame work of cast-iron for supporting the rest of the apparatus, *C* the oil cistern, *D* a cock to allow the oil to enter by the funnel *E*, or by turning it in another direction to make it flow into the retort by the pipe *F*, through the perforated column *G G*, the quantity of oil being regulated by the small index cocks *d*, *d*, which are seen on an enlarged scale in Fig. 17.

After the oil has been distilled by the retorts, which are kept at a moderate red heat, the gas and oil vapours ascend through the perforated column *H H*, by the moveable pipe *I*, into the condenser within the case *J*, which is filled with cold water so as to surround the condenser. The portion of oil vapour which accompanied the gas is now condensed by passing through a worm, and is conducted into the oil-cistern *C* by the short pipe *K*, which by forming a communication between the retorts and the oil-cistern, permits the surface of the oil to be acted upon by the same pressure which exists in the retorts. When the gas is thus freed from the oil vapour, it is conveyed by the pipe *L* to the worm vessel *M*. This vessel has a screw-plug *e* for the introduction of water, and two cocks, one for drawing off the water, and another for regulating the height of the water in the vessel, and for drawing off the condensed oil that may occasionally accumulate. The gas being allowed to bubble through the water in the vessel *M*, is still farther cooled, and is conveyed by the pipe *N* to the gasometer, which is not shewn in the figure. The gasometer is made of sheet iron, and is suspended by a chain passing over pulleys in a brick or cast-iron tank filled with water.

An apparatus of the preceding kind, capable of producing from 12 to 20 argand lights, will occupy a space of only 4 feet by 3, and will require a height of only 8 feet. The great advantage of an oil gas apparatus for private houses is, that there are no disagreeable products in its distillation; and on any particular occasion wax may be distilled by it in place of oil, and the finest species of gas obtained.

The drawing in Fig. 14 represents the double apparatus erected at the Apothecaries-hall, London. It is 10 feet in front, 6 feet deep, and about 8 feet high. It is capable of producing from 1600 to 1800 cubic feet of gas at one operation, or without cleaning out the retorts. This quantity of gas has been computed to supply 300 argand burners for four hours, which is equal to from 3000 to 3600 mould candles.

It has been calculated from pretty correct data, that an establishment for lighting a town with coal gas with 21 retorts, 2 gasometers, and main pipes, &c. which costs £20,000, will supply daily 50,000 cubic feet of gas, and that the annual expenditure will be about £4000, which gives, including interest, for the cost of a quantity of gas that yields the same light as a pound of tallow, $2\frac{3}{4}$d.

An apparatus for oil gas which will yield the same quantity of light[2] has been computed to cost about £15,000, with an annual expenditure of about £8000, which gives 4½d. for the expense of a quantity of gas which yields the same light as a pound of tallow. Hence the following comparison has been made:—

1 lb. of tallow in candles, costs - - -	1s.	0d.
The same quantity of light from spermaceti oil in an argand lamp, - - - -	0	6½
Do. do. from gas from whale oil, - - -	0	4½
Do. do. from coal gas, - - - -	0	2¾

The preceding comparison is obviously unfavourable to oil gas for the purpose of street illumination. We understand, however, it has been ascertained that the illumination of a town by means of oil gas may be effected as cheaply, if not more so, than by means of coal gas. The town of Norwich has already been lighted with it, and we understand that Hull is also to enjoy the same advantage.

Various ingenious contrivances called *Gas Meters*, have been invented for the purpose of ascertaining with accuracy the quantity of gas consumed in a given time; but it would be foreign to our object to give any detailed account of these contrivances. A drawing and description of a very ingenious one by Mr. Malam will be found in the *Edinburgh Philosophical Journal*, vol. v, p. 129.

CHAPTER XII.

ACCOUNT OF THE SAFETY LAMP FOR LIGHTING MINES.

The great number of explosions which took place in coal mines, from the fire damp being inflamed by the lamps of the miners, directed the attention of the coal proprietors to the discovery of some substitute for the ordinary lamp. The light of putrifying fish, and that of phosphorus, had been tried without any success, and the only resource of the miner seemed to be in a *steel mill*, or apparatus which consisted of a steel wheel, made

[2] This calculation supposes that one gallon of whale oil will yield 100 cubic feet of gas.

to revolve rapidly by a wheel and pinion, in contact with flint, so as to produce a succession of sparks, which gave sufficient light to the miner. The expense of the machine, however, and the necessity of another person called a *miller*, to work the machine, and the circumstance of explosions having sometimes taken place even from the sparks of the steel, rendered it a very troublesome and insecure companion.

The dreadful explosion at the Felling colliery,[1] which took place on the 25th May 1802, and by which 92 persons were instantly destroyed, directed the attention of different individuals to the subject of a safety lamp; and in 1813 Dr. Clanny published in the *Philosophical Transactions*, the account of an ingenious apparatus, in which the flame of an air-tight lamp, with a glass front, was sustained by blowing atmospheric air by small bellows, through a stratum of water in the bottom of the lamp, while the heated air passed through water by a recurved tube at the top. This lamp, though perfectly secure, was too complicated for general use, and has, we believe, never been in use.

Sir Humphry Davy, having been invited by Dr. Gray of Newcastle to consider the best methods of preventing accidents from explosion, visited some of the principal collieries in 1815. He found that the fire damp was, as Dr. Henry had previously shewn, light carburetted hydrogen gas; that it was produced in small quantities during the ordinary process of working, but that its principal sources were what are called blowers or fissures in the broken strata near dykes, from which it issues in great quantities, and sometimes for a long course of years; and that old workings when re-opened were generally filled with it.

In continuing his experiments on the fire damp, Sir Humphry found that the mixture which possessed the greatest explosive power was that of 7 or 8 parts of air to 1 of gas; and that neither ignited charcoal, nor iron at the highest degree of heat, nor at an ordinary white heat, were capable of inflaming it. The flame of the explosive mixture refused to pass through metallic tubes $\frac{1}{5}$ of an inch in diameter, and $1\frac{1}{2}$ inch long; it passed more easily through glass tubes, and was al-

[1] A very full and interesting account of this and other catastrophes of a similar kind, will be found in Mr. Bald's curious article on *Coal Mines*, in the *Edinburgh Encyclopædia*, vol. xiv, p. 369, 371.

ways stopped by wire sieves or wire gauze. From these considerations, Sir Humphry Davy was led to the first form of the safety lamp, which was nothing more than a common lamp, with glass or horn windows, and where the flame was supplied with air, by means of safety metallic canals, and where the heated air escaped through similar canals. These safety canals consisted of 3 concentric hollow cylinders, whose depth was 2 inches, the distance $\frac{1}{20}$th of an inch, and the circumference of the smallest $2\frac{1}{2}$ inches. Apertures covered with metallic gauze were substituted in place of these safety canals.[2]

When the safety lamps thus constructed were introduced into explosive mixtures of fire damp, the flame was immediately extinguished.

In the beginning of 1816, Sir Humphry made a great improvementi on the safety lamp. Instead of securing the safety of the miner by the extinction of the lamp, he contrived a method by which the light would burn in any explosive mixture of fire damp, and the light of which arises from the combustion of the fire damp itself. This invention, which is shewn in Plate VIII, Fig. 18, consists in surrounding the flame of a lamp or candle by a wire sieve. The coarsest that he tried with perfect safety contained 625 apertures in a square inch, and the wire was $\frac{1}{70}$th of an inch thick; and the finest 6400 apertures in a square inch, and the wire $\frac{1}{250}$th of an inch thick. When such a lamp is introduced into an explosive mixture of carburetted hydrogen and air, the cylinder becomes filled with a bright flame, and this flame continues to burn as long as the mixture is explosive. When the quantity of air is to that of carburetted hydrogen as 12 to 1, the flame of the wick appears within that of the fire-damp; but when the ratio is as 7 to 1, the flame of the wick disappears. When the thickest gauze is used, it becomes red hot, and yet no explosion takes place.[3]

In some subsequent researches on the combustion of gaseous mixtures, Sir H. Davy proposes to supply the coal-miner with light in mixtures of fire-damp containing too little air to be explosive, by means of a small cage made of platinum wire $\frac{1}{70}$ or $\frac{1}{80}$ of an inch thick, as shewn in Fig. 19.[4] If the flame should be extinguished by the quantity of fire-damp, the glow of the metal will be sufficient to guide him; and by placing the lamp

[2] See the *Philosophical Transactions* for 1816, p. 1.
[3] See *Phil. Trans.* 1816, p. 23. [4] *Id. Id.* 1817, p. 81.

in different parts of the gallery, the relative brightness of the wire will shew the state of the atmosphere in these parts. This beautiful experiment gave rise to Mr. Ellis' *Lamp Without Flame,* in which a coil of platinum wire, the 100dth part of an inch thick, heated red hot, and held above a glass containing æther or alcohol, will retain its red heat, and even reach nearly a white heat, by the invisible combustion of the alcoholic or etherial vapour. A thin sheet of platinum or palladium produces the same effect.

A more particular account of the safety-lamp will be found in the volumes of the *Phil. Trans.* already quoted, or in Sir H. Davy's Treatise "*On the Safety-Lamp for Coal Miners, with some Researches on Flame.*" Lond. 1818.

CHAPTER XIII.

ON THE PRINCIPLES AND CONSTRUCTION OF BALLOONS.

The name *Balloon* is given to a hollow sphere of silk, paper, or any other substance which contains a gas lighter than atmospherical air, or which has the atmospherical air which it contains rarefied and made lighter by heat.

A balloon containing such a gas will manifestly rise in the atmosphere upon the same principles as cork or wood rises in water, and, abstracting the weight of the balloon, will continue to ascend till it reaches a height in the atmosphere where the density of the gas within the balloon is equal to that of the air without.

The late celebrated Mr. Cavendish having discovered in 1766 that hydrogen gas was about *seven* times lighter than common air, our eminent countryman, Dr. Joseph Black, conceived the idea of employing it to raise weights in the atmosphere. He procured the allantois of a calf, and having filled it with hydrogen gas, he found that it ascended to the ceiling of his apartment. With the view of surprising some of his friends, he invited them to supper, and having produced the bag of hydrogen, it immediately ascended and remained attached to the ceiling of the room.[1] Dr. Black, however, was too in-

[1] See the *Edinburgh Encyclopædia*, vol. iii, p. 553, where Dr. Thomson, in his Life of Dr. Black, gives a more minute account of this discovery.

dolent to prosecute his discovery, and no use was made of the lightness of hydrogen gas till the year 1782, when the late Mr. Cavallo, after trying bladders scraped thin, and spheres of Chinese paper, succeeded in elevating soap bubbles by inflating them with hydrogen.

These different attempts, however, can be considered in no other light than as rude and partial efforts at invention; and it cannot be doubted that the real practical merit of the invention of balloons is owing to Stephen and John Montgolfier, of Annonay, in France. In November 1782, they took a silk bag in the form of a square box, open below, and containing about 45 cubic feet, and burning some paper below its mouth, the bag swelled and rose to the height of 75 feet. They next formed a globe 30 feet in diameter, out of coarse linen lined with paper, and having lighted a fire within it, so as to rarefy the air, it rose to a considerable height.

The two brothers were now solicitous to make a public display of their invention. They accordingly invited the public to witness the exhibition on the 5th June 1783, when their fire-balloon ascended to the height of nearly a mile.

The news of the success of this experiment was speedily diffused over Europe. M.M. Charles and Robert resolved to try hydrogen gas, and having constructed a globe, 13 feet in diameter, of thin silk, covered with Caoutchouc varnish, they filled it with hydrogen gas, obtained from the action of sulphuric acid upon iron filings. It ascended from the Champ de Mars on the 27th August 1783, and rose to the height of 3000 feet.

The first time that man trusted himself to this frail machinery was on the 21st November 1783, when M. Pilatre de Rozier and the Marquis d'Arlandes ascended in one of Montgolfier's balloons, of an elliptical shape, 45 feet wide and 75 feet high. The balloon rose 3000 feet, and after describing a track of about six miles above Paris, during the space of about 25 minutes, they descended in safety.

It would be foreign to the nature of this popular notice to give a historical detail of the various aërial voyages which were afterwards performed in every part of Europe. We shall merely add a list of some of the most important.

Height of Ascent. *Feet.*	Aeronaut.	Time and Place of Ascent.	Size of Balloon.
	Nobody	June 5, 1783, Paris;	contained 45 cubic ft.
	Nobody	Aug. 27, 1783, Paris	13 feet.
	A sheep, &c.	Sept. 19, 1783, Paris.	
3,000	Rozier and M. D'Arlandes	Nov. 21, 1783, Paris.	
9,770	M.M. Charles & Robert,	Dec. 1, 1783, Paris	28 feet diameter.
	M. Montgolfier	Jan. 19, 1784, Lyons	109 wide, 134 high.
1,300	Chev. Andreani	Feb. 24, 1785, Milan	70 feet diameter.
a Mile	Blanchard	Mar. 2, 1784, Paris	40 feet diameter.
13,500	M. Fleurant & Madame Thiblé	June 28, 1784, Lyons	75 feet high.
12,520	Rozier and Proust	July 1784, Versailles	Fire-balloon.
	M. Carnus	Aug. 6, 1784, Rhodes	57 feet diameter.
6,000	Duke of Orleans and Roberts	Sept. 19, 1784, Paris	56 feet by 36.
10,465	Morveau and Bertrand	April 25, 1784, Dijon	29 feet.
6,030	Morveau and Virly	June 12, 1784, Dijon.	Same balloon.
	Nobody	Nov. 25, 1783, London	10 feet.
	Lunardi	Sept. 28, 1784, London.	
3,000	M. Testu	June 18, 1786, Paris	29 feet.
3,000	Rozier and Romain	June 15, 1785, Boulogne	40 feet.
	Blanchard & Dr. Jeffries,[2]	Jan. 7, 1785, Dover, across the Channel.	
	Blanchard	August 1785, Lisle; dog descended in parachute.	
	Garnerin & Capt. Sowden,	June 1802, London.	
	Zambecari, Grasetti, and Andreoli[3]	Oct. 7, 1803, Boulogne.	
	Professors Robertson and Sacharof	June 30, 1804, St. Petersburg,	30 ft. diam.
13,000	Gay Lussac and Biot	Aug. 23, 1804, Paris.	
23,040	Gay Lussac	Sept. 5, 1804, Paris.	
	Zambecari and Andreoli,[4]	Aug. 1804, Boulogne.	
	Mosment[5]	April 7, 1806, Lisle.	
15,000	Garnerin[6]	Aug. 4, 1807, Paris.	
	Garnerin[7]	Sept. 21, 1807, Paris.	

Various interesting ascents have been made in England, Scotland, and Ireland, by our enterprising countrymen Messrs. Sadler, senior and junior; and within the last two years a very great improvement in the method of filling balloons has been made by Mr. Green, who employs the carburetted hydrogen gas obtained from coal; so that, by opening a coal gas pipe, a balloon of any magnitude may be filled in little more than an hour.

[2] Obliged to strip themselves of their clothes to lighten the balloon.

[3] Saved by a vessel in the Adriatic.

[4] Balloon took fire. Saved by a boat in the Adriatic.

[5] M. Mosment fell from the balloon and was killed.

[6] Excursion during the night.

[7] Descended during a violent storm 300 miles from Paris.

Description of Balloons, and the Method of filling them.

The balloon in which M. Pilatre de Rozier ascended on the 21st November 1783, is represented in Plate IX, Fig. 4, where the aeronauts are represented at its lower end.

The method of filling balloons with hydrogen gas from iron filings is shewn in Fig. 5, where *F A B* is the balloon in a flaccid state, suspended at *F* on a horizontal rope *E F G.* The strings which surround the balloon are fixed to the hoop *A B*, and to this hoop is suspended the boat or car *C D.* The sulphuric acid and iron filings are put in the casks round *M N*, and from the top of each of these casks is a tin tube, which introduces the gas into the two casks of water *N N*, immediately below the lower open end of a barrel *M M.* A tin tube *T*, inserted in the upper end of each tub *M M*, is connected with silken tubes, which convey the gas to the balloon. When the balloon is about three-fourths full of gas, the silken tubes are removed from the tin ones, the retaining ropes are slipped off, and the balloon escapes into the atmosphere.

The *Parachute* invented by M. Garnerin for descending from the balloon is represented open, and in the act of descending, in Fig. 6. The diameter is about 84 feet, and it is made either of cloth or canvas. It shuts up like an umbrella when it is not used, as shewn in Fig. 7. The aeronaut places himself in a basket *B.* The parachute is attached to the balloon by a rope connected with the network around, and by cutting this rope the parachute fills and expands. M. Garnerin descended in a parachute at Paris in 1797, and at London in 1802.

CHAPTER XIV.

DESCRIPTION OF THE DIVING BELL.

The diving bell is a hollow vessel, in which, when inverted, one or more persons may descend with safety to very great depths under water.

The principle of the diving bell may be very simply illustrated by taking a tumbler glass and fixing a pasteboard rim or circular seat within it, and near the middle of its depth.

If objects easily injured by water are placed upon this seat, and the tumbler immersed with its mouth downwards into a tub of water, it will be seen that a part of the water enters the tumbler and compresses the air within it, but never rises so high as to reach the pasteboard rim and the objects placed upon it. If a large vessel therefore of wood or iron is used, and is loaded so as to sink in water, a person may descend to any depth in the sea without the least risk from the entrance of water into the bell. The deeper he descends, the more will the air be compressed and the more water will enter; but the bell can never be filled with water, even at the greatest depths.

In order to avoid the great inconvenience of breathing air rendered impure by being used, a separate bell filled with fresh air sometimes accompanies the principal bell, or when the depth is not great, a pipe from the surface of the sea supplies the bell with fresh air.

The earliest notice of the diving bell is that of John Taisnier, who was born at Hainault in 1509, and who, in the presence of the Emperor Charles V, saw two Greeks descend at Toledo in a large inverted kettle with a burning light, and ascend again without being wetted. Although the diving bell had been frequently used for the purpose of recovering valuable wrecks, yet it was not till the time of Dr. Halley that it became a safe and a convenient machine. This ingenious philosopher used a bell of wood, containing about 60 cubic feet, and of the form of the upper part of a wine glass. Its diameter was 3 feet at top and 5 at bottom, and it was loaded with lead so as to descend vertically. By means of two barrels cased with lead, and made to descend and ascend alternately from a ship above the bell, Dr. Halley was enabled not only to supply fresh air, but even to keep the water entirely out of the cavity of the bell.[1] Dr. Halley likewise contrived a method of supplying a person with air from the bell when at a considerable distance from it in search of articles. He connected leathern pipes, about 40 feet long, 4½ inches in diameter, and kept open by a spiral brass wire, with the bell, and with a leaden cap which was placed on the head of the diver. The diver coiled this pipe about his arm. There was a cock at the end of the pipe which opened into the cap, so

[1] See *Phil Trans.* 1716, p. 492.

that the person could stop the return of the air whenever it was necessary to stoop down or descend below the surface of air in the bell.[2]

In order to convey to the reader some idea of this interesting machine, we shall give a drawing and description of the diving bell, as improved by Mr. Spalding of Edinburgh.

The bell itself is represented by *A B C D*, Fig. 8, Plate IX. It was made of pipe staves, and was 5 feet long, 5 feet wide at bottom, and $2\frac{1}{2}$ at top. It is suspended by four ropes *e e*, fastened to hooks, and meeting the great rope *Q*. By means of the ballast weights *C C*, and another weight *L*, which may be elevated or depressed by the rope *a M*, the mouth of the bell is kept parallel to the surface of the water, and the bell may be lightened by allowing *L* to reach the ground. The divers stand upon a stage of ropes suspended by hooks *b b*. The two air casks, containing 40 gallons each, with their tackle, are shewn at *T N*, and *O C P* is the flexible pipe by which the air is conveyed from them into the bell. These pipes have cocks as at *P*, and their ends are guided by the ropes *M M*, extending from the bell to the ship above. The heated air is discharged by a cock at *R*. In order to enable the divers to raise the bell, or stop it at any particular depth, Mr. Spalding fixed a second bell *S*, of smaller dimensions, on the large one *A B C D*. This smaller bell contains 25 gallons, and has a cock *t*, which can be opened (the handle *t t* reaching to the lower bell) to allow the air in *S* to escape. Another cock at *v* permits the air to pass from the great bell into the small one. By these means the diver can introduce into the small bell either air or water in any proportion, and thus either increase or diminish the weight of the whole machine.

Dr. Halley has described the effect of the condensed air upon the ears of those who descend in the bell. "The only inconvenience," says he, "that attends it is found in the ears, within which there are cavities opening only outwards, and that by pores so small as not to give admission even to the air itself, unless they be dilated and distended by a considerable force. Hence, on the first descent of the bell, a pressure begins to be felt on each ear, which by degrees grows painful,

[2] See *Phil. Trans.* 1721, p. 177.

like as if a quill were forcibly thrust into the hole of the ear, till at length the force overcoming the obstacle, that which constringes these pores yields to the pressure, and letting some condensed air slip in, present ease ensues. But the bell descending still lower, the pain is renewed, and again eased after the same manner. On the contrary, when the engine is drawn up again, the condensed air finds a much easier passage out of these cavities, and even without pain."

The most interesting account of the influence of the condensed air upon different persons is given by Dr. Colladon, in a curious narrative of a descent in the diving bell at Howth in 1820. "As soon as the bell was immersed in water, we felt about the ears and the forehead a sense of pressure, which continued increasing during some minutes. I did not, however, experience any pain in the ears; but my companion suffered so much, that we were obliged to stop our descent for a short time. To remedy that inconvenience, the workmen instructed us, after having closed our nostrils and mouths, to endeavour to swallow and to restrain our respiration for some moments, in order that, by their exertion, the internal air might act upon the Eustachian tube. My companion, however, having tried it, found himself very little relieved by this remedy. After some minutes, we resumed our descent. My friend suffered considerably; he was pale, his lips were totally discoloured; his appearance was that of a man on the point of fainting; he was in involuntary low spirits, owing perhaps to the violence of the pain, added to that kind of apprehension which our situation unavoidably inspired. This appeared to me the more remarkable, as my case was totally the reverse. I was in a state of excitement resembling the effect of some spirituous liquor. I suffered no pain; I experienced only a strong pressure round my head, as if an iron circle had been bound round about it. I spoke with the workmen, and had some difficulty in hearing them. This difficulty of hearing rose to such a height, that during three or four minutes I could not hear them speak. I could not indeed hear myself speak, though I spoke as loudly as possible; nor did even the great noise caused by the violence of the current against the sides of the bell reach my ears. I thus saw confirmed by experience what Dr. Wollaston had foreseen by theory in his curious and interesting paper on sounds inaudible to certain ears."—

See the *Edinburgh Philosophical Journal*, vol. v, p. 8, for a full account of Dr. Colladon's descent.

CHAPTER XV.

DESCRIPTION OF SOME HYDROSTATIC AND HYDRAULIC ENGINES.

1. *Description of the Water-Blowing Machine.*

This machine is so useful for conveying wind to the furnaces of iron forges, and the principle by which it operates is so curious, as to entitle it to the particular attention of the practical mechanic, as well as the speculative philospher. Although it has been known and generally adopted on the Continent for above a century,[1] yet it has neither been generally introduced into the forges of this country, nor has it found its way into many of our treatises upon machinery.

Let *M N*, Fig. 9, Plate IX, be a cistern of water, with the bottom *A* of which is connected the pipe *B*. The lower extremity *B* of the pipe is inserted into the top of a cask or vessel *C D*, called the condensing vessel, having the pedestal *G* fixed to its bottom, which is perforated with a tube *C I K*. When the water, which comes from the cistern *M N*, is falling through the conical part *a b* of the pipe, it is supplied by the openings or tubes *c*, *d*, with a quantity of air which it carries along with it. This mixture of air and water issuing from the aperture *B*, and impinging upon the surface of the stone pedestal *C*, is driven back and dispersed in various directions. The air being thus separated from the water ascends into the upper part of the vessel, and rushes through the pipe *C I K*, whence it is conveyed to the fire at *K*, while the water falls to the lower part of the vessel, and runs out by the openings *M*, *N*.

In order that the greatest quantity of air may be driven into the vessel *C D*, the water should begin to fall at *b* with the least possible velocity; and the distance of the lowest perforations *o*, *p*, from the extremity of the pipe *B* should be to the length of the vertical tube *A B* as 3 to 8, in order that the

[1] It seems to have been first introduced in Italy in 1672.

air may move in the pipe IK with sufficient velocity. The part of the tube between the lowest perforations and B, and also the vessel CD, must be completely closed, to prevent the escape of the internal air. The water escapes at an aperture above F, a foot in diameter, and sometimes sluices $m\,n$ are used to regulate its discharge.

The wind is supplied from the atmosphere.

Fabri and Dietrich imagined that the wind is occasioned by the decomposition of the water, or its transformation into gas, in consequence of the agitation and percussion of its parts. But M. Venturi,[2] to whom we are indebted for the first philosophical account of this machine, has shewn that this opinion is erroneous, and that the wind is supplied from the atmosphere; for when the lateral perforations were shut, no wind was generated.

Hence the principal object in the construction of these machines is to combine as much air as possible with the descending current. With this view the water is often made to pass through a kind of cullender placed in the open air, and perforated with a great number of small triangular holes. Through these apertures the water descends in many small streams, and by exposing a greater surface to the atmosphere, it carries along with it an immense quantity of air, and is conveyed to the pedestal G by a tube AB, open and enlarged at a, so as to be considerably wider than the end b of the pipe which holds the cullender. According to Venturi, the diameter of the aperture at C should not exceed one half of the diameter of the tube CH.

It has been generally supposed that the waterfall should be very high;[3] but Dr. Lewis has shewn, by a variety of experiments, that a fall of 4 or 5 feet is sufficient, and that when the height is greater than this, two or more blowing machines may be erected, by conducting the water from which the air is extricated into another reservoir, from which it again descends and generates air as formerly. That the air, which is necessarily loaded with moisture, may arrive at the furnace in as dry a state as possible, the condensing vessel CD should be made as high as circumstances will permit; and in order to

[2] Experimental Enquiry concerning the Lateral Communication of Motion in Fluids. Prop. 8.

[3] Wolfius makes the length of the tube CH five or six feet. *Opera Mathematica*, tom. i, p. 830.

determine the strength of the blast, it should be furnished with a gauge filled with water.

Franciscus Tertius de Lanis observes,[4] that he has seen a greater wind generated by a blowing machine of this kind, than could be produced by bellows 10 or 12 feet long.[5]

Cause of the rain wind.

The *rain wind* is produced in the same way as the blast of air in water-blowing machines. When the drops of rain impinge upon the surface of the sea, the air which they drag along with them often produces a heavy squall, which is sufficiently strong to carry away the mast of a ship. The same phenomenon happens at land, when the clouds empty themselves in alternate showers. In this case, the wind proceeds from that quarter of the horizon where the shower is falling. The common method of accounting for the origin of winds by local rarefactions of the air, appears to be pregnant with insuperable difficulties; and there is reason to think that these agitations in our atmosphere ought rather to be referred to the principle which we have now been considering.

The *Ventaroli* or natural blasts which sometimes issue from volcanic mountains, arise from the air carried down the crevices by the falls of water. At the foot of the cascades which descend from the glaciers of Roche Melon upon the naked rock of La Novalese, Venturi found that the force of the wind arising from the air dragged down by the water could scarcely be withstood.[6]

2. *Description of Whitehurst's Machine for raising Water.*

Mr. Whitehurst, an ingenious watchmaker of Derby, appears to have been the first who entertained the ingenious idea of raising water by means of its momentum. A machine upon this principle was erected at Oulton in Cheshire, and was described in the Transactions of the Royal Society for 1755. It was intended for the service of a brewhouse and other offices, and was found to answer effectually the purpose of its erection. This

[4] In Magisterio Naturæ et Artis, lib. v, cap. 3.

[5] If we call a the diameter of the aperture at b, and d the diameter of the tube $A B$, then, according to Venturi, the quantity of air which passes into the tube in one second will be $6.1\, a^2 \sqrt{a + d - 1.4} - 0.4\, a^2 \sqrt{0.1\, a}$.

[6] Those who wish for more information upon the subject of water-blowing machines may consult Lewis's *Commerce of Arts*; the *Journal des Mines*, No. 91; or Nicholson's *Journal*, vol. i, 4to, vol. xii, p. 48.

machine is represented in Plate IX, Fig. , where *A M* is the spring or original reservoir of water, whose surface at *M* is on a level with *B*, the bottom of the reservoir *B N*. The main pipe *A E* is about 200 yards long, and $1\frac{1}{2}$ inches in diameter; the branch pipe *E F* is of the same diameter, and for the service of the kitchen, offices, &c. situated at least 18 or 20 feet below the surface of the reservoir *A M;* the cock *F* is about 16 feet below the surface of the water at *M*. A valve box, with its valve *a*, is shewn at *D*, and *C* is an air vessel into which are inserted the extremities *m*, *n* of the main pipe, which are bent downwards for the purpose of preventing the air from being driven out when the water is forced into it. Now, when the cock *F* is opened, the water will rush out with a velocity of nearly 32 feet per second, corresponding to a pressure of 16 feet perpendicular height. A column of water, therefore, 200 yards long and $1\frac{1}{2}$ inch diameter, is now put in motion, and must have a considerable momentum. Hence, if the cock *F* is suddenly shut, the water will rush through the valve *a* into the air vessel *C*, and condense the included air. This condensation will take place every time that the cock is opened, so that the included air being compressed, will press upon the water in the air vessel, and raise it into the reservoir *B N*. Mr. Whitehurst observes, that the condensation of the air was strong enough to burst the vessel *C* in a few months after it was first constructed, though it was made of sheet lead of about 9 or 10 lbs. to a square foot. Hence he concluded that the momentum of the water was much superior to the simple pressure of the column. This simple and ingenious machine is obviously the same in principle as the hydraulic ram invented by Montgolfier, and which differs from it only in this, that the operation analogous to that of opening the cock *F* is produced by the motion of the water itself, as will be seen in the following description of this ingenious contrivance.

3. *Description of Montgolfier's Hydraulic Ram.*

This interesting machine was first constructed by Montgolfier about the year 1797, and has been brought to a very perfect state by a series of improvements which he has succesively made upon it. The rams which we have represented in Plate IX, Fig. 11, 12, 13, 14, 15, contain the improvements which have been made so late as 1816. The large pipe *A B* (Fig. 11

and Fig. 14), called the body of the ram, passes through the side of the reservoir *P Q*, from which the fall of water is obtained. It has a trumpet mouth at one end *A*, and at the other end an opening *H H*, which can be closed by valves *C* or *D*. When these valves are open, the water will issue at *H H* with a velocity due to the height *A P*; but when the internal valve *C* is closed, as in the figure, the water is prevented from issuing. When the valve *C* opens, it descends into the position shewn by the dotted lines *G G* (Fig. 14) being guided between three or four stems *g*, *g*, which have hooks at the lower ends for supporting the valves. In this case the water has a free passage between these stems, and the width of the passage can be increased or diminished by the screws with which the stems are fixed. The valve *C* is made of metal, and has a hollow cup or dish of metal attached to its lower surface. The seat *H H* of the valve is wider than the diameter of the pipe *A B*. It consists of a short cylinder or pipe screwed by its flanch *h h* (Fig. 14) into the opening of the upper surface of the head *R* of the ram; and the cylinder is so formed as to have an inverted cup or annular space *i i* round the upper part of it for containing air, which cannot escape when it is compressed by the water. A small pipe *k l*, leading from this annular space to the open air, is furnished with small valves *k l*, one of which, *k*, opens inwards to admit the air into *i i*, but to prevent its return, while the other valve *l* admits a certain quantity of air, and then shuts and prevents any farther entrance. The valve *D* is exactly the same as *C*, only it descends as in the figure when it shuts, and rises when it opens.

The upper part of the head of the ram at *E* (Fig. 11) is made flat, and has several valves which allow the water to pass freely from the pipe *A B*, but prevent its return. On each side of the head of the ram, at the part opposite to these valves, is a hollow enlargement, shewn by the dotted lines *K*, forming a circular bason, through the centre of which the pipe *A B R* passes. This part of the construction is shewn more distinctly in Fig. 12, which is a transverse section through *L E Z* in a plane perpendicular to that of the paper. The pipe is here made flat instead of circular, as seen at *E*, Fig. 12, for forming the seats of the valves, and the bason *K K* is covered with an air vessel *F F*. This air vessel communi-

cates all round the pipe *B*, with the bason *K K*, and with the vertical pipe *M*.

The machine being thus constructed, let us suppose the pipe *A B R* (Fig. 11 and 14) full of water, and the valve *C* to be opened, the water will lift the valve *D*, and escape with a velocity due to the height of the reservoir. In a short time, the water having acquired an additional velocity, raises the valve *G*, which shuts the passage, and prevents the escape of the water. The consequence of this is, that all the included water exerts suddenly a hydrostatical pressure on every part of the pipe, compressing at the same time the air in the annular space *i i*, which by its elacity diminishes the violence of the shock. This hydrostatical pressure opens the valves at *E*, and a portion of the water flows into the air vessel *F*, and condenses the air which it contains. The valves at *E* now close, preventing the return of the water into the pipe, and the water recoils a little in the tube with a slight motion from *B* to *A*, in consequence of the reaction or elasticity of the compressed air in *i i*, and also of the metal of the pipe, which must have yielded a little to the force exerted upon it in every direction. The recoil of the water towards *A* produces a slight aspiration within the head *R* of the ram, which causes the valve *D* to descend by its own weight, and prevent the water *X* which covers it from descending into the tube. The air, however, passes through the pipe *l k*, opens the valve *k*, and a small quantity is sucked into the annular space *i i;* but the quantity is very small, as the valve *k* closes as soon as the current of air becomes rapid. During the recoil towards *A*, the valve *C*, being unsupported, falls by its own weight; and when the force of recoil is expended by aeting on the water in the reservoir *P Q*, the water begins again to flow along *A B R*, and the very same operation which we have described is repeated without end, a portion of water being driven into the air vessel *F* at every ascent of the valve *C*. The air in this vessel being thus highly compressed, will exert a force due to its elasticity upon the surface of the water in the vessel *F*, and will force it up through the pipe *M* to a height which is sufficient to balance the elasticity of the included air.

The small quantity of air which is drawn into the annular space *i i* through the air tube *l k* at each aspiration, causes an accumulation of air in the space *i i;* and when the aspiration

of recoil takes place, a small quantity of air passes from $i\,i$, and proceeds along the pipe till it arrives beneath the valves at E, and lodging in the small space beneath the valves, it is forced into the air vessel at the next stroke, and thus affords a constant supply of air to the vessel. The valves make in general from 50 to 70 pulsations in a minute.

When the fall of water, or $P\,Q$, is five feet, and the pipe $A\,B$ six inches in diameter and 14 feet long, a machine with its parts proportioned as in the figure will raise water to the height of 100 feet. It will expend about 70 cubic feet per minute in working it, and will raise about $2\frac{1}{3}$ cubic feet per minute to the height of 100 feet. Mr. Millington observes, that one of these machines is said to have raised 100 hogsheads of water in 24 hours to the height of 134 feet by a fall of $4\frac{1}{2}$ feet.

Form of the ram for raising clean water with foul water. Plate IX. Fig. 13.

The form of the ram represented in Fig. 13 is suited to the case where a current of foul water $A\,B$, is employed to raise clean water from the well $W\,W$. This effect is produced by a bent pipe $O\,P\,Q$, containing a column of air from O to Q, and by another pipe T, with a suction valve t: The mode of action is precisely the same as in Fig. 11. When the valve C shuts, the sudden hydrostatical pressure forces the water up the bent tube at O, compresses the column of air $O\,Q$, which again presses, by its elasticity, on the surface of the water at Q, and forces the clean water up through the valves into the air vessel $F\,F$. The recoil of the water from B to A will produce a rarefaction in the column of air $Q\,O$, in consequence of which, the atmospherical pressure upon the water in the well will raise the valve t, till as much water is admitted as was driven into the air vessel. Montgolfier proposes to substitute a straight pipe, in place of $O\,Q$, and to place a piston, moving freely in the pipe, which will transmit the pressure from the foul water to the clean water, without allowing them to mix.

Form of the machine for produciug a current of air. Fig. 15.

When the ram is employed to produce a current of air, it has the form shewn in Fig. 15. The air is expelled through the air-pipe $w\,m$, in consequence of the mass of water rushing into the air-chamber W, by the shutting of the valve C. The water in W is prevented from following the air by a hollow ball of copper n, which floats on the water, and shuts up the lower end of the pipe, when the water dashes into W. When

things are in the state shown in the figure, and all the air expelled from the chamber *W*, the air compressed in the annular space *p p* (which serves the same purpose as *i i* in Fig. 11), produces a recoil of the water. The valve *D* shuts, *C* opens, the water quits the chamber *W*, and the valve *w* shuts, and prevents the admission of air. At the same time the valve *r* opens, and admits a fresh supply of air into the chamber; but when the water has descended below the float *e*, this float descends, and by its rod *e d* shuts the air-valve *d*. When the force of recoil is spent, the water flows again from *A* to *B*, and the operation which we have described is again repeated, so that there is a constant current of air in the pipe *w m*, which may be equalized by a water regulator, or any other contrivance. See the *Repertory of Arts*, December 1816, and the *Journal of the Royal Institution*, vol. i, p. 216, Lond. 1816.

4. *Description of Bramah's Hydrostatic Press.*

The hydrostatic press is founded on the hydrostatic paradox, which has been already very clearly illustrated in the preceding volume. The celebrated Pascal mentions this application of the principle as a new sort of machine for multiplying forces,[7] and he regards it as a mechanical power equal in value to the lever or the screw.

Although Pascal has the undoubted honour of having first suggested a hydrostatic press, yet no attempt appears to have been made for more than a century and a half to put the suggestion in practice. Mr. Bramah had the great merit of actually constructing the machine, and of applying it practically to a great variety of useful purposes, such as that of working cranes, pulling up the roots of trees, packing cotton and all sorts of goods.

The Hydrostatic Press is represented in Plate IX, Fig. 16, where *A B* is a metallic cylinder of great strength, and bored truly cylindrical. A solid piston *P* is fitted into the top of it by leather packing, so as to be perfectly water tight. A small pipe *g g* is inserted into the bottom *B* of the cylinder, and communicates with a small forcing pump at *C*. This pump stands

[7] *Nouvelle sorte de Machine pour multiplier les Forces.* See his *Traitez de l'Equilibre des Liqueurs et de la Pesanteur de la Masse de l'Air.* Chap. ii. Paris, 1664.

in an iron cistern *H H*, from the side of which rises the bent arm *K K* for supporting the centre of the handle *G*, and keeping the piston rod *L* of the pump in a vertical direction. The weight of the handle *G* is counterpoised by a weight *W*. If the forcing pump *C* and pipe *g g*, and cylinder *A B*, are filled with water, and if one or two men at the handle *G* press down the piston of the forcing pump, the pressure exerted on the surface of the water will be propagated through the pipe *g g* and cylinder *A B* to the lower end of the piston *P*, and the pressure actually exerted on the piston *P* will be to that exerted on the surface of the water in the forcing pump, as the area of the piston *P* is to the area of the piston of the forcing pump. Hence, if the diameter of *P* is 6 inches, and that of the piston in the forcing pump $\frac{2}{10}$ths of an inch, their ratio will be as 30 to 1, and the square of their diameters or their areas as 900 to 1. If a force of 1 ton, therefore, is exerted in the forcing pump, the piston *P* will be pressed upwards by a force of 900 tons.

The enormous power which is thus exerted may be applied in a great variety of ways to produce mechanical effects. It may be conveyed to any distance merely by means of pipes filled with water, and in this way it forms a machine in which there is more simplicity of construction, and less friction, than in any mechanical combination. A full account of the application of this press to a crane and other purposes will be found in the *Edinburgh Encyclopædia*, Art. *Crane*, vol. vii, p. 315.

CHAPTER XVI.

ON THE PRINCIPLES AND CONSTRUCTION OF OPTICAL INSTRUMENTS.

As Mr Ferguson's lecture on optics contains very little practical information on this interesting subject, and as many discoveries and improvements have been made since his time, I shall endeavour to supply this defect in as popular a manner as the subject will admit. In doing this the following arrangement will be observed,—

1. On the Single Microscope.
2. On the Compound Microscope.
3. On the Refracting Telescope.
4. On the Gregorian and Cassegrainian Telescope.
5. On the Newtonian Telescope.
6. On a new Reflecting Telescope.
7. On Achromatic Object-Glasses.
8. On Achromatic Eye-Pieces, &c.
9. On the Camera Obscura.
10. On the Phantasmagoria.
11. On the Dichroscope.
12. On the Kaleidoscope.

1. *On the Single Microscope.*

In the first volume we have described several methods of forming small glass globules for the magnifiers of single microscopes,[1] and have also explained the manner in which the enlarged picture is formed upon the retina. When the lenses are made, either by fusion, or which is by far the most accurate way, by grinding them on spherical tools, they are then to be fitted up for the purpose of examining minute objects. The method which Mr. Wilson has adopted in his pocket microscope is very ingenious, though rather devoid of simplicity, as it obliges us to screw and unscrew the magnifiers, when we wish to view the object with a larger or a smaller power. The simplest and the most convenient method of mounting single microscopes is to fix the lenses in a flat circular piece of brass, which can be moved round a centre, by the action of an endless screw, upon the toothed circumference of the circular plate. After the object has been viewed by some of the magnifiers, it may be examined successively with all the rest, by a few turns merely of the endless screw.

Its magnifying power.

In the first volume, Mr Ferguson has already shewn how to find the magnifying power of single microscopes; but in order to save the trouble of calculation, we have computed the following table (See p. 296) of the magnifying power of convex lenses, from 1 inch to $\frac{1}{100}$ of an inch in focal length, upon the supposition that the nearest distance at which we see distinctly is *seven* inches. The first column contains the focal length of the convex lens in 100ths of an inch.

[1] See Vol. I, p. 167, note 8.

The second contains the number of times which such a lens will magnify the diameters of objects. The third contains the number of times that the surface is magnified; and the fourth the number of times that the cube of the object is magnified. A table of a similar kind, though upon a much smaller scale, has already been published by Mr. Baker; but he supposes the nearest distance at which the eye can see distinctly to be *eight* inches, which is too large an estimate for the generality of eyes.

When we consider, however, that the eye examines very minute objects at a less distance than it does objects of greater magnitude, we shall find that the magnifying power of lenses ought to be deduced from the *distance at which the eye examines objects really microscopic.* This circumstance has been overlooked by writers on optics, and merits our attentive consideration. If we take two specimens of engraving, the one so large that it can easily be read at the distance of 10 inches; and the other so exceedingly minute that it cannot be read at a distance exceeding 5 inches; then it is obvious, that if these two kinds of engraving are seen through the *same microscope,* the one will be *twice* as much magnified as the other. For as the magnifying power of a lens is equal to the distance at which the object is examined by the naked eye, divided by the focal distance of the lens, we shall have $\frac{5}{x}$ for the number of times that the minute engraving is magnified, and $\frac{10}{x}$ for the number of times that the large engraving is magnified, x being the focal length of the lens. It follows, therefore, that the number of times that any lens magnifies objects really microscopic should be determined by making the distance at which they are examined by the naked eye 5 inches.

Upon this principle I have computed the second Table, which contains the magnifying power of convex lenses, when employed to examine microscopic objects.

As microscopic objects are never mathematical lines, and as their solidity or their magnitude in three dimensions is not visible by the microscope, the *third* column in the first table, and the *second* column in the second table, will give the real magnifying power of the microscope, although, from an erroneous analogy with the telescope, the second column in the first table, and the first column in the second, have been generally used.

TABLES of the Magnifying Power of Small Convex Lenses or Single Microscopes,[2]

The distance at which the eye sees distinctly being 7 inches.				*The distance at which the eye sees distinctly being 5 inches.*		
Focal distance of the lens or microscope.	Number of times that the *diameter* of an object is magnified.	Number of times that the *surface* of an object is magnified.	Number of times that the *cube* of an object is magnified.	Number of times that the *diameter* of an object is magnified.	Number of times that the *surface* of an object is magnified.	Number of times that the *cube* of an object is magnified.
100ths of an inch.	Times. Dec.	Times.	Times.	Times. Dec.	Times.	Times.
1 100	7.00	49	343	5.00	25	125
$\frac{3}{4}$ 75	9.33	87	810	6.67	44	297
$\frac{1}{2}$ 50	14.00	196	2744	10.00	100	1000
$\frac{2}{5}$ 40	17.50	306	5360	12.50	156	1953
$\frac{3}{10}$ 30	23.33	544	12698	16.67	278	4632
$\frac{2}{10}$ 20	35.00	1225	42875	25.00	625	15625
19	36.84	1354	49836	26.32	693	18233
18	38.89	1513	58864	27.78	772	21439
17	41.18	1697	69935	29.41	865	25438
16	43.75	1910	83453	31.25	977	30518
15	46.66	2181	101848	33.33	1111	37026
14	50.00	2500	125000	35.71	1275	45538
13	53.85	2894	155721	38.48	1481	56978
12	58.33	3399	198156	41.67	1736	72355
11	63.67	4045	257259	45.55	2075	94507
$\frac{1}{10}$ 10	70.00	4900	343000	50.00	2500	125000
9	77.78	6053	470911	55.55	3086	171416
8	87.50	7656	669922	62.50	3906	244141
7	100.00	10000	1000000	71.43	5102	364453
6	116.66	13689	1601613	83.33	6944	578634
$\frac{1}{20}$ 5	140.00	19600	2744000	100.00	10000	1000000
$\frac{1}{25}$ 4	175.00	30625	5359375	125.00	15625	1953125
3	233.33	54289	12649337	166.67	27779	4629907
$\frac{1}{50}$ 2	350.00	122500	42875000	250.00	62500	15625000
1	700.00	490000	343000000	500.00	250000	125000000

2. *On the Compound Microscope.*

The double microscope is composed of two convex lenses placed at any distance not less than the sum of their focal lengths; and a lens with a large aperture and focal distance is generally fixed a little beyond the anterior focus of the eye-glass, for the purpose of enlarging the field of view. As the focal length of the lenses, as well as their distances, are altogether arbitrary, different opinions have been entertained

[2] For drawings and descriptions of a great variety of single microscopes, see the *Edinburgh Encyclopædia*, Art. *Microscope*, vol. xiv, p. 216.

respecting the most suitable values of these quantities. It has been found, however, from experience, that a good compound microscope may be formed by making the object-glass $\frac{4}{10}$ of an inch in focal length, and the eye-glass 1 inch, their distance being about 8 inches. The amplifying lens or second eye-glass should generally be $1\frac{3}{4}$ inches in diameter, with $2\frac{1}{2}$ inches of focal length, and placed at $1\frac{1}{4}$ inches before the eye-glass ; and the aperture of the object-glass should not exceed *one-tenth* of an inch. If, however, we increase the magnifying power of the microscope by augmenting the distance between the glass next the object and that next the eye, we must likewise enlarge the aperture of the object-glass ; but if we increase the magnifying power by augmenting the curvature, or diminishing the focal length of the object-glass, the aperture must be proportionally diminished. The distance of the eye from the eye-glass should be equal to the focal distance of the latter ; and the hole through which the rays are finally transmitted should not exceed *one-seventh* of an inch.

Its magnifying power.

The method of finding the magnifying power of compound microscopes when only two lenses are employed, has been shewn in the first volume. But as an amplifying lens, or second eye-glass, is always used, we shall shew how to determine the magnifying power of these instruments when three lenses are employed. Divide the difference between the distance of the two first lenses, or those next the object, and the focal distance of the second or amplifying glass, by the focal distance of the second glass, and the quotient will be a first number. Square the distance between the two first lenses, and divide it by the difference between that distance and the focal distance of the second glass: From this quotient subtract the excess of the distance between the first and third glasses above the focal length of the third glass, and divide by the focal distance of the third glass, or that next the eye, and a second number will be had. Multiply together the first and second numbers, and the magnifying power of the object-glass, and the product will be the magnifying power of the compound microscope.

Rule for finding it.

An account of a new Compound Microscope for examining objects of natural history will be found in my *Treatise on New Philosophical Instruments*, p. 401–410.

In another work I have had occasion to give an account of various improvements which I have suggested on the single and the compound microscope.

1. The first of these consists in constructing microscopes on a large scale, from *ten* to *thirty* or *forty* feet in length.

The advantages of large microscopes are very numerous. The errors arising from the internal and superficial imperfections of the glass are diminished. A more correct spherical figure, and better centering or adjustment of the axes of the opposite surfaces can be obtained. The coincidence of the axes of the different lenses is more easily effected. The object may be illuminated in a much higher degree in consequence of its great distance from the object lens. Objects placed in deep cavities may be examined with the utmost facility; and microscopic objects of a perceptible thickness, such as a fly, may be well seen at once, the instrument being adjusted simultaneously to its remotest and its nearest parts. See the *Edinburgh Philosophical Journal*, vol. vi, p. 104, and the *Edinburgh Encyclopædia*, Art. *Optics*, vol. xv, p. 632.

2. The second of these improvements consists in using lenses of coloured glass for diminishing the error of refrangibility. Red and green lenses were found to give very distinct vision, and are infinitely superior to colourless lenses, particularly when the outlines of opaque objects are to be observed.

3. The third improvement consists in illuminating microscopic objects with a *homogeneous flame* of yellow light, obtained from the combustion of diluted alcohol. This apparatus, which I have called a Monochromatic Lamp, is described at great length in the *Edinburgh Philosophical Transactions*, vol. ix. See also the *Edinburgh Encyclopædia*, vol. xv, p. 633.

3. *On the Refracting Telescope.*

Having already described the nature and operation of refracting telescopes, we have now only to lay before the reader a new table of the apertures and magnifying powers of these instruments. It is a remarkable circumstance, that the only table of this kind which has appeared was copied by succeeding writers from Dr. Smith's Optics, as the production of the celebrated Huygens, while it was calculated only by the editors of his Dioptrics An excellent telescope of Huygens, indeed, was the standard upon which the table was constructed; but

this philosopher informs us, in his *Astroscopia Compendiaria*, that he had a refracting telescope with an object-glass 34 feet in focal length, which, in astronomical observations, bore an eye-glass of $2\frac{1}{2}$ inches focal distance, and therefore magnified 163 times, which is considerably greater, in proportion, than the magnifying power of the standard telescope upon which the old table was founded. Since the lenses of these instruments, therefore, may now be wrought with the same accuracy as in the time of Huygens, we have computed the following table according to this new standard, the apertures, magnifying powers, and the focal length of the eye-glass, being severally as the square roots of their focal lengths. As the dimensions of the standard telescope of Huygens were taken in Rhinland measure, the following table is suited to the same measure; but the numbers may be converted into English measure by considering that the Rhinland foot is equal to 1.03 English nearly.

In order to render the common refracting telescope as perfect as possible, without making it achromatic, the exterior surface of the object-glass should be ground to a radius equal to *five-ninths* of its focal length, and the radius of the interior surface, or that next the eye, should be *five* times its focal length. In eye-glasses, the radius of the surface next the object should be *nine* times its focal distance, and that of the surface next the eye *three-fifths* of the same distance. By this means, the aberration arising from the spherical figure of the lenses will be nothing for objects placed in the direction of their axis, and the least possible for objects removed from the axis. According to Huygens, the spherical aberration was the least possible when the radii of the surfaces were as 6 to 1: But though this be true for objects placed in the axis of the lenses, yet a considerable aberration remains when the objects are placed on one side of the axis.

TABLE of the Apertures, Focal Lengths, and Magnifying Power of Refracting Telescopes.

Focal length of the object-glass.	Linear aperture of the object-glass.	Focal distance of the eye-glass.	Magnifying power.
Feet.	Inches. Dec.	Inches. Dec.	Times.
1	0.65	0.50	28
2	1.03	0.62	39
3	1.30	0.75	48
4	1.45	0.87	55
5	1.61	1.00	60
6	1.79	1.07	67
7	1.96	1.15	73
8	2.14	1.21	77
9	2.20	1.30	83
10	2.32	1.38	87
13	2.63	1.58	99
15	2.81	1.70	106
20	3.31	1.95	123
25	3.73	2.15	139
30	4.01	2.40	150
35	4.34	2.58	163
40	4.64	2.76	174
45	4.92	2.93	184
50	5.20	3.08	195
55	5.48	3.22	205
60	5.71	3.36	214
70	6.16	3.64	231
80	6.58	3.90	246
90	7.02	4.12	262
100	7.39	4.35	276
200	10.41	6.17	389
300	12.89	7.52	479
400	14.72	8.71	551
500	16.52	9.71	618

4. *On the Gregorian and Cassegrainian Telescope.*

To the observations already made upon this instrument, we have only to add a few practical remarks. In order to remove the tremors from reflecting telescopes, the springs and screws should be taken away from the back of the speculum, and three small screws employed, which pass through the tube perpendicular to its axis, and touch the back of the mirror merely with their sides. As the speculum is apt to bend when it is supported wholly upon its lower extremity, it should be made to rest upon two points 45 degrees distant from its lowest part, and on each side of it; and if the metal is wedged in at these points with bits of card, it will be

prevented from falling backward or resting upon its lowest point. This improvement was suggested and used by the Rev. Mr. Edwards; but the bending may be still farther prevented by hanging, as it were, the speculum upon a circular piece of metal at the end of the tube. Some reflecting telescopes may be much improved, as Dr. Maskelyne has shewn, by inclining the great speculum about $2\frac{1}{2}$ degrees[3] to the axis of the tube, so that the pencils of rays may fall obliquely on its surface.[4]

The diameter of the small eye-hole may be found by dividing the aperture of the telescope in inches by its magnifying power, but it is generally about $\frac{1}{50}$ of an inch.

The following Table, founded upon the computations of Dr. Smith, contains all the dimensions of Gregorian telescopes, and is more comprehensive and accurate than that which Mr. Edwards published.

Table of the Dimensions, Focal Lengths, and Apertures of Gregorian Telescopes, as constructed by Mr. Short, according to the computations of Dr. Smith. See Plate VIII, Vol. I, Fig. 14.

Focal length of the great speculum.	Breadth of the great speculum.	Breadth of the small speculum, and of the hole in the large one.	Focal length of the small speculum.	Distance between the two specula.	Distance between the large speculum and the plane surface of the first eye-glass.	Focal distance of the first eye-glass, or that next the metals.	Focal distance of the second eye-glass, or that next the eye.	Dist. betw. the plane sides of the 2 lenses.	Distance between the second eye-glass & the small eye-hole.	Diameter of the diaphragm placed in the anterior focus of the lens D.	Magnifying power.
P m. In. Dec.	*D F.* In. D.	*UV—hg.* In. Dec.	*L n.* In. D.	*P L.* In. Dec.	*P R.* In. Dec.	*R.* In Dec.	S. In. D.	*R S.* In. D.	*S c.* In. D.	*a b.* In. Dec.	Times.
5.65	1.55	0.31	1.11	6.78	1.76	2.45	0.81	1.63	0.41	0.27	39.69
9.60	2.30	0.40	1.50	11.25	3.36	3.13	1.04	2.09	0.52	0.35	60
15.50	3.30	0.50	2.15	17.84	3.95	5.97	1.31	2.63	0.66	0.44	86.46
36.00	6.26	0.65	3.43	39.72	1.44	5.12	1.71	3.41	0.85	0.57	165.02
60.00	9.21	0.83	5.01	65.39	2.78	6.43	2.14	4.29	1.07	0.72	242.94
1	2	3	4	5	6	7	8	9	10	11	12

From the following Table of the Dimensions of Cassegrainian Telescopes, founded on Dr. Smith's calculations, it appears that though they are shorter by twice the focal distance of the

[3] This degree of inclination greatly improved the 6 feet Newtonian reflector in the Observatory of Greenwich; but different specula will require different degrees of obliquity, and some may rather be injured by such an inclination.

[4] These observations are also applicable to the metals of Cassegrainian and Newtonian telescopes.

small speculum than those of the Gregorian form with the same focal length, yet they have a greater magnifying power. A Cassegrainian telescope $15\frac{1}{2}$ inches in focal length will magnify, according to the table, 93 times; while a Gregorian one, with a similar speculum, magnifies only 86 times. This great difference between the performance of these instruments does not exist merely in theory; for Mr. Short constructed a telescope of the Cassegrainian form, of 24 inches focus, which, with an aperture of 6 inches, magnified 355 times. With this power, indeed, it was rather indistinct; but it bore a power of 231 times with sufficient distinctness. In the Observatory at Greenwich there is a Gregorian telescope of Short's construction, which magnifies 250 times when the smallest mirror is employed, which is considerably less than the power of the Cassegrainian one of the same size. This superiority of the telescope of Cassegrain has been recently established by the experiments of Captain Kater, who found that the light of a Cassegrainian telescope was to that of a Newtonian one as *seven* to *three*. Conceiving that this superiority arose from the rays not having crossed one another at the focus, he made a series of experiments, for the purpose of determining the difference in the intensity of the light at the same distances within and without the focus of a concave speculum, and he found the intensity within the focus to be 1000, when that without the focus was only about 450. Another set of experiments made the difference much less; but the intensity of the light within the focus was always considerably greater.

Table *of the Dimensions, Focal Lengths, and Apertures of Cassegrainian Telescopes. See Plate VIII, Vol. I, Fig.* 14, *in which the Small Convex Speculum is supposed to be placed at G H.*

Focal length of the great speculum.	Breadth of the great speculum.	Breadth of the small speculum, and of the hole in the large one.	Focal length of the small speculum.	Distance between the two specula.	Distance between the large speculum and the plane surface of the first eye-glass.	Focal distance of the first eye-glass, or that next the metals.	Focal distance of the second eye-glass or that next the eye.	Dist. betw. the plane sides of the 2 lenses.	Distance between the second eye-glass & the small eye-hole.	Diameter of the diaphragm, placed in the anterior focus of the lens D.	Magnifying power.
P m. In. Dec.	*D F.* In. D.	*U V.* In. Dec.	In. D.	In. Dec.	*P. R.* In. Dec.	*R.* In. Dec.	*S.* In. D.	*R S.* In. D.	*S c.* In. D.	*a b.* In. Dec.	Times.
15.50	3.52	0.40	1.97	13.51	1.41	3.17	1.06	2.11	0.53	.18	92.91
36.00	6.57	0.59	3.57	32.75	1.65	4.69	1.56	3.03	0.78	0.26	173.28
60.00	9.61	0.77	3.17	55.21	2.97	6.06	2.02	4.04	1.01	0.34	253.44
1	2	3	4	5	6	7	8	9	10	11	12

5. *On the Newtonian Telescope.*

As the Newtonian telescope was powerfully recommended to the world by the simplicity of its construction, as well as by the name of its illustrious inventor, it is a matter of surprise that its merits should have been so long overlooked. During the last century, Gregorian telescopes seem to have been universally preferred to those of the Newtonian form, till the late celebrated Sir William Herschel introduced the latter into notice, by the splendour and extent of the discoveries which they enabled him to make. This philosopher, equally distinguished by his virtues and his talents, constructed Newtonian telescopes from 7 to 40 feet[5] in focal length, by which he has greatly enlarged our knowledge of the solar system, and disclosed many new and important facts respecting the structure of the heavens.

Plate VIII. Fig. 7.

In the Newtonian telescope, the large parabolic speculum is not perforated with a hole UV. A small elliptical plane mirror, inclined 45 degrees to the axis of the tube, is placed at GH, about as much nearer the speculum than its focus, as the centre of the small mirror is distant from the tube; that is, the distance Gm of the small speculum from the focus of the great one should be nearly equal to PT, half the diameter of the tube. The rays which form the image IK of the object AB, instead of proceeding to form it at m, are intercepted by the plane speculum at GH, and refracted upwards through an aperture in the side of the tube TT, where the image is formed and magnified by a double convex lens of a short focal distance.

Form of the plane mirror.

As the small plane mirror has an oblique position to the eye, it must be of an elliptical form. In order to find its conjugate or shortest diameter, say as the focal length of the great speculum is to its aperture, so is the distance of the small speculum from the focus of the great one to the conjugate diameter of the small mirror; that is, the conjugate diameter of the small mirror is $=\frac{Cm \times DF}{Pm}$. Its

[5] A description and drawing of this noble instrument may be seen in the *Phil. Trans.* 1795, part 2; or in the *Edinburgh Encyclopædia*, vol. xv, p. 642. The diameter of the speculum is 4 feet, its thickness about $3\frac{1}{2}$ inches, and its greatest magnifying power above *Six Thousand Times.*

transverse or longest diameter will be $= \frac{G\,m \times D\,F}{P\,m} \times 1.4142$; that is, equal to the conjugate diameter multiplied by 1.4142; or, which is the same thing, its transverse will be to its conjugate diameter as 7 to 5, which is nearly the ratio of the diagonal of a square to one of its sides.—If a rectangular prism be substituted in place of the small mirror, having its sides perpendicular to the incident and emergent rays, the image will be erect, and a less quantity of light will be lost, than when the reflection is made from a mirror of the common kind.

Dr. Herschel's improvements.

In most of Dr. Herschel's telescopes, the plane mirror is thrown away, and the focal image *I K* is viewed directly with a small eye-glass, placed at *T E*, the lower side of the tube. When the aperture of the speculum is very large, the loss of light occasioned by the interposition of part of the observer's head is trivial; but when the aperture is small, the speculum must be inclined a little to the incident rays. I have frequently taken a Newtonian speculum, $3\frac{4}{10}$ inches in diameter, and 30 inches in focal length, out of its tube, and viewed the moon in this manner with great satisfaction. The superior performance of Newtonian telescopes, without the plane mirror, can be conceived only by those who have made the experiment.

A new finder for Newtonian telescopes.

As it is more difficult to find any of the heavenly bodies with a Newtonian than with a Gregorian telescope, it has been customary to fix a small astronomical telescope on the tube of the former, so that the axes of the two instruments may be parallel. The aperture of its object-glass is large, and cross hairs are fixed in the focus of the eye-glass. The object is then found by this small telescope, which is called the *Finder;* and if the axis of the instruments are rightly adjusted, it will be seen also in the field of the large telescope. When the Newtonian telescope, however, is large, and placed upon its lower end to view bodies in great altitudes, the finder can be of no use, from the difficulty of getting the eye to the eye-piece. On this account I would propose to bend the tube of the finder to a right angle, and place a plane mirror at the angular point, so as to throw the image to one side, or rather above the upper part of the tube, that the eye-piece of the finder

may be as near as possible to the eye-piece of the telescope. If the latter of these plans be adopted, the angular point, where the plane mirror is fixed, should be placed as near as possible to the focal image, in order that only a small part of the finder may stand above the tube; for in this way the eye can be transferred with the greatest facility from the one eye-piece to the other. The advantages of this construction will be understood from Plate X, Fig. 1, where *T T* is part of a Newtonian telescope, *D* the eye-piece, and *A B C* the finder. The image formed by the object-glass *A* is reflected upwards by the plane mirror *B*, placed at an angle of 45 degrees with the axis of the tube, and the image is viewed by the eye-glass at *C*. Those who have been in the habit of using the Newtonian telescope with the common finder, will be sensible of the convenience resulting from this contrivance.

Reasons for computing a new table for Newtonian telescopes.

The only table, containing the apertures, magnifying power, &c. of Newtonian telescopes, which has hitherto been published, was calculated by Dr. Smith,[6] from the middle aperture and power of Hadley's excellent Newtonian telescope, as a standard, the focal length of the great speculum being 5 feet $2\frac{1}{2}$ inches, its aperture 5 inches, and power 208. A speculum, however, 3 feet and 3 inches in focal length, was wrought by Mr. Hauksbee to so great perfection as to magnify 226 times.[7] It shewed the minute parts of the new moon very distinctly, as well as the belts of Jupiter, and the black list or division of Saturn's ring. For these objects, it bore an aperture of $3\frac{1}{2}$ or 4 inches; but in cloudy weather it shewed land objects most distinct when the whole surface of the metal was exposed, which was $4\frac{1}{2}$ inches in diameter. Since the method of grinding specula, and giving them a true parabolic figure, is much better understood at present than it was in the time of Mr. Hauksbee, Newtonian telescopes may be made as perfect as this instrument of his construction. Upon it, as a standard, therefore, we have computed the following new Table, on the supposition that reflecting telescopes of different lengths shew objects equally bright and distinct, when their linear apertures,

[6] *Optics*, vol. i, p. 148. Dr. Smith's table was continued from 17 to 24 feet, by Mr. Edwards.

[7] Smith's *Optics*, vol. ii. *Remarks*, p. 79, col. 2.

and their linear amplifications or magnifying powers, are as the *square square roots*, or *biquadratic roots*, of the cubes of their focal lengths; and consequently, when the focal distances of their eye-glasses are as the square square roots of their lengths.

Explanation of the table. The first column contains the focal length of the great speculum in feet, and the second its linear aperture in inches and 100ths of an inch. The third and fourth columns contain Sir Isaac Newton's numbers, by means of which the apertures of any kind of reflecting telescope may be readily computed.[8] The fifth column exhibits the focal length of the eye-glasses in 1000ths of an inch, and the sixth contains the magnifying power of the instrument.

Table of the Apertures and Magnifying Power of Newtonian Telescopes.

Focal length of the concave speculum.	Aperture of the concave speculum.	Sir Isaac Newton's numbers.		Focal length of the eye-glass..	Magnifying power.
Feet.	Inch. Dec.	Aperture of the speculum.	Focal length of the eye-glass.	Inch. Dec.	Times.
½	1.34	100	100	0.107	56
1	2.23	168	119	0.129	93
2	3.79	283	141	0.152	158
3	5.14	383	157	0.168	214
4	6.36	476	168	0.181	265
5	7.51	562	178	0.192	313
6	8.64	645	186	0.200	360
7	9.67			0.209	403
8	10.44	800	200	0.218	445
9	11.69			0.222	487
10	12.65	946	212	0.228	527
11	13.58			0.233	566
12	14.50	1084	221	0.238	604
13	15.41			0.243	642
14	16.25			0.248	677
15	17.11			0.252	713
16	17.98	1345	238	0.256	749
17	18.82			0.260	784
18	19.63			0.264	818
19	20.45			0.268	852
20	21.24	1591	251	0.271	885
21	22.06			0.274	919
22	22.85			0.277	952
23	23.62			0.280	984
24	24.41	1824	263	0.283	1017

[8] See Gregory's *Optics*, Appendix, p. 229, and the *Philosophical Transactions*, No. 81, p. 4004

The telescope which Sir William Herschel generally used, and with which he made many of his best discoveries, was a Newtonian reflector, with a speculum 7 feet in focal length, having an aperture of $6\frac{1}{4}$ inches, and powers of 227 and 460, though he sometimes employed a power of 6450 for the fixed stars. Sir William Herschel informed me, that he obtained these high powers merely by using small double convex lenses for eye-glasses, and that he had some in his possession less than *one fiftieth of an inch* in focal length.

In one of Sir William Herschel's 7 feet telescopes in the possession of Mr. Sligo, an ingenious gentleman in Edinburgh, an achromatic eye-piece was employed for the smallest magnifying power. The large speculum is well finished, and the the image which it forms remarkably distinct. The contrivances by which the vertical and horizontal movements are effected, are particularly simple and ingenious, and do great credit to their inventor.

6. *Description of a New Reflecting Telescope.*

We believe it will be admitted by every person who has had much experience in the use of reflecting telescopes, that the Newtonian form is decidedly the best. Its construction is so simple, and it is so little liable to derangement from accidental causes, that, for popular use, on a small scale, it is superior to the Gregorian one, while, for instruments of great size, it is the only form that is practicable. But even when we consider it in a scientific point of view, it has the advantage of the Gregorian form. It is more easy to give a perfect figure to an uniform circular piece of metal than to a perforated disc;—the spherical aberration is less, in so far as it is not increased by a second spherical mirror;—and the quantity of light reflected from the oblique small speculum is decidedly greater than when it is reflected at a vertical incidence.

If we could dispense with the use of the small specula in telescopes of moderate length, by inclining the great speculum, and using an oblique, and consequently a distorted reflexion, as proposed first by Le Maire, we should consider the Newtonian telescope as perfect; and on a large scale, or when the instrument exceeds twenty feet, it has undoubtedly this character, as nothing can be more simple than to magnify by a single eye-glass the image formed by a single speculum.

As the *front view* is quite impracticable, and indeed has never been attempted in instruments of a small size, it becomes of great consequence, in a practical point of view, to remove as much as possible the evils which arise from the use of a small speculum. These evils may be thus enumerated.

1. If we suppose the light reflected by the large speculum, or the light incident upon the small one, to amount to 10,000 rays, then the light reflected by a well polished plane speculum will not exceed two thirds of this, or 6666 rays at a vertical incidence, and probably not above 6800 at an incidence of 45°;[9] consequently 3200 rays out of 10,000 are lost by the use of the plane speculum.

2. Besides this great loss of light, we have to encounter all the errors of a second reflexion, arising from imperfections of surface, and also from imperfections of figure, as it is universally admitted that a surface of glass is much superior to a metallic surface; and Newton has himself remarked, "that every irregularity in a reflecting superficies makes the rays stray *five or six times* more out of their due course than the like irregularities in a refracting one."[1]

The construction by which I propose to remedy these disadvantages, is shewn in Plate X, Fig. 2, where $A\,B$ is the speculum reflecting the parallel rays $R\,A$, $R\,B$, to a focus at F. The cone of rays $A\,F\,B$ is intercepted by an *Achromatic Prism* $G\,H$, which refracts them to foci at f, where a distinct image is formed in the anterior focus of the eye-glass E, by which it is magnified. The double prism $G\,H$ being composed of a prism of crown-glass G, and a prism of flint-glass H, united by a cement of a mean refractive power, the loss of light sustained by the pencil in its transmission through the two, will not exceed 600 rays out of the 10,000, as the light transmitted through a lens of glass is, according to Sir William Herschel's experiments, 94850 out of the 10,000 incident rays. Hence the light lost by transmission through the prism is not *one fifth* part of the light lost by reflexion, and the errors of reflexion arising from defects of surface and of figure are

[9] The law of variation is yet undetermined.

[1] The deviation of a ray arising from the imperfections of a reflecting surface is to the deviation arising from the imperfections of a refracting surface as *four* to *one*, when the refraction is made from glass into air, and as *six* to *one* when the refraction is made from air to glass.

also incomparably less. As the refracting angle of the prism G will require to be larger, in order to produce a given deviation $F H f$, when it is opposed by the refraction of the flint-glass prism H, we may place the correcting prism H nearer the focus f, as shewn in Fig. 3, and make it of crown-glass; or it might even be placed at h, beyond the focus, and in contact with the lens E. If this should be found advantageous, the prism and the lens might be formed out of one piece of glass, or a single hemisphere of glass might be used, the eye-hole being placed at such a point of the hemispherical surface, as to obtain a variable prism of the required angle united with a plano-convex lens.

In viewing eclipses of the sun, or the lunar disc, or any other celestial phenomenon, where there is either too much light, or more than is necessary, a telescope may be fitted up to permit more than one person to see through it at the same time, by allowing a portion of the cone of rays to be refracted through another prism to an eye-glass at the opposite side of the tube, or even by having four prisms refracting a fourth part of the cone to each of the four sides of the tube. The same effect might be produced in the common Newtonian telescope, by using a pyramidal small speculum, with the planes meeting at a vertex placed in the centre of the cone of rays, and inclined to one another at angles of 90°, and to the axis of the telescope at angles of 45°. Another person at A might also be made to see the eclipse by means of a parallel plate of glass at $m\ n$, inclined so as to reflect a portion of the cone of rays to A. In this way five persons might see through the telescope at the same instant. This principle, indeed, might be extended indefinitely by placing in the cone of rays any number of plates of parallel glass, so as to reflect to all sides of the tube a great number of images, a portion of the rays transmitted through the *first* glass being reflected from the *second*, and so on to any extent.

7. *On Achromatic Object-Glasses.*

Notwithstanding the claims of foreigners to the invention of the achromatic telescope, we have the most unquestionable evidence that this instrument was invented in 1729, and constructed about the year 1738, by our countryman Chester More Hall, and afterwards in 1758 by Mr. Dollond, who

seems to have been unacquainted with the labours of Mr. Hall.[2] As telescopes of this description are not affected with the prismatic colours, the late Dr. Bevis proposed to distinguish them by the name of *Achromatic*, an appellation which they have hitherto retained, though some have erroneously stated that it was first given them by the astronomer Lalande.

During the 17th century, when every branch of science was cultivated with unwearied assiduity, the attention of philosophers was particularly directed to the improvement of the *refracting* telescope. But as the different refrangibility of the rays of light was then unknown, men of science employed themselves chiefly in trying to remove the spherical aberration, or the error which arises from the spherical figure of the object-glass. For this purpose they ground their object-lenses of a parabolic or hyperbolic figure, or of a spherical form, with the radius of the surface next the object six times greater than that of the surface next the eye, in which case Huygens had shewn that the aberration is less than when the radii of curvature have any other proportion. After all these trials, however, the refracting telescope still retained its former imperfections, and the opticians of those days, completely despairing of bringing it to perfection, turned the whole of their attention to the construction of the *reflecting* telescope.

Imperfections of telescopes.

It was reserved for Sir Isaac Newton to discover the cause of these imperfections, and for Mr. Hall and Mr. Dollond to point out their cure. From Newton's *Theory of Colours*, it plainly appeared that the imperfections of the dioptric telescope arose from the different refrangibility of the rays of light, and that, compared with this, the spherical aberration was extremely trifling. But though Newton, by thus pointing out the cause of the indistinctness of refracting telescopes, contributed indirectly to their improvement, he may certainly be said to have checked the progress of this branch of science, when he stated,[3] "that all refracting substances diverged the prismatic colours in a constant proportion to their mean refractions,—that refraction could not be produced without colour, and, consequently, that no improvement could be expected in

[2] A full account of what was done by Mr. Hall will be found in the *Edinburgh Encyclopædia*, vol. xv, p. 479.

[3] Newton's *Optics*, p. 112.

the refracting telescope." In this conclusion philosophers had acquiesced for above half a century, till Mr Dollond having examined the premises from which it was drawn, obtained a result very different from that of Sir Isaac Newton. He found that substances which had the same refractive power had different dispersive powers, or, in his own words,[4] " that there is a difference in the dispersion of the colours of light, when the mean rays are equally refracted by different mediums;" and thence concluded that the object-glasses of refracting telescopes were capable of being made without being affected by the different refrangibility of the rays of light.

That our readers may understand this illustrious discovery, and the method of its application to the construction of achromatic object-glasses, let *A B C*, Plate X, Fig. 4, be a prism, *O* a beam of white light proceeding in the direction *O N*, but refracted from its rectilineal course by the interposition of the prism, and forming the prismatic spectrum *R M V*. The line *n M* being the direction of the mean refracted light, the angle *N n M* is called the *angle of deviation*, and *R n V* the *angle of dispersion*. In the same medium, the angle of dispersion is always proportional to the angle of deviation, or to the mean angle of refraction, and Newton imagined that this was universally the case in different media, *i. e.* that the angle *N n M* is always proportional to the angle *R n V*, whether the prism be of crown, or flint, or any other kind of glass. Dollond, however, found that these angles were not proportional to each other in different media, but that in some the angle of deviation is larger when the angle of dispersion is smaller, and that in others the angle of deviation is smaller when the angle of dispersion is larger. Thus, if the prism *A B C* be made of crown-glass, the angle of deviation will be *N n M*, and that of dispersion *R n V*; but if a flint-glass prism with a less refracting angle be substituted in its room, the angle of deviation may be *N n M*, while that of dispersion becomes *r n v*.

The application of these principles to the improvement of the refracting telescope will be easily comprehended, if we consider that light is refracted and dispersed by lenses in the same manner as by prisms. Thus, in Fig. 5, let *A B* be a convex lens, *O* a beam of light incident at *n*, emerging at *m*, and proceeding in the direction *m N*; then if we suppose *a b c* to

[4] See the *Philosophical Transactions*, vol. l p. 743.

be a prism, whose sides *a b*, *a c*, are tangents to the surfaces of the lens in the points *n*, *m*, the beam of light *O*, incident at *n*, will emerge at *m*, and proceed in the same direction *m N* as formerly; and if the lens be concave, as *C D*, it will refract and disperse the rays in the same way as a prism *a c d*, placed in a contrary direction with its base *a d* uppermost. Now, if we apply to the prism *a b c* another prism *a c d*, having a similar refracting angle, and the same refractive and dispersive power, or if to the convex lens *A B* we apply the concave one *C D*, having the same curvatnre and the same refractive and dispersive power, then the ray of white light *O*, incident at *n*, will emerge colourless at *p*, and proceed in the direction *p N*, parallel to *O n*, because the change which is produced on the incident ray by the first prism or convex lens, is counteracted by an equal and opposite change produced by the second prism or concave lens. But if the second prism or lens has a different refractive and dispersive power from the first, and if the refracting angle of both is the same, the ray *p N* will be coloured after refraction, because the second prism more than counteracts the effects of the first, and it will be bent to or from the axis of the lenses according as the refracting power of the second prism or lens is greater or less than that of the first.

From these observations, the attentive reader will easily understand the construction of the double achromatic object-glass, in which *A B* is the convex lens of *crown-glass*, and *C D* the concave one of *flint-glass*. As the refractive and dispersive powers of the lens *C D* are greater than those of the lens *A B*, the curvatures of the lenses, or the refracting angles of the corresponding prisms, being equal, the ray *p N* will be bent from the axis of the lenses, and it will be coloured by the excess of the dispersive power of the flint above that of the crown-glass. In this case, therefore, the combined lenses will not have a positive focus. But since, in the same medium, the angle of dispersion increases or diminishes with the angle of deviation, we can diminish the refraction and dispersion of the concave lens by diminishing its concavity, or the refracting angle of the corresponding prism. Now, let the concavity of the lens *C D* be diminished till its dispersion be equal to the dispersion of the lens *A B*, its refraction or power of bending the incident rays will also be diminished; then since the dispersion of the concave lens is equal to the dispersion of the convex one, and its curvature less, the ray *p N* will emerge

perfectly colourless, and it will be bent *towards* the axis of the lenses, as the convergency of the incident ray occasioned by the convex lens is not wholly counteracted by the divergency produced by the concave one. In the same manner, every other ray falling upon the surface of *A B* will be refracted colourless into a positive focus, and an image will be formed perfectly achromatic.

From what has been said concerning the double achromatic object-glass, it will be easy to comprehend how a colourless image is formed by a combination of three lenses, which is now universally adopted for the purpose of diminishing the spherical aberration. In Fig. 6, let *A B*, *C D*, *E F*, be the three lenses which compose the triple object-glass[5] *A B*, and *E F*, being convex, and of crown-glass, and *C D* concave, and made of flint-glass; and let *a b*, *c d*, *e f*, be corresponding prisms, which, if substituted instead of the lenses, would refract and disperse in a similar manner any ray of light which falls upon the points *q*, *m*, *r*, *t*, *g*, *p*, where the sides of the prisms are supposed to touch the surfaces of the lenses. Suppose, also, which is generally the case, that the two convex lenses have equal focal lengths, and that the focal distance of either lens is greater, or their curvature less, than that of the concave one, whose dispersion exceeds that of the lens *A B*; then a ray, *O*, of white light incident at *q*, will, after refraction by the lens *A B*, be separated into its component parts, and proceed in the direction *m r R*, *n s V*; *m r R* being the extreme red ray, and *n s V* the extreme violet. But as these rays are intercepted by the lens *C D*, at the points *r*, *s*, they will undergo another refraction in a contrary direction, and will proceed according to the dotted lines *t r*, *o v*. These rays will diverge after refraction, and be bent *from* the axis of the lenses, since the refraction, as well as the refracting angle of the prism *c d*, or lens *c D*, exceeds the refraction, and refracting angle of the prism *a b*, or lens *A B*; for though the violet ray *q n s* is bent from the red ray *q m r* by the refraction of the lens *A B*, it is again bent towards it by the superior refraction of the concave lens, and they will therefore converge to one another in the direction *t r*, *o v*. In this case, the excess of dispersive power in the

[5] The lenses are placed at a distance from each other in the figure, that the progress of the incident ray may be more easily perceived.

concave lens tends only to delay the mutual convergency of the red and violet rays, or to remove the point where they would meet farther from the lens *e D*. Now it is evident that two rays of different refrangibility falling upon a prism or lens with different angles of incidence, may emerge with the same angle of refraction, or may be united at their emersion from the prism or lens; for in this case their difference of refrangibility counteracts the difference between their angles of incidence. The red and violet rays *o r*, *t v*, therefore, which fall upon the lens *E F* with different angles of incidence, will, after refraction by the third lens, proceed perfectly colourless in the direction *p N*. In the same manner, all the rays which proceed from any object, emerging colourless from the triple object-glass, will unite in one point, and form an image completely achromatic.

Having thus discovered that light could be refracted without colour, the next object of philosophers was to ascertain the curvature which must be given to lenses, in order to produce this effect, and likewise to correct the spherical aberration. This subject has been treated with the greatest ability by several foreign mathematicians, but particularly by Euler,[6] D'Alembert,[7] Clairaut,[8] Boscovich,[9] and Klugel.[1] The writings of these philosophers furnish us with the most complete and accurate information upon this point; and art has in this case received from science all the assistance which she can possibly bestow. It shall be our object at present to reduce the results of their investigations either into tables or into such a form as may be easily comprehended by the practical optician, and thus to furnish the artist with a popular view of this interesting subject.

The only philosophers in our country who have written upon the theory of achromatic object-glasses, are the late learned Dr. Robison of Edinburgh, and Mr. J. F. W. Herschel. From the formulæ contained in the dissertation of the first of these philosophers the following Table is computed.

[6] *Comment. Nov. Acad. Petropol.* tom. xviii, p. 407.

[7] *Mem. de l'Acad. Paris*, 1764, 8vo, p. 139; 1765, 8vo, p. 81; and 1767, 4to, p. 43.

[8] *Mem. de l'Acad. Paris*, 1756, 8vo, p. 612; 1757, 8vo, p. 853; and 1762.

[9] R. J. Boscovichii *Opera Pertinentia ad Opticam et Astronomiam*, Bassani, 1785, tom. i, opusc. 2, p. 169.

[1] *Comment. Reg. Soc. Gotting.* 1795 to 1798, tom. xiii, p. 28.

Table of the Radii of Curvature of the Lenses of a Triple Object-Glass.[2]

Focal length.	Convex lens of crown glass.		Convex lens of flint glass.		Convex lens of crown glass.	
Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
6	4.54	3.03	3.03	6.36	6.36	0.64
9	6.83	4.56	4.56	9.54	9.54	0.92
12	9.25	6.17	6.17	12.75	12.75	1.28
18	13.67	9.12	9.12	19.08	19.08	1.92
24	18.33	12.25	12.25	25.50	25.50	2.56
30	22.71	15.16	15.16	31.79	31.79	3.20
36	27.33	18.25	18.25	38.17	38.17	3.84
42	31.87	21.28	21.28	44 53	44.53	4.48
48	36.42	24.33	24.33	50.92	50.92	5.12
54	40.96	27.36	27.36	57.28	57.28	5.76
60	45.42	30.33	30.33	63.58	63.58	6. 4

The reader will observe, that only three pair of grinding tools are necessary for constructing a telescope according to the preceding table; but the work may be performed by only two grinding tools, if we employ the radii of curvature which are contained in the following Table, computed from the formulæ of Boscovich.

Focal length.	Radii of the four surfaces of the two lenses of crown glass.	Radius of the two surfaces of the concave lens of flint glass.
Inches.	Inches.	Inches.
6	3.84	3.17
9	5.76	4.75
12	7.68	6.34
18	11.52	9.50
24	15.36	12.68
30	19.20	15.84
36	23.04	19.00
42	26.88	23.17
48	30.72	25.36
54	34.66	28.51
60	38.40	31.68

The radii of curvature employed by the London opticians are pretty nearly represented in the following Table, which is calculated from Dr. Robison's measurements.

[2] A telescope 30 inches in focal length, constructed according to this table, bore an aperture of $3\frac{1}{5}$ inches.

Focal length.	Convex lens of crown glass.		Radius of both the surfaces of the concave lens of flint glass.	Convex lens of crown glass.	
Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
6	3.77	4.49	3.47	3.77	4.49
9	5.65	6.74	5.21	5.65	6.74
12	7.54	8.99	6.95	7.54	8.99
18	11.30	13.48	10.42	11.30	13.48
24	15.08	17.98	13.90	15.08	17.98
30	18.34	22.47	17.37	18.34	22.47
36	22.61	26.96	20.84	22.61	26.96
42	26.38	31.45	24.31	26.38	31.45
48	30.16	35.96	27.80	30.16	35.96
54	33.91	40.45	31.27	33.91	40.45
60	37.68	44.94	34.74	37.68	44.94

Two of Dollond's best achromatic telescopes being examined, were found to have their lenses of the following curvatures, reckoning from the surface next the object:—Crown-glass lens 28 inches, and 40. Concave lens 20.9 inches, and 28. Crown-glass lens 28.4, and 28.4. The focal length of the compound object-glass was 46 inches. In the other telescope, whose focal length was 46.3 inches, the curvature of the 1st lens was 28 and 35.5 inches; the 2d lens 21.1 and 25.75; and the 3d 28 and 28. Both these telescopes magnified from 100 to 200 times, according to the powers applied.

The Duc de Chaulnes having in his possession one of Dollond's best telescopes, whose focal length was 3 feet 5 inches 4.25 lines, made a variety of accurate experiments in order to determine the curvature, thickness, and distance of its lenses, and found them to be of the following dimensions.[3] Radius of the 1st surface, or the surface next the object, 25 inches 11.5 lines. Radius of the 2d surface, 32 inches 8 lines. Radius of the 3d surface, 17 inches 10 lines. Radius of the 4th surface, $24\frac{1}{2}$ inches. Radius of the 5th, $24\frac{1}{2}$ inches; and the radius of the 6th, 26 inches and 10.6 lines. Thickness of the first lens at its axis, 2.11 lines; thickness of the second, 1.59 lines; thickness of the 3d, 2.18; and the thickness of the whole lens 5.91 lines.[4]

A very excellent telescope, with a double achromatic object-

[3] These experiments are detailed at great length in the *Mem. de l'Acad. Paris*, 1767, 4to, p. 423.

[4] For the dimensions of the eye-piece of this telescope, see the article on *Achromatic Eye-pieces*, p. 323.

glass, was constructed by M. Antheaulme in 1763, from the formulæ of Clairaut. The lens of flint-glass was placed next the object, and was a meniscus with its convex side outwards. The radius of its concavity was $17\frac{1}{4}$ inches, and the radius of its convex side was 7 feet $6\frac{2}{3}$ inches. The interior surface of the lens of crown glass had a radius of 18 inches, while its exterior surface, or that next the eye, was 7 feet 6 inches. These lenses were separated by a piece of card, and formed a compound object-glass, with a focal length of 7 feet, and an aperture of 3 inches and 4 lines. Its eye-piece consisted of two lenses. That next the object was a double convex lens with 18 lines of focal length, and 9 lines of aperture. Its first surface, or that next the object-glass, had a radius of $11\frac{1}{2}$ lines, and its second surface a radius of 7 inches 2 lines. The second eye-glass, which was a meniscus, had 5 lines of focus and 2 lines of aperture. The radius of its convex surface was $2\frac{1}{4}$ lines, and that of its concave surface, which was next the eye, was 8 lines. The distance between the two eye-glasses was 9 lines.

Tables for Double Achromatic Object-Glasses.

The following Table, calculated from the formulæ of Boscovich, contains the radii of curvature for the lenses of a *double achromatic object-glass.*

Focal length.	Convex lens of crown glass.		Concave lens of flint glass.	
Inches.	Inches.	Inches.	Inches.	Inches.
6	1.94	1.91	1.91	9.49
9	2.91	2.86	2.86	14.24
12	3.88	3.82	3.82	18.99
18	5.82	5.73	5.73	28.48
24	7.76	7.63	7.63	36.99
30	9.70	9.54	9.54	47.47
36	11.64	11.45	11.45	56.97
42	13.58	13.36	13.36	66.46
40	15.51	15.27	15.27	73.98
54	17.45	17.17	17.17	85.47
60	19.39	19.08	19.08	94.95

In the following Table, calculated from Dr. Robison's measurements, the reader will find the radii of curvature which are employed by the London artists in the construction of the double achromatic object-glass. The three first surfaces may, without any perceptible error, be ground on the same tool.

Focal length.	Convex lens of crown glass.		Concave lens of flint glass.	
Inches.	Inches.	Inches.	Inches.	Inches.
6	1.76	2.12	2.07	6.88
9	2.64	3.17	3.10	10.33
12	3.53	4.23	4.13	13.77
18	5.29	6.35	6.20	20.65
24	7.05	8.46	8.26	27.54
30	8.81	10.58	10.33	34.42
36	10.58	12.69	12.39	41.30
42	12.34	14.81	14.46	48.18
48	14.11	16.92	16.52	55.07
54	15.87	19.04	18.59	61.96
60	17.63	21.16	20.66	68.84 [5]

Though it is demonstrable that a telescope constructed according to the preceding tables, and formed of glass, whose refractive and dispersive power is similar to that which was employed in the formulæ upon which these tables are founded, will form an image perfectly distinct and colourless; yet it is so difficult to procure flint-glass of the same refractive and dispersive power, that it is almost impossible for a private individual to succeed, even after several trials. The London opticians have always at hand a number of lenses of various curvatures, and different powers of refraction and dispersion, and by selecting such as answer best upon trial, they are enabled, without much trouble, to construct an object-glass in which the spherical and chromatic aberrations are almost wholly corrected. Those, therefore, who are not furnished with a sufficient number of lenses, must necessarily meet with frequent disappointments in their attempts to construct achromatic telescopes; and the only way of preventing these disappointments, and rendering success more certain, is to have a variety of tables, which being founded on different conditions, give different curvatures to the lenses. If the artist should be unsuccessful, either from the nature of the refracting media which he employs, or from giving the lenses a greater or lesser curvature than the table requires, he may, with very little trouble, sometimes by altering the radius of

[5] In this and the five preceding tables, the sine of incidence is supposed to be to the sine of refraction as 1.526 to 1 in the crown-glass, and as 1.604 to 1 in the flint-glass; and the ratio of the differences of the sines of the extreme rays in the crown and flint-glass 0.6054.

a single surface, adapt the lenses to the conditions of some other table, and in all probability obtain a more favourable result. With the view of facilitating these attempts, we have computed the preceding tables, and for the same purpose we shall subjoin the following different forms of achromatic object-glasses from Boscovich and Klugel.

In these forms, a represents the first surface of the compound lens, or that which is next the object, b the second surface, a' the third, b' the fourth, a'' the fifth, and b'' the sixth; a, b, a'', b'', representing the radii of curvature for the convex lenses of crown-glass, and a', b', the curvature of the concave lens of flint-glass. The focal distance of the first lens, or that whose surfaces are marked a, b, is represented by x, that of the second by y, and that of the third by z, while the focal length of the compound lens is distinguished by the letter F.

FORMS FOR TRIPLE OBJECT-GLASSES.

I.

$$a = b = a'' = b'' = 0.6412 \qquad x = 0.6096$$
$$a' = 0.5227 \qquad y = 0.4384$$
$$b' = 0.5367 \qquad z = 0.6096$$
$$F = 1$$

In this form the two lenses of crown-glass are isosceles,[5] and have the same curvature and focal distance. The middle lens of flint-glass is nearly isosceles, and may be made so in practice, so that only two grinding tools are necessary for this form.

II. *The two first Lenses Isosceles.*

$$a = b = a' = b' = 0.530 \qquad x = 0.5038$$
$$a'' = 1.215 \qquad y = 0.4388$$
$$b'' = 0.3046 \qquad z = 0.7727$$
$$F = 1$$

III. *The first and third Lenses Isosceles.*

$$a = b = a'' = b'' = 0.616$$
$$a' = 0.6356 \qquad F = 1$$
$$b' = 0.3790$$[6]

[5] A lens is called *isosceles* when both its surfaces have the same curvature.

[6] In these two forms the refractive and dispersive power of the glass is supposed the same as in the note on the preceding page.

IV. *The two first Lenses Isosceles.*

$$a = b = a' = b' = 0.4748$$
$$a'' = 0.3514 \qquad F = 1$$
$$b'' = 0.4383$$

V. *The second and third Lenses Isosceles.*

$$a = 0.5721$$
$$b = 1.8744$$
$$a' = b' = a'' = b'' = 0.4748$$

VI. *All the three Lenses Isosceles.*

$$a = b = 0.7963$$
$$a' = b' = 0.4748$$
$$a'' = b'' = 0.5023$$

VII. *Second Lens Isosceles, the first and third of equal Focal Length, but placed in an inverse order.*

$$a = b'' = 0.7306$$
$$a' = b' = 0.4748$$
$$a'' = b = 0.5325$$

VIII. *Second Lens Isosceles, the first and third of equal Focal Length, and placed in a direct order.*

$$a = a'' = 0.7048$$
$$b = b'' = 0.5471$$
$$a' = b' = 0.4748$$[7]

FORMS FOR DOUBLE OBJECT-GLASSES.

I. *First Lens Isosceles.*

$$a = b = 0.3206 \qquad x = 0.3408$$
$$a' = 0.3201 \qquad y = 0.4384$$
$$b' = 1.533 \qquad F = 1.$$

II.

$a = 6943$	Distance between the lenses = 100
$b = 22712$	Aperture = 3000
$a' = 14750$	Thickness of the convex lens = 250
$b' = 18383$	Thickness of the concave lens = 100
$x = 10000$	
$y = 14080$	
$F = 32024$	

[7] In the preceding forms, which are calculated from the formulæ of Boscovich, the sine of incidence is to the sine of refraction as 1.527 to 1 in the crown-glass, and as 1.575 to 1 in the flint-glass, and the ratio of the differences of the sines of the extreme rays 0.6486.

III.[8]

$a = 2138$	Distance between the lenses	$=$	31
$b = 7092$	Aperture	$=$	937
$a' = 4606$	Thickness of the convex lens	$=$	79
$b' = 5740$	Thickness of the concave lens	$=$	31
$x = 3123$			
$y = 4397$			
$F = 10000$			

In making use of the preceding forms, we have only to fix upon the focal length which we intend to give to the object-glass, and multiply the different numbers by this focal length, expressed in feet or inches, and the result will be the proper radii of curvature in feet or inches. Thus, let it be required to construct a double achromatic object-glass according to the first of these forms, whose focal length shall be 20 inches, we shall have

$$a = b = 20 \times 0.3206 = 6 \text{ inches and } \tfrac{4}{10} \text{ nearly};$$
$$a' = 20 \times 0.3201 = 6 \text{ inches and } \tfrac{4}{10} \text{ nearly};$$
$$b' = 20 \times 1.553 = 30 \text{ inches and } \tfrac{7}{10} \text{ nearly};$$
$$x = 20 \times 0.3408 = 6 \text{ inches and } \tfrac{8}{10} \text{ nearly};$$
$$y = 20 \times 0.4384 = 8 \text{ inches and } \tfrac{8}{10} \text{ nearly};$$
$$\text{and } F = 20 \times 1 = 20 \text{ inches.}$$

Mr. Herschell on achromatic object-glasses.

The subject of double achromatic object-glasses has been recently treated with great ability by Mr. J. F. W. Herschel. In order to render the problem determinate, Mr. Herschel has introduced the condition of ensuring the destruction of the spherical aberration not only for parallel rays, but also for those which diverge from objects placed at any moderate finite distance, so as to produce a telescope equally perfect for terrestrial and astronomical purposes. This condition is attended with other advantages. The curvatures are all more moderate than in any other construction hitherto proposed. The two interior surfaces approach so near to coincidence, that no sensible error can occur from neglecting their difference, and grinding them on tools of equal radii; and by any variation in the refractive and dispersive power of the glass, the curvatures vary within extremely narrow limits.

[8] This and the preceding form are calculated from Klugel, and suited to glass, with nearly the same refractive and dispersive powers as that mentioned in the preceding note.

This very interesting result gives a simplicity to the construction of double achromatic object-glasses hitherto unknown, and has enabled Mr. Herschel to lay down the following easy practical rule, applicable in all ordinary cases:—"*A double object-glass will be nearly free from aberration, provided the radius of the exterior surface of the crown lens be* 6.72, *and of the flint* 14.2, *the focal length of the combination being* 10.00, *and the radii of the interior surfaces being computed from these data, by the formulæ given in all elementary works on optics, so as to make the focal lengths of the two glasses in the direct ratio of their dispersive powers.*"

In this construction, the anterior glass, or that which first receives the incident rays, is crown, and is double convex, of unequal convexities; the flatter surface being placed outwards, while the posterior lens, formed of flint-glass, is concavo-convex, having its concave surface applied against the posterior or most convex surface of the crown lens. This combination is represented in Plate X, Fig. 7, where the four surfaces are numbered in the order in which the light traverses them, *O* being the object, and *F* the image formed in the focus.

The rule here stated is given only as approximative, though it is sufficiently exact for ordinary use; but when object-glasses *of great size and value* are to be constructed, their radii must be computed more strictly; and for this purpose Mr. Herschel has calculated a table from the rigorous formulæ, which will be found, with the fullest explanations, in the *Edinburgh Philosophical Journal*, vol. vi, p. 367, or in the *Edinburgh Encyclopædia*, Art. *Optics*, vol. xv, p. 647.

Achromatic object-glasses may sometimes be improved by interposing pure turpentine varnish, or mastich, between the concave and convex lenses. By this means the reflection from the internal surfaces is removed, and that loss of light prevented which arises from an imperfect polish of the surfaces. The adjustment of the axes of the lenses both of double and triple object-glasses is a point of the first importance. Dr. Wollaston has pointed out a very ingenious method of doing this by examining the series of images reflected from the different surfaces of the lenses. An account of this method will be found in the *Phil. Trans.* 1822, p. 32, and in the *Edinburgh Encyclopædia*, Art. *Optics*, vol. xv, p. 653.

Achromatic telescopes have also been constructed by using

only transparent fluids, or substituting them instead of the concave lens of flint-glass. For this discovery we are indebted to the ingenious Dr. Robert Blair, who has given an account of his experiments in the 3d volume of the Transactions of the Royal Society of Edinburgh, to which we must refer the reader, after giving a description of one of these fluid object-glasses.

If pure spirit of turpentine be interposed between two convex lenses of crown glass, having the radii of their surfaces as 6 to 1, with the most convex sides turned inwards, the image formed by this combination will be perfectly achromatic. The spirit of turpentine has the form of a double concave lens, and as its refractive and dispersive powers differ from those of crown-glass, it will act in every respect like a lens of flint-glass. A few years ago I constructed an object-glass of this kind, having 36 inches of focal length, but found it troublesome to keep it in order.—See also the *Edinburgh Encyclopædia*, Art. *Optics*, vol. xv, p. 648.

8. *On Achromatic Eye-Pieces.*

Although a brief account of the achromatic telescope has been given by those who have written upon optics since the invention of that instrument, yet these authors have unaccountably overlooked the construction of achromatic eye-pieces. Dr. Robison, indeed, has treated this subject at considerable length, after Boscovich, but has furnished almost no information to the practical optician. On thisaccount, with the Italian philosopher as our guide, we shall dwell a little longer upon this point than might otherwise be thought necessary in a work like this. In order to correct the error arising from the unequal refrangibility of light in the eye-pieces of telescopes, we are not under the necessity of using compound lenses of crown and flint glass, as this species of aberration can be completely removed by a particular arrangement of the eye-glasses which are employed for erecting the object.

Method of correcting the chromatic aberration by single lenses.

Plate X. Fig. 8.

This will appear from Fig. 8 of Plate X, where $A B$ is an achromatic object-glass, and $D E$ an eye-piece of the kind mentioned in p. 328. Let $C D E$ be the axis of the telescope, and $S T$ a ray passing through the object-glass $A B$. As the object-glass is achromatic, this ray will fall upon the

eye-glass D, without being decomposed into the prismatic colours, through whatever part of the lens it is transmitted. The eye-glass D, however, will separate the ray $S\,T$ into its component colours, and the red part of the ray will be bent into the direction $T\,R$, and the violet part into the direction $T\,V$. But when the second lens is interposed, it will intercept the red ray at the point m, and the violet ray at the point n of its anterior surface. Now, as the red ray $T\,m$ enters the lens E at a point m, farther from the axis than the violet ray, and as the refracting angle of the lens is greater at m than at n, this increase of the refracting angle for the red ray will make up for its inferior refrangibility, and the rays $T\,m$, $T\,n$, will emerge parallel from the lens E in the direction $m\,r$, $n\,v$. The chromatic aberration, therefore, which is always proportional to the angle formed by the resulting rays $m\,r$, $n\,v$, will be destroyed. In small pocket telescopes, as opera glasses, &c. where it would be very inconvenient to apply a long eye-piece, compound lenses of crown and flint glass should be adopted, and may consist either of two or three glasses, with the following curvatures.

FORMS FOR A DOUBLE EYE-GLASS.

I. *Both Lenses Isosceles.*[9]

$$a = b = 0.320 \qquad x = 0.304$$
$$a' = b' = 0.529 \qquad y = 0.438$$

II. *First Lens Isosceles.*

$$a = a' = b = 0.320 \qquad x = 0.304$$
$$b' = 1.517 \qquad y = 0.438$$

FORMS FOR A TRIPLE EYE-GLASS.

I. *All the three Lenses Isosceles.*

$$a = b = a'' = b'' = 0.640 \qquad z = x = 0.608$$
$$a' = b' = 0.529 \qquad y = 0.438$$

II. *First Lens Isosceles.*

$$a = b'' = 0.810 \qquad z = x = 0.608$$
$$b = a' = b' = a'' = 0.529 \qquad y = 0.438$$

[9] The letters a, b, x, y, &c. represent the same quantities as in page 319.

If it is required to erect the object as in the Galilean telescope, the middle lens of flint-glass must be made convex, and the other lenses concave, but with the same radii of curvature, so that the concavity of the compound lens may predominate.

On Eye-Pieces with three Lenses, which remove the Chromatic Aberration.

The three lenses must be made of the same kind of glass, and may be of any focal length. The distance between the first and second, or the two next the object, must be equal to the sum of their focal distances, and the distance between the second and third must exceed the sum of their focal distances, by a quantity which is a third proportional to the distance between the first and second, and the focal length of the second lens; or, in other words, the distance between the second and third lenses must be equal to the sum of their focal distances, added to the quotient arising from the square of the focal distance of the second lens, divided by the sum of the focal distances of the first and second. These, and other circumstances which should be attended to in the construction of achromatic eye-pieces, will be better understood by expressing them algebraically.

Formulæ for achromatic eye-pieces.

Thus, let F be the focal length of the object-glass, and $x\ y\ z$ the focal distances of these eye-glasses, reckoning from that which is nearest the object. Then we shall have

1, The distance between the first and second lenses, $x + y$

2, The distance between the second and third, $y + z + \dfrac{y^2}{x+y}$

3, Distance of the first lens from the focus of the object-glass, - - - $\dfrac{xy}{x+y}$

4, Magnifying power of the eye-piece, - $\dfrac{Fy}{xz}$

5, Focal distance of a single lens, with the same magnifying power, - - $\dfrac{xz}{y}$

6, Distance of the eye from the third lens, - z

7, Length of the whole eye-piece, - $x + 3y + 2z$

8, Length of the whole telescope, $F + x + 3y + 2z$

9, Aperture of the lenses[1] a, a', a'' .... $a' = a''$, $a = \frac{x\,a}{y}$

10, The aperture of the diaphragm, or field bar, or m, should be a little less than - - a

And should be placed in the focus of the object-glass.

11, The field of view is nearly - - $\frac{3438\,m}{F}$

Although the aberration of colour will be completely removed by making the lenses of any focal length, and placing them at the distances indicated by the preceding formulæ, yet it is preferable to make the first and second lenses of the same focal length, and to give the third a less focal distance, and make its distance from the second equal to its own focal length, added to $1\frac{1}{2}$ the focal distance of one of the other lenses; for, in this case, where x and y are equal, the expression $\frac{y^2}{x+y}$ which, when added to $y + z$, expresses the distance between the second and third lenses, becomes $\frac{1}{2}\,y$.[2] Beside the simplicity of this combination, it has another advantage, for the magnifying power of the eye-piece is always equal to the magnifying power of the third lens. This is evident from the fifth formula $\frac{xz}{y}$, which becomes $= z$ when x and y have the same value. So that in this construction, when we wish to give a certain magnifying power to a telescope, we have only to take such a focal length for the third lens as will produce this magnifying power, and make the focal length of the other two a little greater than that of the third. By increasing the focal lengths of the two first lenses, the image is not injured by any particles of dust which may be lying on their surface, and the spherical aberration is also diminished. By augmenting the curvature of the third lens, however, we contract the field of view, which ought, if possible, to be avoided. This may be avoided, indeed, as Boscovich has shewn, by making the third lens consist of two convex ones of the same glass, their surfaces being in contact, and their focal lengths equal. From long experience, he found that eye-pieces of this construction are superior to all others,

[1] The apertures of the lenses may be made equal to one another, but should never be greater than half the focal distance of the third lens.

[2] Since $x = y$ in this case, $\frac{y^2}{x+y}$ is $= \frac{y^2}{2y} = \frac{y}{2}$ or $\frac{1}{2}y$ for $\frac{1}{2}y \times 2y = y^2$.

and that the error arising from the spherical figure of the glass is greatly diminished by making all the lenses plano-convex, and turning the plane sides to the eye, excepting the second lens, whose plane surface should be turned to the object. All the lenses may be made of the same focal length, and then the distance between the first and second, and the second and third, will be equal to the sum of their focal distances. In this case the third and fourth lenses, which are joined together, are considered as a single lens, whose focal length is equal to one half the focal length of either of the two. The apertures too may be all equal, and the field bar must be a little less than any of the apertures.

In all kinds of achromatic eye-pieces which are composed of single lenses, flint-glass should be employed, because it has the greatest refractive power, and therefore requires a less curvature to have the same focal distance. The spherical aberration, consequently, which always increases with the curvature of the lenses, will be less in a flint-glass eye-piece than in one of crown-glass. Flint-glass, indeed, produces a greater separation of colours, but the error arising from this cause is completely removed by the proper arrangement of the lenses.

On Eye-Pieces with Four Lenses, which remove the Chromatic Aberration.

A good achromatic eye-piece may be made of four lenses, if their focal lengths, reckoning from that next the object, be as the numbers 14, 21, 27, 32, their distances 23, 44, 40, their apertures 5.6, 3.4, 13.5, 2.6, and the aperture of the diaphragm placed in the anterior focus of the 4th eye-glass 7. The focal length of the equivalent single eye-glass is $15\frac{2}{5}$.

In one of Ramsden's small telescopes, whose object-glass was $8\frac{1}{2}$ inches in focal length, and its magnifying power 15.4, the focal lengths of the eye-glasses were 0.77 of an inch, 1.025, 1.01, 0.79, and their respective distances, reckoning from that next the object, were 1.18, 1.83, 1.10.

In the excellent telescope of Dollond's construction, which belonged to the Duc de Chaulnes, the focal lengths of the eye-glasses, beginning with that next the object, were $14\frac{1}{4}$ lines, 19, $22\frac{3}{4}$, 14; their distances 22.48 lines, 46.17, 21.45; and their thickness at the centre 1.23 lines, 1.25, 1.47. The fourth lens was plano-convex, with the plane side to the eye, and the rest were double convex lenses.

On Achromatic Eye-Pieces for Astronomical Telescopes.

In eye-pieces of this kind, which invert the object, the focal length of the first lens should be triple that of the second, and their distance double the focal length of the second, or $\frac{2}{3}$ of the focal length of the first. The lenses should be plano-convex, the plane surfaces turned to the eye, in order that the aberration of sphericity may be diminished as much as possible.

The telescope of Dollond's, belonging to the Duc de Chaulnes, had two astronomical eye-pieces, one of which was furnished with a micrometer. In the eye-piece which carried the micrometer, the first lens was $12\frac{3}{4}$ lines in focal length, and 1.62 lines thick; the second was 5.45 lines in focal length, and 1.25 thick; the distance between their interior surfaces 4.20 lines, and the distance of the first lens from the focus of the object-glass $13\frac{3}{4}$ lines. In the other eye-piece, the focal length of the first lens was 8.30 lines, and its thickness 1.60; and the focal length of the second was 3.53, and its thickness 0.97 lines. In both these eye-pieces the lenses were plano-convex, with the plane surfaces turned to the eye.

On Single Eye-Pieces.

Single eye-pieces consist of single lenses, either convex or concave; and their form should be either plano-convex or plano-concave, or exactly such as will give a minimum error of sphericity.

The great importance of doing this will appear from the following results obtained by Mr. Herschel. The minimum aberration produced by an eye lens of the best form being 1, that of the other lenses will be,

	Aberration.
Plano-convex, plane side first, - -	4.2
Plano-concave, plane side first, -	4.2
Do. convex, or concave surface first, -	1.081
Double convex or double concave, -	1.567
Best form, - - - -	1.000

As it is desirable, however, to reduce the spherical aberration below the amount of its minimum in single lenses, we must have recourse to two lenses placed together. Mr. Herschel has shewn that the minimum aberration which can be produced by two convex lenses placed together will be 0.2481, or one-fourth of that of a single lens in its best form; and that

this diminution will be effected by two plano-convex lenses, as in Plate X, Fig. 9, the focal length of the first being to that of the second as 1 to 2.3. If the lenses are of equal foci, the aberration will still be so low as 0.6028, or reduced nearly one-half.

In order, however, to destroy the spherical aberration altogether, by two lenses put together, Mr. Herschel has shewn that they must have the form represented in Figs. 10 and 11, the convex sides being turned to the eye when used as lenses, and to the parallel rays when used as burning-glasses. The following are the focal lengths and radii of these lenses:—

	Fig. 10.	Fig. 11.
Focal length of the first lens,	+10.000	+10.000
Radius of its first surface, -	+ 5.833	+ 5.833
Radius of its second surface,	—35.000	—35.000
Focal length of the second lens,	+17.829	+ 5.497
Radius of its first surface, -	+ 3.688	+ 2.054
Radius of its second surface,	+ 6.291	+ 8.128
Focal length of the combination,	+ 6.407	+ 3.474

Mr. Herschel has remarked, that in a burning-glass the destruction of the aberration of rays parallel to the axis is of the greatest importance. In order to try this, he had two lenses ground to the radii in the first column of the above table. They were about three inches in aperture, and when combined as directed above, the aberration was almost totally destroyed. Their combined effect as a burning-glass was decidedly superior to the first lens used alone.

In eye-glasses and magnifiers, when we wish to examine a *minute object*, Mr. Herschel is of opinion, that perfect distinctness in the central point is of more consequence than extent of field, and that too much pains cannot be taken to destroy the central aberration.

In those cases, on the contrary, where periscopic effects are required, such as in spectacles, reading-glasses, magnifiers of moderate power, and eye-glasses for certain astronomical purposes, the correction of the central aberration may be sacrificed without inconvenience. The best periscopic combination which Mr. Herschel found was a double convex lens of the best form, but placed in its worst position for the lens next the eye; and a plano-concave, whose focal length is to that of the other as 2.6 to 1, or as 13 to 5, placed in contact with its flatter surface, and having its concavity towards the object, as in Fig. 12.

The aberration of such a lens for rays parallel to the axis is no less than twenty-two times that of a single lens in its best position; yet upon forming such a combination accidentally, Mr. Herschel was surprised at the remarkable extent of oblique vision—the absence of fatigue in reading with it—and the great destruction of colour arising from the opposition of the prismatic refractions of the two lenses. The focal length of the compound lens was 1.84 inch. The field of tolerably distinct vision was 40° from the axis, and the letters of a book might be read with management as far as the 75th degree. In making such combinations, the lenses should be very thin, and the eye applied as close as possible.

9. *On the Camera Obscura.*

The Camera Obscura, which is one of the simplest and most amusing of our optical instruments, has already been described in the first volume, p. 184. The improvements which have been made upon it since its first invention, regard chiefly its external form, and no attempts have been made to increase the brilliancy and perfection of the image. When we compare the picture of external objects, which is formed in a dark chamber by the object-glass of a common refracting telescope, with that which is formed by an achromatic object-glass, we shall find the difference between their distinctness much less than we would have at first expected. Although the achromatic lens forms an image of the minutest parts of the landscape, yet when this image is received on paper, these minute parts are obliterated by the small hairs and asperities on its surface, and the effect of the picture is very much impaired.

In the construction of the camera obscura, the splendour of the picture depends on various circumstances which have not hitherto been sufficiently attended to. It has been found from experience, that a common lens is preferable to the best achromatic object-glass; and Dr. Wollaston has suggested the application of the periscopic principle to the lens of the camera. M. Cauchoix has found, that the most advantageous ratio of the radii of curvature is that of 5 to 8, the shortest of the two being that which is turned towards the image.

Next to the optical perfection of the image, is the nature of the surface upon which it is received. When paper is employed, it should be made as smooth as possible by smoothing its surface; and when a stucco surface is used, the greatest care

must be taken to remove all asperities, and make it perfectly uniform. With the view of obviating the imperfection of the ordinary opaque grounds, I made a great variety of experiments. Though I did not find any white surfaces superior to those in common use, I was surprised to observe the brilliancy and distinctness with which the pictures were represented when received upon the silvered back of a looking-glass; and I succeeded in giving additional perfection to the images, by removing the protuberances and roughness of the tinfoil by carefully grinding the surface with a soft kind of hone. Notwithstanding the bluish colour of the metallic ground, white objects are represented in their true tints, and so brilliant is the colouring, and so rich the verdure of the foliage, that the image seems to surpass in beauty even the object itself. I have also made various experiments to discover a good *transparent surface* as a substitute for ground glass, which would permit the application of an eye-glass. The loss of light, when the image is received on ground glass, is enormous. In order to prevent this, and give brilliancy to the colouring, I placed another plate of glass above the ground one, and introduced between them water, or any other fluid of a different refractive power from the ground glass;—the dispersion of the light at the separating surface of the ground glass and the fluid being sufficient to detain the convergent pencils without greatly weakening their intensity. As such a ground, however, cannot be used for copying, I put a slightly opaque varnish upon the surface of a glass plate, and also upon thin squares of mica, and found that these varnishes might be marked with the finest lines of a pencil, and that an impression of the sketch might be conveyed by the slightest pressure of the hand to a piece of paper. One of the simplest and best of all the varnishes which I used was that of milk dried upon the glass, after it had been freed entirely of its butyraceous particles. The beauty and distinctness is such, that it will even admit the application of a lens to magnify the image, and when properly made, will receive the mark of a black lead pencil.

10. *On the Phantasmagoria.*

The term *Phantasmagoria*, or the raising of spectres, has been given to the exhibition of an optical apparatus similar to the magic lantern, which was some years ago publicly shewn

in this country. No description, so far as we know, of the apparatus actually used, has ever been published; but a very excellent arrangement for a phantasmagoria has been proposed by Dr. Young, which we shall describe in his own words. "The light of the lamp *A* (Plate X, Fig. 13), is thrown by the mirror *B*, and the lenses *C* and *D*, on the painted slider at *E*, and the magnifier *F* forms the image on the screen at *G*. This lens is fixed to a slider, which may be drawn out of the general support or box *H*, and when the box is drawn back on its wheels, the rod *I K* lowers the point *K*, and by means of the rod *K L* adjusts the slider in such a manner, that the image is always distinctly painted on the screen *G*. When the box advances towards the screen, in order that the images may be diminished and appear to vanish, the support of the lens *F* suffers the screen *M* to fall, and intercept a part of the light. The rod *K N* must be equal to *I K*, and the point *I* must be twice the focal length of the lens *C* before the object, *L* being immediately under the focus of the lens; the screen *M* may have a triangular opening, so as to uncover the middle of the lens only, or the light may be intercepted in any other manner.

"In order to favour the deception, the sliders are made perfectly opaque, except where the figures are introduced, the glass being covered in the light parts with a more or less transparent tint, according to the effect required. Several pieces of glass may also be occasionally placed behind each other, and may be made capable of such motions as will nearly imitate the natural motions of the objects which they represent. The figures may also be drawn with water colours on thin paper, and afterwards varnished. By removing the lantern to different distances, and altering at the same time, more or less, the position of the lens, the image may be made to increase or diminish, and to become more or less distinct at pleasure, so that, to a person unaccustomed to the effects of optical instruments, the figures may appear actually to advance and retire. In reality, however, these figures become much brighter as they are rendered smaller, while in nature the imperfect transparency of the air causes them to appear fainter when they are remote than when they are near. This imperfection might be easily remedied by the interposition of some semi-opaque substance, which might gradually be caused to admit more light as the figure became larger, or by uncovering a larger or a

smaller portion of the lamp, or of its lens. Sometimes, by throwing a strong light upon an actual opaque object, or on a living person, its image is formed on the curtain, retaining its natural motions; but in this case the object must be considerably distant, otherwise the images of its nearer and remoter parts will never be sufficiently distinct at once, the refraction being either too great for the remoter, or too small for the nearer parts; and there must also be a second lens placed at a sufficient distance from the first to allow an inverted image to be formed between them, and to throw a second picture of this image on the screen in its natural erect position, unless the object be of such a nature that it can be inverted without inconvenience. This effect was very well exhibited at Paris by Robertson: He also combined with his pictures the shadows of living objects, which imitate tolerably well the appearance of such objects in a dark night, or by moonshine; and while the room was in complete darkness, concealed screens were probably let down in various parts of it, on which some of the images were projected; for they were sometimes actually situated over the heads of the audience." See Dr. Thomas Young's *Lectures on Nat. Phil.* vol. i, p. 426, 785.

11. *Description of the Dichroscope.*

The name *Dichroscope* may be appropriately given to those instruments which exhibit the complementary colours of polarised light, from δυο, *two*, χροα, *colour*, and σκοπεω, to *see*.

The dichroscope, in its original state, is nothing more than a portable apparatus for exhibiting the phenomena of the colours of chrystallized plates. The apparatus, as brought from Paris in 1814, consisted of a piece of black glass, or black japan, *M N* (Plate X, Fig. 14), which was carried in the pocket, and of a separate tube *A B E*, containing a plate *A B*, of rock crystal, cut perpendicular to the axis, and one of Rochon's doubly refracting prisms, *c d*, made of rock crystal, so as to give only two images polarised in opposite planes. When the tube *A B E* was directed to the black plate *M N*, at an angle of about 57°, as nearly as could be guessed, the complementary colours of the crystallized plate *A B* were seen, which varied upon turning round the tube. If the aperture *A B* was small and round, the two images of it coloured with the opposite complementary colours were completely separated; but if the

aperture was large, and either round, or in the form of a star, or any other shape, the two images overlapped each other, and the overlapping parts were always white, in consequence of the mixture of the two complementary tints.

The form in which I constructed the dichroscope is shown in Plate X, Fig. 15, where the plate *M N*, composed of six or more of the thinnest and most transparent plates of glass, is made to move round a hinge at the end *A C* of the tube *A B D C*, so as to reflect or transmit a strong beam of polarised light, *R S*, along the axis of the tube ; *n* is a convex eye-glass placed next the eye ; *o* an aperture of a circular or any any other form, in the focus of the lens *o* ; *m* a flat piece of topaz or rock crystal,[3] not much larger in diameter than the pupil of the eye, and cut in the proper direction from the crystal ; and *B D* a prism of nearly the same diameter, formed out of rock crystal in the manner long ago described by M. Rochon,[4] so as to produce the greatest separation of the images, or what is still simpler, a prism of calcareous spar having the refraction and dispersion as much as possible corrected by an opposite prism of glass, or of balsam of Tolu, or indurated Canada balsam. When the instrument is thus fitted up, the rays *R S*, polarised by reflexion from the plates *M N*, or by transmission through them, are arranged into their complementary colours by the crystallized plate *m*, and are afterwards separated into two distinct pencils by the double refraction of the prism *B D*. An eye, therefore, placed at *n*, will see two distinct images of the aperture *o*, and the colour of the one image will be complementary to that of the other. These images will exhibit alternate variations of tint by turning round either the tube or the polarizing plates *M N*. If the two images overlap, the parts that overlap will be white, in consequence of the combination of the two opposite colours. The object of using the lens *n* is to shorten the tube, but if we remove the eye-glass, the aperture *o* may be made of any size, and placed at any distance from the eye.

In Fig. 16 is represented another dichroscope which I have

[3] A thin film of sulphate of lime is much better than any other mineral, as it requires no trouble to prepare it. Topaz is preferable to rock crystal, as the latter very often gives false tints, from a want of uniformity of structure.

[4] *Journal de Physique*, 1801. *Memoire sur le Micrometre de Crystal de Roche*. Paris, 1807.

constructed, and which is so very simple that any person can make it for himself; *M N O P* is a tube about two inches long, attached to a ball and socket. The end *M O* of the tube carries an aperture of any form, and the ball *C D* contains two prisms of calcareous spar, separated by a thin film of sulphate of lime, so placed that each pair of the four images is tinged with the complementary colours, as described in a future chapter. A lens *L* is cemented either upon the anterior or posterior surface of the compound prism, or may be kept separate from the prism at *L*, but whatever be its position, it must always ennable the eye at *E* to see the aperture with perfect distinctness; and the focal length of the lens must be so adapted to the magnitude of the aperture, that the images of it can be sufficiently separated by the universal motion of the ball *C D*. The interior of the tube being covered with a black pigment, the instrument is ready for use. If we direct it to the sky, or to any luminous object, four brilliantly coloured images of the aperture will be distinctly seen, the colour of the two middle images being complementary to that of the two extreme images. By moving the ball in the socket, the colours will constantly change, and the images will sometimes overlap, and sometimes separate, exhibiting the finest variety of hues, and pleasing the eye by their combinations, and by the soft harmony of their contrasts.[5]

If we use a perfect crystal of calcareous spar, with an interrupting vein, as described in *Philosophical Transactions*, 1815, p. 281, and in the *Edinburgh Encyclopædia*, vol. xv, p. 611, and substitute it in place of *C D*, we have a *natural dichroscope* of the most perfect kind.

In the instrument where it is necessary to polarize the light by black glass, or japanned metal, there is no less than $\frac{1}{1}\frac{2}{3}$ths of the incident pencil lost by reflexion, while in the preceding instrument the light lost by transmission is very small. From this cause, the brightness of the colours is incomparably greater, and they may even be distinctly seen in candle light, by directing the aperture to a piece of white paper held near the candle.

[5] The phenomena will admit of many beautiful variations, by using several films of sulphate of lime, having their axes variously inclined to one another.

12. *Description of the Kaleidoscope.*

The term *Kaleidoscope*, from καλος, *beautiful*, ειδος, a *form*, and σκοπεω, to *see*, is the name of an instrument which I invented some years ago for the purpose of creating and exhibiting an endless variety of beautiful forms.

This instrument, in its simplest form, consists of two reflecting planes, made of glass or metal, from 5 to 10 or 12 inches long, and about an inch broad. When these reflectors are put together as in Plate X, Fig. 17, with two of their edges in contact, and their reflecting faces inclined at an angle of 60°, or the sixth part of a circle,—when they are put in a tube, and when the eye is placed at *E*, as near the angular point as possible, it will observe the opening *A O B* multiplied six times, and arranged round the centre *O*.

If any object, however shapeless, is placed before the opening *A O B*, and near the ends of the reflectors, the eye at *E* will observe in the two adjoining sectors an inverted image of this irregular object, apparently facing the direct image in *A O B*, and the direct and inverted image will form an object perfectly symmetrical; and these symmetrical images being multiplied by successive reflexions, the whole circular space, composed of six sectors, will present to the eye the most perfect picture that can be conceived.

If the objects placed in *A O B* are pieces of coloured glass, lying in a cell bounded by discs of glass, the pictures increase in beauty, and as the glass fragments change their position by the motion of the cell, a succession of the most perfect pictures will be displayed, which are literally infinite both in number and variety.

The objects which give the finest outlines by inversion are those which have a curvilineal form, such as circles, ellipses, looped curves like the figure 8, curves like 3 and the letter S, spirals, and other forms. Glass, both spun and twisted, and of all colours and shades of colours, should be formed into the preceding shapes, and when these are mixed with pieces of flat coloured glass, blue vitriol, native sulphur, yellow orpiment, differently-coloured fluids moving in small enclosed vessels of glass, &c., they will make the finest transparent objects for the kaleidoscope. A very fine effect is produced when only

two colours are used, viz. those that harmonise with each other, such as red and green, blue and gold yellow. Pieces of glass of these two colours may be mixed with twisted pieces of colourless glass with great effect.

In the simple kaleidoscope the two reflectors may be fixed at a constant angle, so as to be an aliquot part of a circle, or 360°, or they may be made to vary their inclination by various simple contrivances.

If the eye is raised above *E*, or if the objects are placed at any distance from the ends of the reflectors, the symmetry of the picture and the uniformity of the light vanish, so that it is essentially necessary to the production of forms perfectly beautiful and symmetrical that the eye be placed as near as possible to the angular point, and the objects as near as possible to the ends of the reflectors.

Telescopic Kaleidoscope.

The utility of the kaleidoscope may be greatly extended by the addition of a lens, as in Fig. 18. Without such an extension of its power, the kaleidoscope could only be regarded as an instrument of amusement; but when it is made to embrace objects of all magnitudes, and at all distances, it takes its place as a general philosophical instrument, and becomes of the greatest use in the fine, as well as the useful arts.

In considering how this change might be effected, it occurred to me, that if *M N*, Fig. 18, were a distant object, either opaque or transparent, it might be introduced into the picture by placing a lens, *L L*, at such a distance before the aperture *A O B*, that its image may be distinctly formed upon the plane passing through *A O B*. By submitting this idea to experiment, I found it to answer my most sanguine expectations. The image formed by the lens at *A O B* became a new object, as it were, and was multiplied and arranged by successive reflexions, in the very same manner as if the object *M N* had been reduced in the ratio of *M L* to *L A*, and placed close to the aperture.

In the compound kaleidoscope, thus constructed, the furniture of a room, books and papers lying on a table, pictures on the wall, a blazing fire, the moving branches and foliage of trees and shrubs, bunches of flowers, horses and cattle in a park, carriages in motion, the currents of a river, moving

insects, and in short every object in nature, may be introduced by the aid of the lens into the figures created by the instrument. When the flames of a blazing fire constitute the object, the kaleidoscope creates from it the most magical fire-works, in which the currents of flame which compose the picture may be turned into every possible direction.

The theory of this instrument; the various forms of annular, parallel, and polycentric kaleidoscopes; its application to the magic lanthorn and the solar microscope; and the mode of employing it in the fine and useful arts, have been explained at great length in my *Treatise on the Kaleidoscope.*

CHAPTER XVII.

ON THE METHOD OF FORMING THE LENSES AND SPECULA OF REFRACTING AND REFLECTING TELESCOPES.

SECT. I.—*On the Method of grinding and polishing Lenses.*

HAVING fixed upon the proper aperture and focal distance of the lens, take a piece of sheet copper, and strike a fine arch upon its surface, with a radius equal to the focal distance of the lens, if it is to be equally convex on both sides, or with a radius equal to half that distance, if it is to be plano-convex, and let the length of this arch be a little greater than the given aperture. Remove with a file that part of the copper which is without the circular arch, and a *convex gage* will be formed. Strike another arch with the same radius, and having removed that part of the copper which is within it, a *concave gage* will be obtained. Prepare two circular plates of brass, about $\frac{1}{10}$ of an inch thick, and half an inch greater in diameter than the breadth of the lens, and solder them upon a cylinder of lead of the same diameter, and about an inch high. These tools are then to be fixed upon a turning-lathe, and one of them turned into a portion of a concave sphere, so as to suit the convex gage; and the other into a portion of a convex sphere, so as to answer the concave gage. After the surfaces of the brass plates are turned as accurately as possible, they must be ground upon one another alternately, with flour emery; and when the two surfaces exactly coincide, the grinding tools will be ready for use.

Procure a piece of glass of a straw colour, whose dispersive power is as small as possible, if the lens is not for achromatic instruments, and whose surfaces are parallel ; and by means of a pair of large scizzars or pincers, cut it into a circular shape, so that its diameter may be a little greater than the required aperture of the lens. When the roughness is removed from its edges by a common grind-stone,[1] it is to be fixed with black pitch to a wooden handle of a smaller diameter than the glass, and about an inch high, so that the centre of the handle may exactly coincide with the centre of the glass.

The glass being thus prepared, it is then to be ground with fine emery upon the concave tool if it is to be convex, and upon the convex tool if it is to be concave. To avoid circumlocution, we shall suppose that the lens is to be convex. The concave tool, therefore, which is to be used, must be firmly fixed to a table or bench, and the glass wrought upon it with circular strokes, so that its centre may never go beyond the edges of the tool. For every 6 circular strokes, the glass should receive 2 or 3 cross ones along the diameter of the tool, and in different directions. When the glass has received its proper shape, and touches the tool in every point of its surface, which may be easily known by inspection, the emery is to be washed away, and finer kinds[2] successively substituted in its room, till by the same alternation of circular and transverse strokes, all the scratches and asperities are removed from its surface. After the finest emery has been used, the roughness which remains may be taken away, and a slight polish superinduced by grinding the glass with pounded pumice-stone, in the same manner as before. While the operation of grinding is going on, the convex tool should, at the end of every five

[1] When the focal distance of the lens is to be short, the surface of the piece of glass should be ground upon a common grindstone, so as to suit the gage as nearly as possible ; and the plates of brass, before they are soldered on the lead, should be hammered as truly as they can be done into their proper form. By this means much labour will be saved both in turning and grinding.

[2] Emery of different degrees of fineness may be made in the following manner :— Take five or six clean vessels, and having filled one of them with water, put into it a considerable quantity of flour emery. Stir it well with a piece of wood, and when it has stood for 5 seconds, pour the water into the second vessel. After it has remained about 12 seconds, pour it out of this into a third vessel, and so on with the rest ; and at the bottom of each vessel will be found emery of different degrees of fineness, the coarsest being in the first vessel, and the finest in the last.

minutes, be wrought upon the concave one for a few seconds, in order to preserve the same curvature to the tools and the glass. When one side is finished off with the pumice-stone, the lens must be separated from its handle by inserting the edge of a knife between it and the pitch, and giving it a gentle stroke. The pitch which remains upon the glass may be removed by rubbing it with a little oil, or spirits of wine; and after the ground side of the glass is fixed upon the handle, the other surface is to be wrought and finished in the very same manner.

When the glass is thus brought into its proper form, the next and the most difficult part of the operation is to give it a fine polish. The best, though not the simplest, way, of doing this, is to cover the concave tool with a layer of pitch, hardened by the addition of a little rosin, to the thickness of $\frac{1}{15}$ of an inch. Then having taken a piece of thin writing paper, press it upon the surface of the pitch with the convex tool, and pull the paper quickly from the pitch before it has adhered to it; and if the surface of the pitch is marked every where with the lines of the paper, it will be truly spherical, having coincided exactly with the surface of the convex tool. If any paper remains on the surface of the pitch, it may be removed by soap and water; and if the marks of the paper should not appear on every part of it, the operation must be repeated till the polisher, or bed of pitch, is accurately spherical. The glass is then to be wrought on the polisher by circular and cross strokes, with the oxide of tin, called the *flowers of putty* in the shops, or with the red oxide of iron, otherwise called *colcothar of vitriol*, till it has received on both sides a complete polish.[3] The polishing will advance slowly at first, but will proceed rapidly when the polisher becomes warm with friction. When it is nearly finished, no more putty or water should be put upon the polisher, which should be kept warm by breathing upon it; and if the glass moves with difficulty, from its adhesion to the tool, it should be quickly removed, lest it spoil the surface of the pitch. When any particles of

[3] As *colcothar of vitriol* is obtained by the decomposition of sulphate of iron, it sometimes retains a portion of this salt. When this portion of the sulphate of iron, Mr. Edwards remarks, is decomposed by dissolution in water, the yellow ochre which results penetrates the glass, forms an incrustation upon its surface, and gives it a dull and yellowish tinge, which is communicated to the image which it forms.

dust or pitch insinuate themselves between the glass and the polisher, which may be easily known from the very unpleasant manner of working, they should be carefully removed, by washing both the polisher and the glass, otherwise the lens will be scratched, and the bed of pitch materially injured.

The operation of polishing may also be performed by covering the layer of pitch with a piece of cloth, and giving it a spherical form by pressing it with the convex tool when the pitch is warm. The glass is wrought as formerly, upon the surface of the cloth, with putty or colcothar of vitriol, till a sufficient polish is induced. By this mode the operation is slower, and the polish less perfect; though it is best fitted for those who have but little experience, and would therefore be apt to injure the figure of the lens by polishing it on a bed of pitch.

In this manner the small lenses of simple and compound microscopes, the eye-glasses, and the object-glasses, of telescopes, are to be ground. In forming concave lenses, Mr. Imison (*School of Arts*, part ii, p. 145) employs leaden wheels with the same radius as the curvature of the lens, and with their circumferences of the same convexity as the lens is to be concave. These spherical zones are fixed upon a turning lathe, and the lens, which is held steadily in the hand, is ground upon them with emery, while they are revolving on the spindle of the lathe. In the same way convex lenses may be ground and polished, by fixing the concave tool upon the lathe; but these methods, however simple and expeditious they may be, should never be adopted for forming the lenses of optical instruments where an accurate spherical figure is indispensable. It is by the hand alone that we can perform with accuracy those circular and transverse strokes, the proper union of which is essential to the production of a spherical surface.

SECT. II.—*On the Method of Casting, Grinding, and Polishing the Specula of Reflecting Telescopes.*

The specula of reflecting telescopes are generally composed of 32 parts of copper and 15 of grain tin, with the addition of two parts of arsenic, to render the composition more white and compact. The Reverend Mr. Edwards found, from a variety of experiments, that if one part of brass and one of silver be added to the preceding composition, and only one part of arsenic used, a most excellent metal will be obtained, which is the whitest, hardest, and most reflective that he ever

met with. The superiority of this composition, indeed, has been completely evinced by the excellence of Mr. Edwards's telescopes, which excel other reflectors in brightness and distinctness, and shew objects in their natural colours. But as metals of this composition are extremely difficult to cast, as well as to grind and polish, it will be better for those who are inexperienced in the art, to employ the composition first mentioned.

Method of casting the metal.

After the flasks of sand[4] are prepared, and a mould made for the metal by means of a wooden or metallic pattern, so that its face may be downwards, and a few small holes left in the sand at its back, for the free egress of the included air;—melt the copper in a crucible by itself, and when it is reduced to a fluid state, fuse the tin in a separate crucible, and mix it with the melted copper, by stirring them together with a wooden spatula. The proper quantity of powdered arsenic, wrapt up in a piece of paper, is then to be added, the operator retaining his breath till its noxious fumes are completely dissipated; and when the scoria is removed from the fluid mass, it is to be poured out as quickly as possible into the flasks. As soon as the metal is become solid, remove it from the sand into some hot ashes or coals, for the purpose of annealing it, and let it remain among them till they are completely cold. The ingate is then to be taken from the metal by means of a file, and the surface of the speculum must be ground upon a common grindstone, till all the imperfections and asperities are taken away. When Mr. Edwards's composition is employed, the copper and tin should be melted according to the preceding directions, and, when mixed together, should be poured into cold water, which will separate the mass into a number of small particles. These small pieces of metal are then to be collected and put into the crucible, along with the silver and brass; after they have been melted together in a separate crucible, the proper quantity of arsenic is to be added, and a little powdered rosin thrown into the fluid metal before it is poured into the flasks.

Method of forming the grinding-tools, &c.

When the metal is cast, and prepared by the common grindstone for receiving its proper figure, the gages and grinding tools are to be formed in the same manner as for convex lenses, with this differ-

[4] The finest sand which I have met with in this country is to be found at Roxburgh Castle, in the neighbourhood of Kelso.

ence only, that the radius of the gages must always be twice the focal length of the speculum. In addition to the convex and concave brass tools, which should be only a little broader than the metal itself, a convex elliptical tool of lead and tin should also be formed with the same radius, so that its transverse may be to its conjugate diameter as 10 to 9, the latter being exactly equal to the diameter of the metal. On this tool the speculum is to be ground with flour emery, in the same manner as lenses, with circular and cross strokes alternately, till its surface is freed from every imperfection, and ground to a spherical figure. It is then to be wrought with great circumspection, on the convex brass tool, with emery of different degress of fineness, the concave tool being sometimes ground upon the convex one, to keep them all of the same radius, and when every scratch and appearance of roughness is removed from its surface, it will be fit for receiving the final polish. Before the speculum is brought to the polisher, it has been the practice to smooth it on a bed of hones, or a convex tool made of common blue hones, covered with a number of grooves, at right angles to each other, and about the 30th of an inch broad and deep. This additional tool, indeed, is absolutely necessary when silver and brass enter into the composition of the metal, in order to remove that roughness which will always remain after the finest emery has been used; but when these metals are not ingredients in the speculum, there is no occasion for the bed of hones. Without the intervention of this tool, I have finished several specula, and given them as exquisite a lustre as they could possibly have received. Mr. Edwards does not use any brass tools in his process, but transfers the metal from the elliptical leaden tool to the bed of hones. By this means the operation is simplified, but we doubt much if it is in the least degree improved. As a bed of hones is more apt to change its form than a tool of brass, it is certainly of great consequence that the speculum should have as true a figure as possible before it is brought to the hones; and we are persuaded, from experience, that this figure may be better communicated on a brass tool, which can always be kept at the same curvature by its corresponding tool, than on an elliptical block of lead. We are certain, however, that when the speculum is required to be of a determinate focal length, this length will be obtained more precisely with the brass tools than without

Bed of hones.

them. But Mr. Edwards has observed, that these tools are not only unnecessary, but "really detrimental." That Mr. Edwards found them unnecessary, we cannot doubt, from the excellence of the specula which he formed without their assistance; but it seems inconceivable how the brass tools can be in the least degree detrimental. If the mirror is ground upon twenty different tools before it is brought to the bed of hones, it will receive from the last of these tools a certain figure, which it would have received, even if it had not been ground on any of the rest; and it cannot be questioned, that a metal wrought upon a pair of brass tools is equally, if not more, fit for the bed of hones, than if it had been ground merely on a tool of lead.

When the metal is ready for polishing, the elliptical leaden tool is to be covered with black pitch,[5] about $\frac{1}{20}$ of an inch thick, and the polisher formed in the same way as in the case of lenses, either with the concave brass tool, or with the metal itself. The colcothar of vitriol should then be triturated between two surfaces of glass, and a considerable quantity of it applied at first to the surface of the polisher. The speculum is then to be wrought in the usual way upon the polishing tool, till it has received a brilliant lustre, taking care to use no more of the colcothar, if it can be avoided, and only a small additional quantity of it, if it should be found necessary. When the metal moves stiffly on the polisher, and the colcothar assumes a dark muddy hue, the polish advances with great rapidity. The tool will then grow warm, and would probably stick to the speculum, if its motion were discontinued for a moment. At this stage of the process, therefore, we must proceed with great caution, breathing continually on the polisher, till the friction is so great as to retard the motion of the speculum. When this happens, the metal is to be slipped off the tool at one side, cleaned with soft leather, and placed in a tube for the purpose of trying its performance; and if the polishing has been conducted with care, it will be found to have a true *parabolic* figure.

In polishing the small speculum for his Newtonian reflectors, Sir. W. Herschel always employed two or more tools considerably larger than the speculum to be polished; but he did not

[5] In summer, or when the pitch is soft, it should be hardened by the addition of a little rosin; and should always be strained through a piece of linen, in order to free it from impurities and rough particles.

use them till after the speculum was made nearly flat with emery upon a small tool of lead. These tools, or beds of hones, should not be less than six inches in diameter, and the figure of the tools is not considered as completed till the speculum can be first highly finished upon one of them, and afterwards be applied to another, without experiencing any change of figure. The last half dozen strokes should be performed in the direction of the larger axis of the ellipse. When the speculum is thus brought to a perfect figure, it must be polished upon a pitch polisher of a circular form, whose diameter is but *one-tenth* greater than the transverse axis of the speculum.

CHAPTER XVIII.

ON THE PHENOMENA OF DOUBLE REFRACTION.

In the different phenomena of refraction which have been described both in this and in the preceding volume, the rays of light were supposed to fall upon water or glass, or some substance that had no external crystalline shape. In these cases, the ray was refracted *singly*, or was not divided into two rays, and consequently the images produced by such refractions were all *single.*

It has been found, however, that a great number of regular crystals, such as *Iceland spar*, *ruby*, *emerald*, *zircon*, *titanite*, and *rock crystal*, give *two* refracted rays when there is only one incident ray, or produce *two* images when there is only one object; and they are therefore said to possess the property of *double refraction.*

In all crystals which possess this property, one of the rays or images is refracted according to the *ordinary* law of the sines, discovered by Snellius, and therefore the direction of this one ray or image, which is called the *ordinary ray* or the *ordinary image*, may be found by the rule in Vol. I, p. 156.

The second ray or image is formed according to an entirely different rule, and is therefore called the *extraordinary ray* or *image.*

In all doubly refracting crystals, there is one or more lines along which the double refraction, or the separation of the ordinary and extraordinary images, is nothing, or vanishes, or along which these images perfectly coincide. This line is called

the *axis of the crystal*, or the *axis of double refraction.* Those crystals which have ONE such axis are called *crystals with* ONE *axis of double refraction*, and those crystals which have TWO such axes are called *crystals with* TWO *axes of double refraction.*

SECT. I.—*On Crystals with one Axis of Double Refraction.*

After examining a great variety of crystallized bodies, I succeeded in determining that all crystallized bodies which have the form of their primitive crystals, the *obtuse rhomboid*, such as *Iceland spar ;* the *acute rhomboid*, such as *Ruby ;* the *regular six-sided prism*, such as *Emerald ;* the octohedron with a square base, such as *Zircon ;* the *right prism with a square base*, such as *Titanite ;* and the *bipyramidal dodecahedron*, such as *Rock crystal*,—have all *one axis* of double refraction, coincident with the crystallographic axes of these different solids.

In *Iceland spar*, for example, the axis of double refraction is the short diagonal, which joins the two obtuse solid angles of the rhomboid. In *Ruby* it is the long diagonal, which joins the acute summits of the rhomboid ; and so on with the rest.

Let us now suppose that each of the crystals above named is shaped into a sphere, *A B C D*, Plate XI, Figs. 1, 2, and that *AB* is the axis of the crystal, and consequently the axis of double refraction. Then, if a ray of light is incident in the direction *A B* or *B A*, it will not be divided, or there will be no double refraction. If the incident ray is a little inclined to *A B*, it will be divided into two rays, and the separation of the two rays, or the quantity of double refraction, will gradually increase as the inclination of the ray to the axis *A B* increases ; and when the ray comes into the direction *C D* at right angles to *A B*, the double refraction is a *maximum*, or the greatest possible. If we now measure this refraction at different inclinations, and determine its index (See Vol. I, p. 157, note 4), as well as the index of the ordinary refraction, we shall find that in some crystals the extraordinary refraction increases from *A* and *B* to *C* and *D*, while in others it diminishes. It becomes necessary, therefore, to divide crystals with one axis into two classes, *Positive* and *Negative*,[1] as in the following Table, deduced principally from my observations.

[1] M. Biot, who first observed this difference in two or three crystals, calls them *Attractive* and *Repulsive*, names which involve a hypothesis.

Negative Crystals, with One Axis.

Carbonate of lime.	Idocrase.
Carbonate of lime and magnesia.	Wernerite.
Carbonate of lime and iron.	Mica from Kariat.
Tourmaline.	Phosphate of lead.
Rubellite.	Phosphato-arseniate of lead.
Corundum.	Hydrate of strontites.
Sapphire.	Arseniate of potash.
Ruby.	Muriate of lime.
Emerald.	Muriate of strontian.
Beryl.	Nitrate of soda.
Phosphate of lime.	Subphosphate of potash.
	Sulphate of nickel and copper.

Positive Crystals, with One Axis.

Zircon.	Dioptase.
Quartz.	Apophyllite.
Oxide of tin.	Sulphate of potash and iron.
Tungstate of lime.	Super-acetate of copper & lime.
Titanite.	Hydrate of magnesia.
Boracite.	Ice.

Let us now suppose that in the *Positive crystals* the *maximum* index of extraordinary refraction is to the *minimum* index, or that of ordinary refraction, as $\frac{1}{O c}$ is to $\frac{1}{O C}$ or $\frac{1}{O A}$ (Fig. 16), 1 and in *Negative crystals* that the minimum index of extraordinary refraction is to the maximum index, or that of ordinary refraction, as $\frac{1}{O c}$ is to $\frac{1}{O C}$ or $\frac{1}{O A}$ (Fig. 15); then, if $A B$ is the axis of a spheroid of revolution, and $c d$ its equatorial diameter, the index of extraordinary refraction, at any point a of the solid, will be $\frac{1}{O a}$. 2

The following are the maximum and minimum indices of refraction for two crystals of the Negative and Positive class :—

Iceland spar,...	Ord.	1.6543	According to the observations of Malus.
	Ext.	1.4833	
Rock crystal,...	Ext.	1.5582	
	Ord.	1.5494	

Sect. II.—*On Crystals with two Axes of Double Refraction.*

Those eminent philosophers who had hitherto written on double refraction considered the law above described (which has always been regarded as the finest discovery of Huygens), as the universal law of double refraction. In the year 1816, however, I discovered that by far the greatest number of crystals possessed *two* axes of double refraction, and I succeeded in determining the law by which all the phenomena could be accurately calculated.

After examining an immense number of such crystals, I ascertained that all regular crystals which belong to the prismatic system of Mohs, or whose primitive forms are the *Right prism*, with its base a rectangle, a rhomb, or an oblique parallelogram; the *Oblique prism*, with its base a rectangle, a rhomb, or an oblique parallelogram; or the *Rectangular* and *Rhomboidal Octohedron*, have *two* rectangular axes of double refraction, coincident with some permanent line in the primitive form.

Thus, in Fig. 3, if AB, DE, be two rectangular axes of different intensities, then the action of these axes will be such that there will be two apparent axes passing through $P\ P'$, along which the double refraction is nothing. The angular distance between P and P' will diminish as the difference between the intensities of the two axes diminishes; and when the difference vanishes, P and P' will coincide at O, and there will be only one apparent axis of double refraction passing through O.

It would be unprofitable in a popular work to attempt to explain the mode in which the double refraction at any point of the sphere may be determined by combining the separate actions of the two axes AB, DE, but the general law will be made apparent in the next Chapter, on the Polarisation of Light.

Before concluding this Chapter, it may be proper to mention that all those crystals whose primitive form is the *Cube*, the *Regular Octohedron*, and the *Rhomboidal Dodecahedron*, have a double refraction, or, as I have endeavoured elsewhere to prove, they have *three equal rectangular axes* of double refraction, which are *in equilibrio* at every point of the sphere.

CHAPTER XIX.

ON THE POLARISATION OF LIGHT.

In the preceding chapter, we have considered merely the separation of the two images as the characteristic of *double refraction;* but this property of bodies is invariably accompanied with another no less remarkable, namely, the *polarisation* of the light of which the two pencils are composed.

The word *polarisation* is employed to denote a property which may be communicated to ordinary light, and in virtue of which it exhibits the appearance of having *polarity* or *poles* possessing different properties.

The discovery of the polarisation of light in doubly refracting crystals, was made by the celebrated Huygens.

Having taken two pieces of Iceland crystal, he found that if all the sides of the one piece were parallel to those of the other, as in Plate XI, Fig. 3, the two pencils, *B C*, *B D*, 4 formed by the double refraction of the ray *A B*, were not doubly refracted in passing through the second crystal in the lines *E F*, *G H;* the pencil *D G*, which had been *regularly* or *usually* refracted by the first crystal, being now only *regularly* refracted by the second, while the pencil *C E*, which had been *extraordinarily* or *unusually* refracted by the first, was only *extraordinarily* refracted by the second. The same result took place in all other positions, where the principal sections[1] of the two crystals were in the same plane, and whether the two surfaces were parallel or not. "Now, it is wonderful," says Huygens, "why the rays *C E* and *D G*, incident from the air upon the lower crystal, should not divide themselves like the first ray *A B*. One would say that the ray *D G*, in passing through the upper crystal, must have lost what was required to move the matter which served for the extraordinary refraction; and, in like manner, that *C E* had lost what was necessary to move the matter which served for the ordinary refraction. There is, however, another fact which overturns this reasoning, which is,

[1] The principal section of a crystal is any plane passing through the axis *A B*, Plate XI, Fig. 1, 2.

that when we arrange the two crystals, so that the planes which form the principal sections cut one another at right angles; then, whether the surfaces are parallel or not, *the ray D G, which proceeds from the ordinary refraction, suffers only the extraordinary refraction in the lower crystal, and the ray C E which proceeds from the extraordinary refraction, suffers only the ordinary refraction.* But in all the other positions, infinite in number, besides those which I have determined, the rays *D G*, *C E*, divided themselves into two by the refraction of the lower crystal, so that from the single pencil *A B* there are formed four pencils, sometimes of equal brightness, and sometimes of different brightnesses, according to the different positions of the crystals, but which do not seem to have more light when taken altogether than the single ray *A B*."

From these facts Sir Isaac Newton concluded, that every ray may be considered as having *four sides* or *quarters*, two of which, opposite to one another, incline the ray to be refracted after the usual manner, as often as either of them are turned towards the coast or side of double refraction, and the other two, whenever either of them are toward the coast of unusual refraction, do not incline it to be otherwise refracted than after the usual manner. The two first may therefore be called the *sides* of unusual refraction, and every ray of light may be considered as having two opposite sides originally endowed with a property on which the unusual refraction depends, and the other two opposite sides not endowed with that property.

It is obvious from these facts, that the difference between *Direct light* and light that is *Polarised* by the action of a crystal, is, that the *former* may always be divided into two pencils by a doubly refracting crystal, whereas, in the *latter*, this division depends on the angle formed by the principal sections of the two crystals.

The effects above described may be produced by combining any two *positive*, or any two *negative* crystals; but if the one is *positive*, and the other *negative*, then the same effects are produced when the principal sections are at right angles, as when they are parallel in the other cases, and *vice versa*.

Sect. I.—*On the Polarisation of Light by Reflexion.*

Not a single fact was added to our knowledge of the polarisation of light from the time of Huygens till the year 1810,

when Malus made the important discovery that when a ray of light is reflected at a particular angle from the surfaces of transparent bodies, whether solid or fluid, it is *polarised* like *one of the pencils* formed by doubly refracting crystals.

Let a ray of light $R\ S$ (Fig. 5), proceed from the sun or the sky, and fall upon a piece of plate glass $A\ B$, at an angle $R\ S\ P$ of incidence of about 54° 35′. If the reflected ray $S\ T$ is received upon the first surface of a rhomb of Iceland spar, it will not be divided into two pencils, either when the principal section of the rhomb is parallel to the plane of reflexion $R\ S\ T$, or when it is perpendicular to this plane; but in every intermediate position of the rhomb, the pencil $S\ T$ will be divided into two, exactly as if it had been one of the pencils formed by another rhomb of Iceland spar.

In order to analyse this remarkable fact, Malus placed the principal section of a crystal of calcareous spar in a vertical direction, and having divided by it a ray of direct light into *two* pencils, he allowed these two pencils to fall upon the surface of *Water* at an angle of 52° 45′, the angle at which he had found *Water* to polarise the incident light. The ordinary ray was refracted at the surface of the water, and part of it reflected like all other light; but the extraordinary ray *penetrated the water entirely; not one of its particles escaped refraction, and none of them suffered reflexion.* When the principal section of the crystal, however, was perpendicular to the plane of incidence, the *extraordinary* ray only produced a partial reflexion, and the *ordinary* ray penetrated the water entirely, without suffering any reflexion.

When the ray of *direct* light $R\ A$, Fig. 6, fell upon the first surface $M\ N$ of a plate of glass, $M\ N\ P\ Q$, and was polarised after reflexion in the direction $A\ D$, Malus observed that the refracted portion $A\ C$ reflected at C from the second surface, and emerging at M in the direction [redacted], parallel to [redacted], A D M T was also polarised simultaneously with $A\ D$, although the angle was much less than [redacted], the sine of the first angle being RAP to the sine of the second as the sine of incidence is to that of refraction. Hence it follows, that a plate of glass, or any other transparent substance, polarises simultaneously the light that falls upon both its surfaces, when it is incident on the first at the proper angle, and it may therefore be substituted in place of a surface of water in all experiments of this kind.

Effects of different bodies.

Malus found that *black marble*, *ebony*, and other opaque bodies, polarised light by reflexion like transparent ones. He found that the angle of incidence at which this property was communicated was greater in bodies of a high refractive power; and he measured, with considerable accuracy, the polarising angles of glass, water, and other bodies. In order to discover the law which regulated the phenomena, he compared these angles with the dispersive powers of glass and water, and finding that there was no relation between these properties of transparent bodies, he draws the following general conclusion: "The polarising angle neither follows the order of the refractive powers nor that of the dispersive forces. It is a property of bodies independent of the other modes of action which they exercise upon light."

In order to determine the manner in which a polarised ray is reflected from a plate of glass, having a rotatory motion round the polarised ray, and forming with it a constant angle, let $S\,T$, Fig. 5, be the ray polarised by reflexion from $A\,B$, the incident ray $R\,S$ being supposed, for the sake of illustration, to proceed from the sun when due *south*, and let it be received upon a second plate $C\,D$, so that the angle of reflexion from $C\,D$, viz. $P\,T\,V$, may be always equal to the angle of reflexion from $A\,B$, viz. $P\,S\,T$, and let us suppose that the plate $C\,D$ revolves round the ray $S\,T$, preserving its inclination to it invariable. Then, in the case represented in the figure, where the second plate $C\,D$ faces the *south*, and causes the plane of the second reflexion from $C\,D$ to be parallel to the plane of the first reflexion from $A\,B$, the ray $S\,T$ will suffer partial reflexion in the direction $T\,V$, as if it had been common light. If the plate is now turned round $S\,T$ in different azimuths, so that its face begins to turn to the *east*, the intensity of the reflected pencil $T\,V$ will gradually diminish; and when it has performed a revolution of 90°, or has its face turned towards the *east*, the pencil $T\,V$ will *disappear entirely, not a single ray of it suffering partial reflexion*, in which case the plane of reflexion from $C\,D$ is at right angles to the plane of reflexion from $A\,B$. By continuing to turn the plate, the ray $T\,V$ will re-appear, and will gradually increase in intensity till the plate has turned round 180° from the beginning of its motion, in which case the plane of reflexion from $C\,D$ will be again parallel to the

plane of reflexion from *A B*, and the reflected ray *T V* will then be a maximum, as it was at the commencement of its motion. By continuing to turn the plate, the reflected ray will *again disappear* at an azimuth of 270°, and will regain its maximum upon returning to 360°, or 0°, the point from which it set out. The ray of light *S T* is said to be *polarised in the plane P S T*.

If, in the azimuth of 90°, when the ray *T V* entirely disappears, we breathe upon the plate *A B* or *C D*, so as to communicate to either a thin film of aqueous vapour capable of reflecting light, the ray *T V* will be revived. This effect arises from the polarising angle for water being 52° 45′, whereas it is 56° 45′ for glass.

If *A B* is made to receive the ray of a candle *R S*, at an angle of 52° 45′, and *C D* the ray *S T* at an angle of 56° 45′, then it is obvious that the image of a candle will be seen in the direction *V T*, because the ray *S T* is not polarised by the plate *A B*. If we now, however, breathe upon *A B*, the image of the candle seen along *V T* will disappear, because the ray *S T* being reflected from water is now polarised. Hence, if we place beside one another two sets of reflectors arranged as above described, we may, by breathing upon the two contiguous ones, exhibit the paradoxical effect of reviving and extinguishing the rays by the same breath. See *Edin. Phil. Journal*, vol. vii, p. 146.

Law of the polarisation of light by reflexion.

While repeating the experiments of Malus in the summer of 1811, I measured the polarising angles of a great number of transparent bodies, and in 1813 and 1814 I endeavoured to connect them by some general principle. The measures for *Water* and the *precious stones* afforded a surprising coincidence between the cube roots of the indices of refraction and those of the tangents of the polarising angles, but the results for glass resisted every method of classification. Having afterwards found, however, that glass often acquires by exposure to the air an incrustation, or experiences a decomposition, which alters its superficial action upon light, and consequently its polarising angle, without altering its refractive power, I neglected it altogether, and employed only the surfaces of other bodies. In this way I obtained the results in the following Table, and in November 1814 was enabled to simplify the law of the tangents, as stated in the following page.

Table containing the Calculated and Observed Polarising Angles for various Bodies.

Names of the Bodies.	Calculated polarising angles for the *first* surface.	Observed polarising angles for the *first* surface.	Difference between the calculated and observed angles.	Calculated polarising angles for the *second* surface.
	° ′ ″	° °		° ′ ″
Air,[2]	45 0 32	45 or 47		44 59 28
Water,	53 11 0	53° 6′	0° 5′ —	36° 49′
Fluor spar,	55 9 0	54 50	0 19 —	34 51
Obsidian,	56 6 0	56 3	0 3 —	33 54
Birdlime,	56 40 0	56 46	0 6 +	33 20
Sulphate of lime,	56 45 0	56 28	0 17 —	33 15
Rock crystal,	56 58 0	57 22	0 24 +	33 2
Hydrate of magnesia,		57 25		
Sulphate of barytes...	58 44 0	58 29	0 15 +	31 16
Opal-coloured glass,	58 33 0	58 1	0 32 —	31 27
Topaz,	58 34 0	58 40	0 6 +	31 26
Mother of pearl,	58 50 0	58 47	0 3 —	31 10
Iceland spar,	58 51 0	58 23	0 28 —	31 9
Orange-coloured glass,	59 28 0	59 12	0 16 —	30 32
Spinelle ruby,	60 25 0	60 16	0 9 —	29 35
Zircon,	63 0 0	63 8	0 8 +	27 0
Glass of antimony,	64 30 0	64 45	0 15 +	25 30
Sulphur,	63 45 0	64 10	0 25 +	26 15
Diamond,	68 1 0	68 2	0 1 +	21 59
Chromate of lead,	68 3 0	67 42	0 21 —	21 56

From the observations contained in the preceding Table, it follows, that a ray of white light will be completely polarised by a single reflexion from transparent bodies of all kinds, or the quantity of polarised light in it will be a maximum when the reflexion is made at such an angle that *the Tangent of the angle of incidence is equal to the Index of refraction.*

The results obtained from this general law are given in column 2d of the preceding table, and the accuracy with which it represents the experiments will be seen by the differences in the 4th column.

If we suppose $M N$ (Fig. 7), to be the reflecting surface, and $B A$ a ray of light polarised by reflexion in the direction AD ~~$A B$~~, $A C$ the refracted ray, and $E K$ perpendicular to the

[2] This observation on *Air* was made by M. Arago, who communicated it to me in August 1814, as *between* 45° *and* 47°, and of which a written memorandum was taken in his presence. M. Biot, in his *Traité de Physique*, tom. iv, p. 289, has *omitted entirely the* 47°, which we trust he will replace in the next edition of his work.

surface at the point of incidence A, then it follows, from the above law,

1. That the *sum of the angles of incidence and refraction is always a right angle*, when the reflected ray is completely polarised; that is, the angles $B A E$, and $C A K$, are together equal to 90°.

2. That the *complement of the polarising angle is always equal to the angle of refraction*, or $D A M$ is equal to $C A K$. And

3. That when the reflected ray is completely polarised, the *reflected ray forms a right angle with the refracted ray*, or $D A C$ is equal to 90°.

When the light is incident on the second surface $P Q$ of a transparent body $M N Q P$, as at $A C$ (Fig. 6), the reflected ray $C M$ is completely polarised when the co-tangent of the angle of incidence is equal to the index of refraction, as will appear from the last column in the table; and in this case the angle of polarisation at the second surface, or $M C O$, is the complement of the angle of polarisation $D A P$ at the first surface. In this case, also, the polarised ray $C M$ forms a right angle with the refracted ray $C F$.

The laws of polarisation now explained are equally applicable to the separating surfaces of two media having different indices of refraction, and are likewise true for the differently refrangible rays of the prismatic spectrum. For a full account, however, of these subjects, the reader is referred to the *Phil. Trans.* 1816, p. 60, or to the *Edinburgh Encyclopedia*, vol. xv, p. 580–585.

SECT. II.—*On the Polarisation of Light by Refraction.*

Although the polarisation of light by refraction had escaped the notice of Malus in his early experiments, yet he was led, in the prosecution of them, to the discovery of this curious fact in the year 1811. Some time after this, M. Biot made the same discovery; and, in the summer of 1813, ignorant of what had been done abroad, I discovered the same fact, and succeeded in determining also the law of the phenomena. Malus and I were led to the discovery of this property by entirely different methods. He first observed it during an examination of the light transmitted by uncrystallized plates, and it presented itself to me by noticing the *polarisation in one*

plane of the light transmitted by a piece of mica, whose laminæ must have been partially detached, the mica having absorbed or reflected all the light of one of its images. From this remarkable fact, I was led to try a number of thin plates of glass, which I found to polarise the transmitted light in one plane, exactly like the piece of mica.

Malus's experiments on the polarisation of light by transmission.

Plate XI. Fig. 7, 8.

In order to explain this property of light, let *B A* (Fig. 7) be the ray incident at the polarising angle, and *A D* the pencil polarised by reflexion; then the refracted pencil *A C* will contain a portion of light polarised in an opposite manner to *A D*. If *A C* falls upon a second refracting surface, another portion of it will be polarised by reflexion from that surface, and a similar portion by refraction, so that the transmitted ray will now contain a greater quantity of polarised light. In like manner, by transmitting the light *O A* through a bundle or pile of glass plates, such as *A B C D* (Fig. 8), the transmitted ray will always contain more and more polarised light, till the resulting pencil *E′ E* is wholly polarised in one plane.

If the ray *S T*, instead of being polarised by the plate *A B*, (Plate XI, Fig. 5) is polarised by a bundle of glass plates, parallel to *A B*, and acting upon a ray *R S* transmitted through them, it will be found, that before it is received upon the second plate *C D*, it will entirely penetrate the plate in the azimuths 0° and 180°, and will be reflected like common light in the azimuths 90° and 270°, which is exactly the reverse of what took place when the ray *S T* was polarised by reflexion from *A B*.

The operation of bundles of glass plates will be still more strikingly seen by referring to Fig. 9, where *A B* is a bundle of glass plates, receiving the incident ray *R S* at the polarising angle, so as to polarise the transmitted ray *S T*. Then, if a second and similar bundle *C D* receives the polarised ray *S T*, so that the plane of refraction in *C D* is parallel to the plane of refraction in *A B*,—in this case the light will be *wholly transmitted* through the second bundle *C D*, as the ray *S T* falls upon the reflecting planes of *C D*, under the same circumstances as it did in Fig. 8, when no light was reflected. The same effect will be produced at an azimuth of 180°, by turning the bundle *C D* round the ray *S T*, so as to preserve the same inclination to it. At the azimuths of 90° and 270°, on the contrary, it will be wholly reflected at its various inci-

dences upon the plates, and not a single ray will be transmitted through the bundle.

From the various observations made by Malus, he deduces the following general conclusion:—

"When a ray of light falls upon a plate of glass at an angle of 35° 25′, all the light which it reflects is polarised in one direction (or *plane*). The light which traverses the glass is composed, 1*st*, of a quantity of light polarised in a direction (or *plane*) opposite to that which is reflected, and proportional to it; and, 2*dly*, *of another portion not modified, and which preserves the character of direct light.*"

In my paper on the polarisation of light by oblique transmission (*Phil. Trans.* 1814), I have given an account of experiments analogous to those of Malus, and I observed the same phenomena with the thinnest *films of glass*, with the elementary *films of mica*, with thick *plates of mica*, with folds of *gold-beaters' skin*, and with the films of *gold leaf*.

In order to determine the law of the phenomena, as depending on the number of refracting plates, and the angle of incidence, I made numerous experiments, and obtained the following results:—

1. That the number of plates which polarise a maximum of light by transmission at different angles of incidence, are to one another as the co-tangents of the angles of incidence.

2. That when a pencil of light is incident at any angle except a right angle, upon the surface of a transparent body, a certain portion of the transmitted light is completely polarised, while the remaining portion has *suffered a physical change*, approaching more or less to that of complete polarisation.[3]

[3] It appears from experiment, that 24 plates are necessary to polarise completely a given pencil of light, at an angle of 61°; consequently 12 plates will not polarise the whole pencil at the same angle. Let us now suppose that the portion not polarised by the 12 plates amounts to 20 rays out of 100; then, if these 20 rays were *absolutely unpolarised, and in the same state as direct light*, they would require to pass through 24 plates at an angle of 61°, in order to be completely polarised. But the experiments prove that they require to pass only through other 12 plates at that angle in order to be completely polarised. It therefore follows, that the 20 rays have been *half polarised* by the first 12 plates, and the polarisation completed by the other 12. Hence arises the mistake of Malus (which is repeated by Biot), who observes, that the light transmitted obliquely through glass consists, 1st, of a quantity of polarised light; and, 2d, *of another portion, not modified, and which preserves the characters of direct light.*

3. That if a pencil of light is *partly polarised* by transmission at any angle, it will be more and more polarised at every successive transmission in the same plane till its polarisation is complete, whether the transmissions are made at angles all above or all below the polarising angle, or at angles, some of which are above and some below the polarising angle.

4. That when a pencil of light is transmitted through one or more parallel plates of the same refractive power, the quantity of light polarised by transmission will be a maximum when the tangent of the angle of incidence is equal to the index of refraction.

Sect. III.—*On the Colours produced by Transmitting Polarised Light through thin Crystallized Plates, or near the resultant axes of Doubly Refracting Crystals.*

In the year 1811 the colours of thin crystallized plates were discovered by M. Arago, and the same discovery was made by me, and communicated in 1812 to the Royal Society of Edinburgh, before any person in this country had any knowledge of what had been done in France. But though the general fact which we discovered was the same, yet I was led, by a peculiar mode of observation, to examine the phenomena seen along the real and the resultant axes of crystals, and was thus conducted to the discovery of the Single and Double Systems of Coloured Rings, which had escaped the observation of others, and which have led to the general laws of polarisation and double refraction.

Before we begin to explain this class of phenomena, we shall describe the method by which they may be most easily observed.

Let *R S*, Fig. 10, be a ray of direct light, from the sky or from a candle, and *S T* the pencil polarised by reflexion from the plate of glass *A B*; then, as we have before seen, the ray *S T* will not suffer reflexion, if it is incident at the polarising angle upon a second plate *C D*, provided the plane of the second reflexion is at right angles to that of the first. Let the two plates of glass, therefore, be capable of being fixed in this position, so that the light is reflected in the direction *T V*, which will be known by the appearance of an undefined dark spot on the image of the part of the sky which is reflected from *A B*. The plate *A B* may be called the *polarising plate*, and *C D* the *analyzing plate*.

If we now take a thin plate of *sulphate of lime* or *gypsum*, such as *e f*, sliced from a crystal by means of a lancet or penknife, and expose it to polarised light by holding it in the reflected ray *S T*, it will be found, by turning it round in a plane perpendicular to *S T*, and looking into the plate *C D*, in the direction *V T*, that there are four positions, distant 90° from each other, viz. 0°, 90°, 180°, and 270°, where no change is produced; while at every intermediate position the surface of the plate of sulphate of lime will appear covered with a brilliant colour, suppose *red*. The intensity of this colour is a *maximum* at the points 45°, 135°, 225°, 315°, and it disappears entirely at 0°, 90°, 180°, and 270°.

Let the analyzing plate be now gradually turned from the position in Fig. 10, in the azimuth of 90°, to the azimuth of 0°, which we shall call the zero of the circle in which it revolves round the polarised ray *S T*, the plate of sulphate of lime *e f* remaining fixed in the azimuth of 45°, so as to give the brightest *red;* then by looking into the analyzing plate *C D*, it will be seen that the red gradually becomes fainter and fainter, till *C D* comes into the azimuth of 45°, when no colour whatever appears. Beyond this point, or 45°, a faint *green* appears, which gradually becomes more and more intense, and reaches its maximum at 90°. From 90° the *green* becomes paler, till it entirely disappears at the azimuth of 135°, when the *red* again commences, and reaches its maximum of brilliancy at 180°. The very same changes are repeated from 180° to 360°, or 0°.

From these simple facts, it is manifest that the plate *e f* transmits two sets of differently coloured rays, viz. *red* and *green*, which are the *Extraordinary* and the *Ordinary*, which we shall call *E* and *O*, and which are always complementary to each other, or together make white light. Its colour is always *red* when the analyzing plate *C D* is in the azimuth of 90° and 270°, and always *green* when *C D* is in the azimuth of 0° and 180°.

Now, as none of the *green* rays *O* are reflected in the azimuth of 90°, it is quite clear that they are polarised in the same manner, or in the same direction or plane, as the ray of white light *S T*, none of which is reflected from *C D* in that azimuth; that is, the green rays *O preserve their primitive polarisation*, or are not acted upon by the plate *e f*. In like

manner, it is manifest, that as the *red* rays E are reflected, they must have a polarisation different from that of $S\,T$, or these rays must have been *depolarised*, or have had *their polarisation altered by the plate* $e\,f$. When $e\,f$ was in the azimuth of 45°, the red rays were most copiously reflected when $C\,D$ was in the azimuth of 90° and 180°, and not reflected at all in the azimuth of 0° and 270°. Hence they must in this case have been polarised in a plane at right angles to the polarisation of $S\,T$, that is, their polarisation must have suffered a change of $90° = 2 \times 45° = 2\,a$, calling a the azimuth of the plate $e\,f$.

In order to determine if the change of polarisation in the tint E is equal to $2\,a$ in every other azimuth, let the plate $C\,D$, or the plane of reflexion from it, be placed in an azimuth equal to a, the azimuth of the crystalline plate; then if the pencil E is polarised in the azimuth $2\,a$, the plane of reflexion from the plate $C\,D$ will form equal angles, viz. a with the plane $R\,S\,P$, in which the pencil O is polarised, and the new plane in which E is supposed to be polarised, and consequently it will reflect an equal portion of these two kinds of rays, that is, of the *red* and *green*, so that the resulting pencil will be white. By making this experiment at all azimuths, or for all values of a, M. Biot found this to be the case.

In the preceding illustrations we have supposed the polarised light, after it has been acted upon by the sulphate of lime $e\,f$, to be analysed into its two *colours* by means of a reflecting surface $C\,D$. The same effect may be produced by a prism of Iceland spar, or any other doubly refracting crystal, where the two images are distinctly separated. When the principal section of Iceland spar is in the plane $R\,S\,T$, and the plate $e\,f$ in the azimuths of 45°, 135°, 225°, and 315°, the *extraordinary* image formed by the spar will exhibit the *red* tint, while the *ordinary* image will exhibit the complementary *green*. The light may also be analysed by transmitting it through a bundle of glass plates, as in Plate XI, Figs. 8 and 9; or by a singly polarising crystal, such as *Agate* or *Tourmaline*, or the artificial crystals described in Sect. VII, page 367.

In order to see the surface of the plate $e\,f$, the distance of the eye from it, or $V\,T + T\,e$, must be equal to 7 or 8 inches; but if the eye is armed with a magnifying-glass, $e\,f$ may be brought nearer to T, till $V\,T + T\,e$ is equal to the focal dis-

tance of the lens. In this case the surface of the plate *e f* will be magnified. In order to see minute structures, such as that of *Apophyllite*, &c. (see p. 377) the application of a lens is necessary; but in this case the analysing plate *C D* should be very small, in order to admit *e f* on one side near the point of reflection *T*, and the eye on the other side, so that *V T* + *T e* may not exceed an inch or half an inch. In such cases, however, it will be found advantageous to use an agate or a tourmaline microscope, which may consist of a thin plate of either, cemented to the flat side of a plano-convex lens.

SECT. IV.—*On the System of Rings produced by Crystals with one Axis of Polarisation and Double Refraction.*

If we take either a positive or a negative crystal, having one axis of double refraction, and hold it at *e f*, Fig. 10, so that the polarised ray *S T* passes through the axis of double refraction, and then placing the eye as near *D* as possible, so as to receive the reflected ray *T V*, and bringing the crystal near to *C*, and as near the eye as possible, a beautiful system of coloured rings will be seen along the axis of double refraction, as shewn in Fig. 11. In order to see these rings to great advantage, the light *R S* must be a broad luminous surface, such as that of the sky seen through a window, or the surface of a ground glass globe, illuminated by a lamp. These rings are intersected with a *black cross A B C D*, through the centre of which, *O*, passes the axis of double refraction. The following are the colours of the rings, reckoning from the centre *O*, with the values of the tints, as given by Newton, for the analogous colours of thin plates of glass, or any body whose index of refraction is 1.55. The numbers are millionths of an inch, and shew that similar colours would be produced by films of glass, &c. that have the thicknesses in the table.

FIRST RING.	Values of the Tints.	SECOND RING.	Values of the Tints.
Black, - - -	0	Violet, - - -	$7\frac{1}{5}$
Blue, - - -	$1\frac{1}{2}$	Blue, - - -	9
White, - - -	$3\frac{2}{5}$	Green, - - -	$9\frac{5}{7}$
Yellow, - - -	$4\frac{2}{5}$	Yellow, - - -	$10\frac{2}{5}$
Orange, - - -	$5\frac{1}{6}$	Orange, - -	$11\frac{1}{9}$
Red, - - -	$5\frac{1}{5}$	Scarlet, - - -	$12\frac{2}{3}$

THIRD RING.	Values of the Tints.	FOURTH RING.	Values of the Tints.
Purple,	$13\frac{1}{2}$		
Blue,	$15\frac{1}{10}$	Bluish Green,	22
Green,	$16\frac{1}{4}$	Green,	$22\frac{3}{4}$
Yellow,	$17\frac{1}{2}$	Yellowish Green,	$23\frac{2}{9}$
Red,	$18\frac{5}{7}$	Red,	26
Bluish Red,	$20\frac{2}{3}$		

If the plate of glass $E D$ is transparent, and if the eye is placed behind it so as to receive the transmitted rays, it will perceive a system of rings in which all the colours are complementary to those of the reflected system, the *cross* being now *white*, and the colours as follows.

First Ring.	Second Ring.	Third Ring.	Fourth Ring.
White,	White,	Green,	Red,
Yellowish Red,	Yellow,	Yellow,	Bluish Green.
Black,	Red,	Red,	
Violet,	Violet,	Green.	
Blue.	Blue.		

When the thickness of the crystal which produces these rings is *increased*, the rings become *smaller* in diameter, but retain the same series of colours; and when the thickness of the crystal is *diminished*, the rings *increase* in diameter.

If we take a *positive* crystal which produces a system of rings, and place it upon a *negative* crystal which produces a system of the same size, the difference of their effects will be produced; the one will destroy the effects of the other, and no ring whatever will be produced; if, on the contrary, a positive crystal is joined with a positive, or a negative with a negative crystal, the sum of their effects will be produced. Hence it follows that positive and negative crystals produce systems of rings of an opposite character, like positive and negative electricity, or like north polar and south polar magnetism.

The systems of rings now described differ in size in different crystals, even at the same thickness. In order, for example, to produce with *Beryl* a system of rings of the same size as those produced by a given thickness of *Calcareous spar*, we must take a much greater thickness of beryl. The following are nearly the relative intensities of the forces which produce the rings in a few crystals:—

Calcareous spar,	35
Rock crystal,	2
Beryl,	1

The best crystal for exhibiting the rings without any trouble, is the *Iceland spar* or *Carbonate of lime*, called *basée* by Hauy, and consisting of thin triangular plates, whose faces are perpendicular to the axis of the rhomb, or that of double refraction.

The systems of rings above described, I discovered in the year 1813, in *Ruby*, *Emerald*, and *Beryl*. Dr. Wollaston found them in carbonate of lime in 1814, and I discovered them in all the other crystals in the Table given in p. 354.

SECT. V.—*On the Double System of Rings produced by Crystals with two Axes of Double Refraction.*

In the year 1813 I discovered the double system of rings in *Topaz*, *Nitre*, and other crystals; and in the *Philosophical Transactions* for 1814 I have described the different phenomena which they exhibit in *Topaz*. This system of rings, however, is best seen in plates of *Nitre* or *Saltpetre* about $\frac{1}{12}$th or $\frac{1}{15}$th of an inch thick, and cut by planes perpendicular to the axis of the hexahedral prism. When any of these plates is placed at $c f$, Fig. 10, and as near C as possible, the eye at V will, in four positions of the plate of *Nitre*, 90° from one another, see the *double system of rings* shewn in Fig. 12, each system surrounding the poles $P P'$, and the whole intersected by a black cross $A B$, $C D$. At other four positions of $e f$, 90° from each other, and 45° distant from each of the first four positions, the eye will observe the system shewn in Fig. 13, where the black cross is now changed into two hyperbolic branches $M P N$, $M' P' N'$, passing through the poles P, P'.

In different crystals the distance between the poles $P P'$ varies, being very small in some, and very great in others, as in the following Table :[4]—

Carbonate of lead, -	5°	15′	Sulphate of strontian,	50°	0′
Nitre, - -	5	20	Sulphate of lime,	60	0
Arragonite, -	18	18	Topaz, - -	65	0
Cymophane, -	27	51	Carbonate of soda,	70	0
Sulphate of barytes,	37	42	Carbonate of potash,	80	0
Stilbite, -	41	42	Muriate of copper,	85	0
Mica, - -	45	0	Sulphate of iron, about	90	0

In examining the phenomena of crystals with two axes, I have been led to consider the poles $P P'$, or the centres of the

[4] A Table containing 58 crystals will be found in the *Edinburgh Encyclopædia*, Art. *Optics*, vol. xv, p. 592.

rings, as *apparent* or *resultant axes*, and to regard the real axes as always at right angles to one another, the one passing through *O* being always the principal one. In this way I was led to the following general law, by which all the phenomena could be calculated and represented in the most accurate manner.

The tint produced at any point of the sphere by the joint action of two *rectangular axes, is equal to the diagonal of a parallelogram whose sides represent the tints produced by each axis separately, and whose angle is double of the angle formed by the two planes passing through that point of the sphere and the respective axes.*

From this general law, it follows that one system of rings may be produced by two equal rectangular axes of an opposite name, so that the above law includes the law of Huygens for one axis.

It follows also from the above law, that *Three* equal and rectangular axes will be *in equilibrio*, and will destroy one another's action.

SECT. VI.—*On Circular Polarisation in Rock Crystal, Amethyst, and certain Fluids.*

When we look at the positive system of rings along the axis of a *Quartz* crystal, the rings are distinctly seen, but the black cross *A B*, *C D*, Fig. 11, is obliterated as in Fig. 14, and the central space is filled up with an uniform tint of the colours mentioned in p. 361, the tint passing through all the series of these colours, as the plate of quartz increases in thickness from a thin film. When the plate is extremely thin, the black cross is seen; as it becomes thicker, a pale bluish-white light next covers it, then white, yellowish-white, &c. Upon examining these colours, it was observed that they differed in their properties from those which composed the rings, since the tints along the axis of quartz varied in colour as the plate *C D* was turned round the polarised ray *S T*.

M. Biot endeavoured to explain this fact by supposing the axes of polarisation of the luminous particles to have a progressive rotation, and that the rapidity of this rotation increased with the refrangibility of the rays. He concluded also that this property belonged to the ultimate particles of silex, and could not be taken from them. M. Biot also found that

some crystals of quartz, which have been called *left-handed* by Mr. Herschel, cause the tints to descend by turning the plate *C D* from *right to left*, while others, called *right-handed*, cause the tints to descend by turning the plate *C D* from *left to right*.

In examining the structure of the *Amethyst*, I found that this mineral was composed of *two kinds of quartz*, one of which turned the planes of polarisation from *left to right*, and the other from *right to left*. These two kinds of quartz are arranged in veins, as represented in Fig. 15. The shaded veins, which correspond to each alternate face of the pyramid, turn the planes of polarisation from *right to left*, while all the rest of the crystal turns the same planes from *left to right*; and, what is very interesting, the black lines where these two structures unite have no action whatever on the planes of polarisation. In some specimens these opposite veins are so minute, that they destroy each other's action upon the polarised ray, and when this happens, the single system of rings appears with its black cross, and entirely free of any of the tints of circular polarisation. The *colouring matter* of the *Amethyst* is arranged in a very singular manner in relation to these veins; and the fracture across the veins exhibits a beautiful, and sometimes a regular rippled structure, resembling the engine-turning on a watch, and affords an infallible mineralogical character of the amethyst, whether it is *yellow*, *orange*, *olive*, *green*, *blue*, or perfectly *colourless*.

In order to determine whether or not the phenomena of circular polarisation, as M. Biot maintained, arise from the ultimate particles of silex, I examined *Opal* and *Tabasheer*, but could not detect the slightest trace of such a property. I then conceived the idea of examining *quartz that had been melted by heat*, and having received from Dr. Hope a piece which that eminent chemist had reduced to fusion, I found that it did not exhibit the slightest trace either of common or circular polarisation.

A very important discovery relative to circular polarisation was some time ago made by Mr. Herschel. Impressed with the idea that this property might be related to some crystalline structure in quartz, he examined different specimens of that variety which is called by Hauy *Plagiedre*, and which possesses unsymmetrical faces x, x, x, x', x', x', (Fig. 16) that always *lean in one uniform direction* round the summit A, but

sometimes to the *right* and sometimes to the *left;* he submitted these crystals to examination; and though M. Biot, as Mr. Herschel remarks, assures us that no peculiarity in the crystalline form can lead us to conjecture the direction of rotation, yet he found in every case, after examining *fifty-three* different crystals, that the direction in which they turned the planes of polarisation was invariably the same as the direction in which the plagiedral faces leant round the crystal. Hence Mr. Herschel concludes that *these faces are produced by the same cause which determines the displacement of the plane of polarisation of a ray traversing the crystal parallel to its axis.*

M.M. Biot and Seebeck discovered about the same time that certain fluids exhibited in a very weak degree the phenomena of circular polarisation; but it is to the former that we owe the analysis of this class of phenomena. Having filled a tube six or seven inches long with oil of turpentine, and placed it in the apparatus shewn in Fig. 5, he observed the complementary colours, which had the same property as those produced by a plate of rock crystal, which turned the planes of polarisation from right to left. By trying other fluids, he obtained the following results:—

From Right to Left.	From Left to Right.
Oil of turpentine.	Essential oil of lemons.
Essential oil of laurel.	Solution of natural camphor in alcohol.
Solution of artificial camphor in alcohol.	Syrup of sugar.
Vapour of turpentine.	

M. Biot found that the tints rose in the scale as the thickness of the fluid was increased; that oil of turpentine, mixed with another fluid that has not the power of acting upon light, gave a tint proportional to the number of particles of turpentine in the mixture; and that, when two fluids that turned the planes of polarisation in opposite directions were mixed in quantities reciprocally proportional to the intensity of their action, they neutralised one another. He found also that oil of turpentine required to have a thickness of $68\frac{1}{2}$ millimetres to give the same tint as a single millimetre of quartz. For farther information on this subject see the *Mem. de l'Institut*, 1818; *Memoirs of the Cambridge Society*, vol. i, part 1; *Edinburgh Philosophical Journal*, vol. iv, p. 371–433; vol. vi, p. 379; and the *Edinburgh Encyclopædia*, vol. xv, p. 597, &c.

SECT. VII.—*On Singly Polarising Crystals.*

In all the crystals of which we have hitherto spoken, two images polarised in opposite planes are produced by double refraction. I found, however, in 1812, that the *Agate* had the singular property of producing only one image, polarised in one plane; and I afterwards discovered an analogous property in the *massive Carbonate* of *Barytes*, in certain plates of *Mica*, and in *Mother of Pearl.*

In all these cases, I found that the light of the second image was either scattered or absorbed, or that it was converted into a nebulous mass of light; and I observed some very singular appearances produced by this cause in *Oil of mace*, melted and cooled between two plates of glass.

In 1815, M. Biot and M. Seebeck discovered an analogous property in the *Tourmaline.*

In prosecuting these experiments on singly refracting crystals, I found that the property of single refraction could be communicated to them artificially, and I succeeded in doing this with *Calcareous spar*, *Arragonite*, *Nitre*, and many other crystals.

This effect is produced by inducing a great degree of roughness on the surfaces, and cementing upon them a plate of glass, by means of an oil or a balsam of the same refractive power as either the ordinary or the extraordinary ray. By this means the other ray is either diminished in intensity, rendered nebulous, or obliterated. See *Phil. Trans.* 1819, p. 146, or *Edinburgh Encyclopædia*, vol. xv, p. 600.

SECT. VIII.—*On the Dichroism of Crystals, and the Absorption of Polarised Light.*

The term *Dichroism*, from δυο *two*, and χροα *colour*, I have applied to denote the two colours which Dr. Wollaston long ago observed in *Tourmaline* and certain crystals of *Palladium*, which Cordier afterwards observed in the *Iolite* or *Dichroite*, and which Bournon noticed in several specimens of *Mica*. The same phenomenon I found in *Augite*, where one of the pencils is a *deep blood red*, and the other a *brilliant green*, and in *Sapphire*, *Idocrase*, and many other minerals. In Iolite, the two colours are a beautiful *blue* and a *yellowish-brown*. These two colours are seen by looking through the crystal

in different directions. The caüse of this phenomenon was entirely unknown till I investigated its origin in the *Iolite*, and shewed that the two colours were related to its two resultant axes of double refraction, and arose from the absorption of *common light*, according to laws depending on the position of the incident ray in reference to its axes of double refraction.

The absorption of *polarised light* is still more interesting. If we take a prism of certain specimens of *Beryl* of a bluish-green colour, and expose it to polarised light, as at *e f* (Fig. 10), we shall find that it transmits only a beautiful *blue* light when its axis is horizontal or perpendicular to the plane of polarisation, and only a *greenish-white* light when the axis is perpendicular to the horizon or coincides with that plane, the transmitted light passing from the *former* to the *latter* tint, while the crystal is moving from the *first* into the *second* position. Hence it is obvious, that the *green* light is absorbed in *one* position, and the *blue* light in the *other* position. This absorption varies with the angle which the polarised ray forms with the axis of the prism, being a maximum when that angle is 90°, and vanishing altogether when the ray passes along the axis.

If we now cut the crystal of *Beryl* into a prism, so as to separate its two images, we shall find that these two images have different colours, the one having the same colour that would have been produced by exposing it in one position to polarised light, and the other image having the colour that would be obtained by turning it round 90°. Here, then, we have two singular properties of this class of crystals, which always appear to accompany one another, viz. the property which the extraordinary refracting force possesses of selecting certain rays out of the compound beam of common incident light, and the property of absorbing these rays in one position, and the complementary rays in another position, when the incident light has been previously polarised.

The property which has now been described as belonging to *Beryl*, I have found in other twelve crystals with *one axis*, and in a great variety of crystals with *two axes*, such as *Topaz*, *Mica*, *Sulphate of Barytes*, *Kyanite*, *Epidote*, &c. See *Phil. Trans.* 1819, p. 11, and *Edin. Phil. Journal*, vol. ii, p. 346.

SECT. IX.—*On the Polarising Structure produced by the Transmission of Heat through Glass.*

The different phenomena which are described in this section were discovered by me in the years 1814 and 1815, and have been described at very great length in the *Phil. Trans.* for 1816. A very brief notice of them only can be given in a popular work like the present.

If we take a plate of glass, *A C B D* (Fig. 11) perfectly cylindrical, and transmit heat from its circumference to its centre, we shall find that it will exhibit, when exposed to polarised light, and when the heat has reached its centre, the system of rings shown in the figure traversed by a black rectangular cross *A B*, *C D*. This system of rings is precisely the same, both in appearance, and in the character of its tints, as the system seen along the axis of *Zircon*, *Ice*, and other crystals of the *positive* class.

If the circular plate of glass, on the contrary, is immersed in boiling oil, or equally heated in any other way, and is allowed to cool rapidly, it will exhibit a similar system of rings; but this system has a *negative* polarisation, like the rings formed by *Calcareous spar*, *Beryl*, &c. and other crystals of the *negative* class.

This opposition in the character of the two plates may be well observed, by combining them together, as already described in page 361.

By comparing the value of the tints (see p. 361) with their distances from the centre of the plate, I found that they vary as the squares of their distances from the axis.

Let a well annealed rectangular plate of glass, *E F C D*, having no polarising structure (Fig. 17), be now placed with its lower edge *C D* upon a thick bar *A B* of red hot iron, and exposed to polarised light in the usual way. The entrance of the heat into the glass at *C D* will be marked by fringes of coloured light at *C D*; and *nearly at the same instant, and before a single particle of heat has reached the upper edge E F, or even a b, corresponding fringes will appear at E F*, and also at *a b*, and the fringes will become more distinct till they resemble those seen in Fig. 17, where *M N*, *O P*, are two black lines, where there is neither double refraction nor polarisation. Be-

tween *O P* and *M N* the doubly refracting and polarising structure is *negative*, because the fringes descend in the scale when crossed by the axis of a plate of sulphate of lime ; while the structures between *O P* and *E F*, and between *M N* and *C D*, are *positive*.

If the plate of glass is immersed in boiling oil, and then allowed to cool rapidly, it will exhibit the same fringes, &c ; but the central structure will now be *positive*, and the two external structures *negative*.

When the heat is uniformly distributed over the plate, in both the preceding cases, the fringes are no longer visible. In all these cases, similar fringes and structures are seen through the thickness of the plates, and also in the direction of their length. When the plate of glass is nearly of a square form, and its thickness about one-third of the length of one of its sides, these fringes assume the form shewn in Fig. 18, and when the length of the piece of glass is about three times that of one of the sides, the fringes have the form shewn in Fig. 19.

All the tints displayed in these fringes resemble, in every respect, the tints polarised by doubly refracting crystals, and follow the same laws in their combination.

If we make a cut with a diamond across the line *a b* of a plate of glass *C D E F* (Fig. 17), and having placed it on the hot iron *A B*, break it into two pieces when the fringes are fully developed, we shall find that *each half of the plate has the same structure as the whole plate had before*. The heat, whose unequal distribution, producing an inequality of density (according to a certain law indicated by the progression of the tints), has its distribution instantaneously altered, and a portion of it escapes during the change. This effect, which is one of the most singular facts in natural philosophy, is shown in Fig. 20, which represents the arrangement of the tints, when the glass is divided along the line *r s*, each plate having two lines of no polarisation, and *one negative* between *two positive* structures, exactly like the original plate before it was divided. If the plate has been divided in the direction *C D*, the angular fringes will start up at the two new angles which are formed by the division of the plate.

This experiment may be performed in a still simpler manner, by taking two separate plates of glass, *E F s r*, *G H s r*, Fig. 20. When these plates are pressed together with the force of

a screw, and then laid with the edge GH on a plate of hot iron, the heat will distribute itself as if the whole constituted one plate, and produce the effect shewn in Fig. 17; but the moment the plates are separated, the heat will distribute itself as in Fig. 20.

By *combining* a *positive rectangular plate*, or one in which the principal axis is *positive*, with another positive rectangular plate, so that the lines of no polarisation are parallel, the effect of each will be combined as if a plate had been used equal to the sum of their thicknesses, and the fringes on each side of the black lines will increase in number.

If we combine a *positive rectangular plate* with a similar *negative rectangular plate*, the effect of the one will counteract that of the other. If they have equal actions, the polarising structure of the one will destroy that of the other; but if their actions are unequal, the effect will be the same as if a plate had been used of the same thickness as the difference of their thicknesses, and having the structure of the thickest of the two combined plates.

When two positive or *two negative* rectangular plates are crossed, as shewn in Fig. 21, the tints are in some places *raised* in the scale, and in others *depressed*, according as opposite or similar structures are opposed to one another, the crossing of two positive or two negative structures sinking the tints in the scale, and the crossing of a positive with a negative structure raising them. By finding the tint at any given point in each plate from the preceding formulæ, and combining these tints according to the rule already given, it will be found that the *isochromatic curves* (or curves of equal tint) at the intersectional space $A\ B\ C\ D$ are *Hyperbolas*, which will be *equilateral* when the breadths and the maximum tints of the two plates are the same.

When a *positive rectangular plate crosses a negative plate*, it will be found, by the same process, that the isochromatic curves are *Ellipses*, as in Fig. 22, when the plates are of unequal breadths; and that they become *circles* when the plates and maximum tints are equal, as in Fig. 23.

All these different phenomena, which I first observed, are deducible mathematically from my formula in the *Edinburgh Transactions*, vol. viii, p. 357.

We have already seen that *circular plates*, or *cylinders* of

glass, have *one positive axis*, like *zircon* ; but when the cylinder has the form of a tube, like *A C B D* (Fig. 24), the polarising force is distributed in a very remarkable manner. A black circular fringe *m p n o* forms the line of no polarisation, and the coloured fringes are placed on each side of this dark ring, and concentric with it. The structure on the outside of *m p n o* is *positive*, like *zircon*, &c. and the structure on the inside *negative*, like *calcareous spar*.

If a tube of glass is brought to a red heat, and then cooled by inserting in its bore a cylinder of iron, or any other conducting body, the structure will then be the same as is represented in Fig. 25.

If a solid cylinder of glass which has only one structure is perforated in its centre, it will exhibit the appearance in Fig. 24.

When the tints are arranged in a glass cylinder, as in Fig. 24, take a file with a very sharp edge, and cut the tube entirely through by a notch *E F* (Fig. 25). By this operation the particles will be freed from the state of violence in which they are held, and will assume the very same arrangement which they never fail to take in rectangular plates of glass. By exposing the tube thus divided to polarised light, it will exhibit the appearance shewn in Fig. 25, where *m p n o*, *m′ p′ n′ o′*, are two dark fringes having a negative structure on the outer side of each, and a positive structure between them, as in plates of glass with two axes.

Sect. X.—*On the Communication of the Polarising Structure by Compression and Dilatation.*

On the 3d of January 1815, I discovered that soft animal substances, such as calves' foot jelly and isinglass, acquire from simple pressure that peculiar structure which enables them to form two images polarised in an opposite manner, like those produced by doubly refracting crystals, and to exhibit the complementary colours of regularly crystallized minerals. This effect was observed in a cylindrical piece of calves' foot jelly which could scarcely support its own weight, and which had no action upon polarised light; but whenever it was pressed between the finger and the thumb, or even touched gently by the finger, it displayed the properties of the polarising structure.

During subsequent experiments on this subject, in October 1815, I observed that *compression* produced a *negative* po-

larising structure, and *dilatation* a *positive* polarising structure; and by dilating isinglass I created a polarising structure more powerful than that which is possessed by beryl.

These experiments, which had been confined to soft substances, I extended, on the 1st November 1815, to plates of solid glass; but finding it difficult to apply regular forces to such a hard body, I thought of developing the polarising structure by *bending plates of glass, whose edges were ground and polished.* In this way I succeeded in exhibiting the phenomena in the most simple manner.

If we take any slip of glass, cut merely with a diamond, and holding one end of it in each hand, bend it slightly, we shall observe, through its edges, when exposed to polarised light, two separate structures *A B N M*, *C D N M*, Fig. 26, separated by a dark line *M N*; and each of them covered with coloured fringes, the scale of which commences at *M N*. When the axis of a plate of sulphate of lime is made to cross these fringes, those in *C D N M* on the concave side will *rise* in the scale, and will therefore be *positive*, while those in *A B N M* on the convex side will *fall* in the scale, and will therefore be *negative*. By measuring the breadth of the fringes, the tints were found to vary as their distance from the axis. By the application of great forces I succeeded also in altering the polarising structure of regularly crystallized bodies, and in communicating that structure where it did not previously exist.

For a full account of these experiments, see the *Phil. Trans.* 1816, p. 156, and the *Edin. Trans.* vol. viii, p. 281.

Sect. XI.—*On the Polarisation of Light by Metals, and by the Second Surfaces of Transparent Bodies.*

The discovery of the polarisation of light in two opposite planes by polished metals was independently made by Malus and myself, but the priority is due to Malus, who concluded from his observations, that while transparent substances refract all the light polarised in one plane, and reflect all the light polarised in the opposite plane, metallic bodies reflect what they polarise in both planes.

In examining the effects produced by successive reflexions from metallic surfaces, I discovered that they possessed the singular property of producing, when exposed to polarised light,

the phenomena of the complementary colours, and of moveable polarisation, like crystallized bodies.

Let us suppose that two parallel plates of highly polished silver, about three or four inches long, and half an inch broad, are fixed at the distance of about half an inch, and are interposed at *e f*, Fig. 10, between the polarising plane *A B* and the analyzing plate *C D*, and that the silver plates can be turned round, so that the plane of reflexion may form any angle with the plane of primitive polarisation *A S T*. If the plane of reflexion from the silver plates is either parallel or perpendicular to the plane of primitive polarisation, the action of the plates upon the polarised ray will be nothing, that is, the ray will retain its primitive polarisation, and will be colourless, however great be the number of reflexions. In every other position, however, of the plane of reflexion from the silver plates, and at every angle of incidence, the polarised ray will be divided into two portions, *O* and *E*, one of which, *O*, retains its primitive polarisation, while the other, *E*, is polarised in an angle equal to 2 *a*, or twice the azimuth of the plane of reflexion. When *a* is 45°, the tint *E* is a maximum, just as in plates of regularly crystallized bodies; the azimuthal angle of the plane of reflexion in the former, replacing the azimuth of the axis in the latter.

When a polarised ray is reflected from a single metallic surface in the manner now described, it experiences the same modification as if it had passed through a plate of any crystallized body of a certain thickness. If the action of the metallic surface is combined with that of a plate of sulphate of lime, having its axis coincident with the plane of reflexion, the colour polarised by the system will be that which is due to the sum of the thicknesses of the crystallized plate, and the equivalent plate of the same substance; but if the axis of the plate is at right angles to the plane of reflexion, the colour polarised by the system will be that which belongs to the difference of the thicknesses of the crystallized plate and the equivalent plate. The same is true of two or more metallic reflexions, each reflexion being equivalent to a plate of a crystallized body of a given thickness, their thickness varying with the angle of incidence; and if the angle of incidence varies, the thickness of the equivalent plate always increases as the angle of incidence upon the metal diminishes, or the depth to which the incident

ray penetrates the metallic surface increases as it approaches to the perpendicular.

The same effect is produced by successive reflexions from *Gold*, and, in an inferior degree, from *Platinum*, *Steel*, *Brass*, *Copper*, *Tin*, *Lead*, *Mercury*, *Metal for specula*, and *Amalgam of Bismuth*.

When a ray of *common light* has suffered a number of reflexions from polished plates of silver, I found, that even when the number of reflexions was thirty-six, the emergent pencil consisted of two pencils polarised in opposite planes. A portion of the most refrangible rays was absorbed at each reflexion, so that the resulting pencil was of a *deep red* colour. As one of the images was decidedly fainter than the other, the pencil would have ultimately been all polarised in the plane of reflexion. Hence it follows, that the intensity of the pencil polarised in the plane of reflexion is greater than that of the pencil polarised in the opposite plane; but these two intensities approach nearer to equality in silver than in any other metal.

If common light is incident upon *Steel*, and all the other metals except *gold* and *silver*, *five* or *six* reflexions at an angle of 70° are sufficient to polarise the whole of the incident pencil in the plane of reflexion. Hence it follows, that in all these metals the pencil polarised in the plane of reflexion exceeds greatly in intensity that which is polarised in an opposite plane, a great portion of this last pencil having been absorbed by the substance of the metal.

On the polarisation of light by total reflexion.

The discovery of the polarisation of light in two opposite pencils by the action of the forces which produce total reflexion, was made by me in 1814, and explained in my paper on the Polarisation of Light by Reflexion.

The experiments by which I ascertained this property were conducted in a manner similar to those of Malus upon polished metals. A ray of polarised light was found to be depolarised by total reflexion, when the plane of total reflexion was inclined 45° to the plane of primitive polarisation, and in intermediate degrees at different azimuthal angles, excepting when the azimuths are 0°, 90°, 180°, and 270°, or when the plane of total reflexion is parallel or perpendicular to the plane of primitive polarisation.

During these experiments I likewise discovered that the complementary colours of moveable polarisation were produced by total reflexion in a manner analogous to those produced by metallic surfaces; one reflexion appearing to correspond to an equivalent plate of a crystallized body of a certain thickness. The effect is increased by increasing the number of reflexions in the same plane; but when the two reflexions are in planes at right angles to each other, the actions counteract each other. These effects I attributed to the circumstance of one of the pencils being later in suffering reflexion than the other, the first being under the influence of the force that produces the usual partial reflexion, and the other, after beginning to be refracted, being caused to return by the continued operation of the same power.

For farther information respecting Metallic Polarisation, See my *Treatise on New Phil. Instruments*, Pref. p. xiii, xiv, and p. 347; and *Biot's Traité de Physique*, tom. iv.

Sect. XII.—*On the Influence of the Surfaces of Crystals on the Light which they reflect.*

It has been remarked by Malus, " that the action which the first surface of *Iceland spar* exercises upon light, is independent of the position of its principal section; that its reflecting power extends beyond the limits of the polarising forces of the crystal; and that, as light is only polarised by penetrating the surface, the forces which produce extraordinary refraction begin to act only at this limit." He also observes, that " the angle of incidence at which Iceland spar polarises light by partial reflexion is 56° 30′; that it then comports itself like a common transparent body; and that, whatever be the angle comprehended between the plane of incidence and the principal section of the crystal, the ray reflected by the first surface is always polarised in the same manner."

In order to examine with care the superficial action of *Calcareous spar*, I exposed several surfaces by cleavage, and having selected the one that had the most perfect polish, I covered all the other sides of the rhomb with black wax, and measured the polarising angles in planes variously inclined to the principal section. The following are the results of a great number of observations:

Position of the Crystal.	Azimuth.	Polarising angle.
Short diagonal in plane of reflexion -	0°	57° 14′
One of the edges in plane of reflexion	50 57½′	58 32
Long diagonal in plane of reflexion -	90	59 32

Difference between the greatest and least polarising angle 2° 18′

In these experiments the results were the same, whether the obtuse angle of the rhomb was nearest or farthest from the eye, or whether it was to the right or left hand of the observer.

Change in the plane of polarisation by weakening the reflecting force.

Since the extraordinary force in calcareous spar was thus shewn to extend to such a distance beyond the surface as to modify the polarising angle produced by superficial reflexion, it became extremely probable that the polarisation of the reflected ray might suffer some change from the same cause. I accordingly introduced a film of oil of cassia between a glass prism and the surface of the spar, and having inclined the prism at a very small angle to that surface, I thus separated the image formed at the common surface of the prism and the oil, from the image formed at the common surface of the oil and the spar. The effect was exactly what I had anticipated. The influence of the ordinary reflecting force was reduced almost to nothing, and the light reflected from the separating surface of the oil and the spar, was polarised at an angle of about 45½°, and was almost entirely under the dominion of the force which emanated from the axis. The following were the results obtained with an ordinary surface, inclined 45° 23½′ to the axis.

1. *Azimuth* 0°. When the plane of the principal section is in the plane of reflexion, the light reflected at the surface of the oil and the spar is polarised in the plane of reflexion, the obtuse solid angle being farthest from the eye. The light of the image is of a faint red colour, and has very little intensity.

2. *Azimuth* 12°. The obtuse angle being farthest from the eye, the reflected pencil is polarised about 45° out of the plane of reflexion.

3. *Azimuth* 42°. The reflected pencil is polarised transverse or at right angles to the plane of reflexion, or 90° out of it. The light is now of a yellowish white ttin, and is much more intense than in azimuth 0°.

4. *Azimuth* 90°. When the plane of reflexion is perpendicular to the plane of the principal section, the obtuse solid angle being either to the right or left hand, the reflected pencil is polarised a little more than 135°, or — 45° out of the plane of reflexion. The intensity of the pencil is now intermediate between that of azimuth 0° and 45°.

5. *Azimuth* 180°. The obtuse angle being now next the eye, the pencil is polarised 180° out of the plane of reflexion, or it has again returned into that plane.

In passing through the last 45° of azimuth, the polarisation varies very slowly, the change being only about 10° : whereas, in passing through the first 42° of azimuth, the polarisation varies no less than 90°, indicating that this change depends upon the angle which the incident ray forms with the axis of the crystal. See *Phil. Trans.* 1819, p. 145, or the *Edinburgh Encyclopædia*, vol. xv, p. 608.

Sect. XIII.—*On the Structure of the Tesselated Apophyllites.*

In ordinary crystals every portion, however small, has the same property as the whole, and the forces of double refraction and polarisation have the same character and the same intensity in all parallel directions. A very singular exception to this general law presented itself to me while examining the *Apophyllites* of Faroe.

When we remove the uppermost slices from each of the two summits of the crystal, to the thickness of the 100th of an inch or more, and examine it either by the microscope or by polarised light, we perceive no tesselated structure, and this slice has only *one axis* of double refraction.

If we now remove the next slice, and all subsequent slices, we shall find that they exhibit by polarised light, and also by the microscope, under favourable circumstances of illumination, the beautiful figure represented in Fig. 27. The outer case $M\,O\,N\,P$, which binds the interior parts together, is composed of a great number of parallel veins, which, from their minuteness, display the colours of striated surfaces. This external coating envelopes no fewer than *nine* separate crystals, viz. the central lozenge $a\,b\,c\,d$, which has *one axis* of double refraction ; the four prisms A, B, C, D, with trapezial bases, which have *two axes* of double refraction ; and the four triangular prisms $e\,h\,l$, $l\,m\,n$, $n\,k\,g$, $g\,f\,e$, all of which are separated

from one another by distinct veins. The inflected lines *e h l*, *l m n*, *n k g*, *g f e*, are most easily seen by the microscope. The central lozenge is seen much less frequently, and the radial lines *h a*, *c k*, *f d*, *b m*, require a particular mode of illumination to be distinctly recognised, though they are easily seen by polarised light. In the quadrants *A M* and *D N*, the planes of the two resultant axes are coincident, and lie in the line *M N*; and in like manner in the quadrants *B O*, *C P*, the plane of the axes is in the line *O P*. The consequence of this is, that when a plate of sulphate of lime has its axis coincident with *M N*, it depresses the tints in *A* and *B*, and raises the tints in *C* and *D*, while the central lozenge and the radial lines have the same tint as the plate of sulphate of lime.

One of the Faroe varieties of *Apophyllite* which occurs in complete and transparent crystals, exhibits through both its pair of parallel longitudinal faces the very remarkable tesselated structure shewn in Fig. 28, consisting of the most splendid colours, arranged like the finest mosaic, and with the most perfect symmetry in relation to the centre of the crystal. The existence of the curvilineal solid in the centre;—the gradual diminution in the length of the circumscribing plates, in consequence of which they taper, as it were, from the angles of the central square to the truncated angles at the summits; but, above all, the reproduction of similar tints on each side of the central figure, and at equal distances from it, cannot fail to strike the observer with surprise and admiration. A full account of the singularities of apophyllite will be found in the *Edinburgh Philosophical Journal*, vol. i, p. 1, and *Edinburgh Transactions*, vol. ix, p. 317. See also the *Transactions of the Cambridge Society*, vol. ii, p. 1, for an account of the tesselated structure of Brazilian Topaz.

Sect. XIV.—*On the Multiplication of Images, and their accompanying Colours in Calcareous Spar and other Crystals.*

The multiplication of the images in some specimens of *Calcareous spar* was first observed by Bartholinus, and afterwards described by Huygens, Benjamin Martin, Dr. Robison, Mr. Brougham, and Malus.

The crystals of *Calcareous spar* which possess this property are intersected with one or more planes *A B C D*, Fig. 29, *e b c g*, *a f h d*, perpendicular to the long diagonals *E F*, *G H*,

of the rhomboidal faces, and parallel to the edges *E G*, *F H*. When we look through the two parallel faces of the crystal at a candle, so that the light may pass through one of these planes, we observe the two principal images *A*, *B*, Fig. 30, two secondary images *a*, *b*, sometimes *four*, viz. *a*, *b* ; *a′*, *b′*, and sometimes *six*, *a*, *b* ; *a′*, *b′* ; *a″ b″*. When there are more than one interrupting plane, the images are often doubled, or multiplied to such a degree that the eye observes, in favourable positions of the rhomboid, heaps of images of the most beautiful kind, sometimes varying in the intensity of their light, sometimes vanishing, and sometimes reappearing. These images are in general highly coloured with all the tints of the spectrum, though in some specimens no colours at all appear, excepting the prismatic tinges at their edges, arising from refraction ; and by varying the inclination of the rhomboid, the colours which affect the different images change in the most rapid and beautiful manner, running through the whole series of the colours of the polarised rings.

In order to explain these phenomena, Malus supposes *that there is a fissure, or crack*, within the crystal, and he considers the colours with which the images are tinged as the colours of a plate of air in the fissure or crack, and quite similar to the Newtonian colours of thin plates.

By examining with the microscope, and subsequently breaking up the specimens which produce the preceding phenomena, I found, that in *no case whatever* is there a *fissure, or a crack*, in the crystal. I ascertained, however, that the phenomena were produced by veins, or *strata of calcareous spar*, such as *A B C D*, &c. Fig. 29, always parallel to the long diagonal *E F*. I found these veins to vary from almost imperceptible films to plates of considerable thickness, and succeeded in determining that these strata were regular rhomboidal plates of the spar, having their axes in a constant position in relation to the axis of the rhomboid which contained them. After these results were established, all the phenomena produced by such interrupted crystals admitted of immediate explanation.

Let *M N*, Fig. 31, be the vein, or stratum, of calcareous spar interposed between the two prisms *A E B*, *C F D*, and, though it adheres firmly to these prisms by both its surfaces, let us suppose an open interval on each side of it. Let *R b* be a ray incident at *b*. This ray, after being refracted doubly,

will emerge in two pencils at c, c', and will enter the plate of spar $M N$. As the axis of this stratum is neither parallel nor perpendicular to the plane in which the rays $b c$, $b' c'$, are polarised, and as it is so thin as to produce the phenomena of moveable polarisation, each of these pencils will be divided into two complementary tints—suppose *red* and *green*, shewn by the double lines $d e$, $d' e'$ in the figure, one of which will retain its primitive polarisation, while the other is polarised in a double azimuth, as in all ordinary cases. These double pencils, emerging at e, e', and incident upon the second prism $C D F$ at f, f', will be divided, as in the figure, and will emerge at $g h$, $g' h'$, in the directions $g n$, $h m$, $g' n'$, $h' m'$, the two images formed by the rays m, m' being *red*, and the other two images formed by n, n' *green*. The various colours, in short, which affect these images, are portions of the single system of coloured rings which surround the axis of the plate $M N$; and the second prism $C D F$ is precisely an analyzing prism, which separates the tints polarised by the plate $M N$.

If any doubt could remain of the certainty of this explanation of the phenomena, I have removed it entirely, by actually composing artificial rhomboids, and interposing a plate of calcareous spar, having the same position as it has in the natural crystal. This artificial rhomboid produced all the various and beautiful phenomena which are displayed in the compound crystal. See the *Phil. Trans.* 1815, p. 270; *Edin. Trans.* vol. viii, p. 165; and the *Transactions of the Geological Society*, vol. v.

CHAPTER XX.

ON NEW DIALS.

1. *Description of an Analemmatic Dial, which sets itself.*

The analemmatic dial is represented by $C D$ in Fig. 32 of Plate XI, and is generally described upon the same surface with a horizontal dial $A B$, for the purpose of ascertaining its proper position, without the assistance of a meridian line or compass. It is always of an elliptical form, approaching to that of a circle, as the place for which it is made recedes from the equator. Its stile is perpendicular, and has different positions in the line ♋ ♑, changing with the declination of the sun, and

indicated by the names of the months marked upon its surface. From the obliquity of the stile of the one dial, and the rectangular position of the other, the motion of their shadows is so different, that the dial may be reckoned properly placed when the shadows of both stiles indicate the same hour.

Theory of the Dial. In order to understand the theory and construction of this dial, let $B E$ (Fig. 34) be its length perpendicular to the direction of the meridian. Having bisected $B E$ in A, make $A O$ equal to the sine of the latitude of the place; and with the cosine of the latitude as radius, set off $A D$ and $A C$ equal to the tangent of $23^{\circ} 28'$, the sun's greatest declination. The points D and C are the places of the stile in the time of the solstices, on the 21st of June and December; and if the tangent of the sun's declination for the first day of every month is set off in a similar manner between A and D and A and C, the points thus found will be the place of the stile on those days, and the radius $B C$ drawn from all these points to B will be the hour line of six at these different times.

In order to prove this, let $Z Æ N H$ (Fig. 35), be the meridian, $P p$ the six o'clock hour circle, and $P H$ the height of the pole, then $A Z S$ is the azimuth of the sun, and $P Z S$ its complement, $A S$ the sun's declination, and $P S$ its complement. Now, in the spherical triangle PZS right angled at P, we have by spherical trigonometry (Playfair's Euclid, Prop. XVIII.) Radius : Sin. $P Z =$ Tang. $P Z S$: Tang. $P S$, that is, Radius : Sin. $P Z =$ Co Tang. Azimuth : Co Tang. declination, for $P Z S$ is the complement of the azimuth, and $P S$ the codeclination; but as radius is a mean proportional between the tangent and cotangent (Def. IX, Cor. 1, plane trigonom.), the tangents will be in the reciprocal ratio of the cotangents, and consequently cotang. azimuth : cotang. declin. = Tang. declin. : Tang. azimuth. Therefore, Rad. : Sin. $P Z =$ Tang. declin. : Tang. : azimuth; and the sine of $P Z$ the colatitude, is the same as the cosine of the latitude.

Now, if $A C$ represents the six o'clock hour line when the sun is in the equator, and $A C$ the tangent of the sun's declination, for a radius equal to the cosine of the latitude, or $A C =$ Tang. declin. $\times$ cosin. latitude, the angle $A B C$ will be equal to the sun's azimuth, for from the last analogy, Tang. declin. $\times$ cos. latitude = Rad. $\times$ Tang. azimuth, therefore $A C =$ Rad. $\times$ Tang. azimuth; that is, $A C$ is equal to the

tangent of the sun's azimuth when $A B$ is radius; and consequently $A B C$ is the sun's azimuth since $A C$ is its tangent. If the sun were in the equator and the stile at A, his azimuth from the south would be $O A B$, whereas when the stile is at C, his azimuth is $O C B$, which is equal to $O A B - A B C$; therefore $A B C$ is the sun's azimuth from the east or west at six o'clock, and $B C$ the six o'clock hour line. In the same way it might be shewn, when the stile is placed in any point between C and D, that a line drawn from it to the point B will be the six o'clock hour line for that declination, and that the angle at B, comprehended between this line and $A B$, will be equal to the azimuth of the sun.

In order to determine the horary points and the circumference of the dial, we must consider, that if the equator be projected upon the horizon of any place, it will form an ellipse whose conjugate or shortest diameter is equal to the sine of the latitude of that place. Let $B M F$ (Fig. 36), therefore, be the equator projected on the horizon of a given place, so that $A M$, half the conjugate axis, is to $A B$, half the transverse axis, as the sine of the latitude of that place is to radius. Then having described the semicircle $B\,XII\,F$, divide the quadrants $B\,XII$, and $XII\,F$, into six equal parts for the hours, into 12 for the half hours, and into 24 for the quarters, each hour being 15 degrees in the daily motion of the sun, each half hour $7° 30'$, and each quarter $3° 45'$, and from these points, from the point III, for example, draw $III\,C\,E$ parallel to $A\,XII$, or perpendicular to $A B$, the point C where this line cuts the ellipse will be the horary point, and $D C$ will be the three o'clock hour line when the stile is at D.

As there is some difficulty, however, in describing an ellipse with accuracy, we shall shew how to find the horary points without describing this conic section. Take $B C$ (Fig. 33) equal to the breadth of the dial, and having bisected it in A, draw A 12 perpendicular to $B C$, and equal to the sine of the latitude, $A C$ being radius. Then upon the centre A, with the distance A 12, describe the semicircle D 12 F, and with the distance $A B$ the semicircle $C H B$. Divide the quadrant $H B$ into six equal parts for hours in the points m, n, o, p, q, and the quadrant 12 E into the same number of equal parts in the points a, b, c, d, e; and through a, b, c, &c. draw a 11, b 10, c 9, &c. parallel to $C B$; and through m, n, o, &c. draw

m 1, n 2, o 3, parallel to HA;—the points of intersection, 1, 2, 3, 4, 5, wi l be the horary points, and will be in the circumference of an ellipse. The horary points being thus known, it is not necessary to trace the ellipse, otherwise it might be easily done with the hand. If the divisions Hm, mn, &c. are subdivided into half hours and quarters, or even lower, the corresponding points in the ellipse 12 B may be determined in a similar manner

In order to demonstrate that C is the horary point of three o'clock, and DC the hour-line when the sun is at his greatest north declination, we must find from the construction the angle CDM, or the sun's azimuth, reckoned from the south, and see if the triangle PZS (Fig. 37), furnishes us with a similar expression of the angle Z, or sun's azimuth. In Fig. 36, CH, or its equal AE, is evidently the sine of the horary angle, AB being radius; and since CE or AH is the cosine of the horary angle, in a circle whose radius is AM, or the sine of the latitude, we will have CE or $AH =$ Cos. horary angle $\times$ Sin. lat. But according to the first part of the construction $AD =$ Tan. declin. $\times$ Cos. lat.; therefore DH, the difference between AD and AH, will be $=$ Cos. hor. angle $\times$ Sin. lat. — Tang. declin. $\times$ Cos. lat.; and the tangent of the angle CDH or $\frac{CH}{DH}$ will then be equal to

$$\frac{\textit{Sin. Hor. Angle}}{\textit{Cos. Hor. Angle} \times \textit{Sin. Latit.} - \textit{Tang. Decl.} \times \textit{Cos. Lat.}}$$

Now, in order to find a similar expression for the angle PZS (Fig. 37), let SO be a perpendicular upon PZ; and the sines of the segments PO, ZO, will be reciprocally proportional to the angles at the base P and Z (Playfair's *Spher. Trig.* Prop. XXVII); that is, Sin. ZO : Sin. $PO =$ Tang. P : Tang. Z; and therefore, Tang. $Z = \frac{\textit{Sin. } PO \times \textit{Tang. } P}{\textit{Sin. } ZO}$. But, Sin. $ZO =$ Sin. $\overline{PZ - PO}$[1] $=$ Sin. $PO \times$ Cos. $PZ -$ Sin. $PZ \times$ Cos. PO. Now, since Rad. : Tang. $=$ Sin. : Cosine, and since Cos. : Sin. $=$ Rad. : Tang. we have, by the rule of proportion, Sin. $PO =$ Cos. $PO \times$ Tang. PO; and Tang. $PO = \frac{\textit{Sin. } PO}{\textit{Cos. } PO}$

[1] See Trail's *Algebra*, Appendix, No. 6, *on the Arithmetic of Sines*, theorem ii.

Therefore, $\frac{Sin.\ P\ O}{Sin.\ Z\ O} = \frac{Cos.\ P\ O \times Tang.\ P\ O}{Sin.\ P\ O \times Cos.\ P\ Z - Sin.\ P\ Z \times Cos.\ P\ O}$

Dividing by Cos. $P\ O$ we have

$$\frac{Sin.\ P\ O}{Sin.\ Z\ O} = \frac{Tang.\ P\ O}{\frac{Sin.\ P\ O \times Cos.\ P\ Z}{Cos.\ P\ O} - Sin.\ P\ Z};$$ and since

$Tang.\ P\ O = \frac{Sin.\ P\ O}{Cos.\ P\ O}$, we shall have, by substitution,

$$\frac{Sin.\ P\ O}{Sin.\ Z\ O} = \frac{Tang.\ P\ O}{Tang.\ P\ O \times Cos.\ P\ Z - Sin.\ P\ Z}$$

Again, by Playfair's *Spher. Trigon.* Prop. XXI, Cos. P : Rad. = Tang. $P\ O$: Tang. $P\ S$, consequently Tang. $P\ O$ = Tang. $P\ S$ × Cos. P. Substituting, therefore, this new value of Tang. $P\ O$ in its room, in the last equation, multiplying the whole by Tang. P, and dividing by Tang. $P\ S$,[2] we shall have

$$\frac{Tang.\ P \times Sin.\ P\ O}{Sin.\ Z\ O} = \frac{Cos.\ P \times Tang.\ P}{Cos.\ P\ Z \times Cos.\ P - Sin.\ P\ Z \times Cos.\ P\ S}$$

But since Tang. : Rad. = Cos. : Sin.; Sin. P = Cos. P × Tang. P. By substituting Sin. P in place of its value, we shall have Tang. Z, or its equal,

$$\frac{Tang.\ P \times Sin.\ P\ O}{Sin.\ Z\ O} = \frac{Sin.\ P}{Cos.\ P \times Sin.\ P\ Z - Sin.\ P\ Z \times Cos.\ P\ S}$$

that is, by substituting the names of the symbols

$$Tang.\ Z = \frac{Sin.\ Hor.\ Angle}{Cos.\ Hor.\ Ang. \times Sin.\ Lat. - Tang.\ Dec. \times Cos.\ Lat.}$$

which is the same expression of the tangent of the sun's azimuth, or angle Z, as was deduced from the former construction.

The analemmatic dial being thus demonstrated, its construction will be better understood by taking an example. Let it be required, therefore, to construct one of these dials for latitude 56 degrees north, which nearly answers to Edinburgh. Let $A\ C$ (Fig. 33), be taken for half the breadth or radius of the dial, and let it be divided into 1000 parts, then A 12, which must be equal to the sine of the latitude, or 56 degrees, will be 829, which are the three first figures of the natural sine of 56 degrees in a table of sines. In order to find the points D, C, (Fig. 34) where the stile is to be placed at the solstices on the

[2] Since the tangents are in the inverse ratio of the cotangents, multiplying any number by the cotangent, is the same as dividing it by the tangent.

21st of June and December, take the tangent of 23° 28′, the sun's declination at that time, and it will be 434, if the radius were *A C* or 1000; but as the radius is the cosine of the latitude, which is 559, we must say as 1000 : 559 = 434 : 243, the length of *A D* and *A C*. On the 21st of February, April, August, and October, the sun's declination is nearly 11° 19′, the tangent of which for a radius of 1000 is 200; but for a radius of 559, the cosine of the latitude, it will be 112, which is the distance of the stile from *A* on both sides on the 21st of the months already mentioned. On the 21st of January, May, July, and November, the sun's declination is nearly 20° 8′ the tangent of which, for the radius 1000, is 367; but for the radius 559 it will be 205, which is the distance of the stile from *A*, on both sides, on the 21st of these months, the names of the months being inserted beside the points, as in Fig. 32. The horary points are now to be determined in the manner already mentioned, and the dial will be finished. In order to place the dial, we have only to turn it round till the stile of the analemmatic dial indicates the same hour with that of the horizontal one, and it will then be properly placed.

Description of a New Dial in which the Hours are at Equal Distances in the Circumference of a Circle.[3]

With any radius describe the circle *F X I I B* (Plate XI, Fig. 36), draw *A X I I* for the meridian, and divide the quadrants *F X I I*, *B X I I*, each into six equal parts for hours. To the latitude of the place add the half of its complement, or the height of the equator, and the sum will be the inclination of the stile, or the angle *D A C*. Thus, at Edinburgh, the latitude is 55° 58′, the complement of which, or the altitude of the equator, is 34° 2′; the half of which, 17° 1′, being added to 55° 58′, gives 72° 59′ for the inclination of the stile or the angle *D A C*. The position of the stile in the figure is that which it must have on the 21st of March and September, when the sun crosses the equator; but when the sun has north declination, the point *A* must move towards *D*, and when he is south of the equator, it must move in the opposite direction. In order to find the position of the point *A* for any declination of the sun, multiply together the radius of the dial, the tangent

[3] This dial was invented by M. Lambert, and is described and demonstrated in the *Ephemerides* of Berlin, 1777, p. 200.

of half the height of the equator at the place for which the dial is constructed, and the tangent of the sun's declination, and the product of these three quantities, divided by the square of the radius of the tables, will give the distance of the moveable point *A* from the centre of the circle *F XII B*.

Let it be required, for example, to find the position of the point *A* on the 21st of December and June, when the declination of the sun is a maximum, or 23° 28′, the radius *A B* of the dial being divided into 100 equal parts.

$$\begin{aligned} \text{Log. } 100 &= 2.0000000 \\ \text{Log. Tang. } 17°\ 1' &= 9.4857907 \\ \text{Log. Tang. } 23°\ 28' &= 9.6376106 \\ \hline \text{Sum} \quad & 21.1234013 = \text{Log. of product.} \end{aligned}$$

From this logarithm substract 20, the logarithm of the square of the radius, and the remainder will be 1.1234013 = Log. 13.29. Take $13\frac{1}{4}$ parts, therefore, in your compasses, and having set them both ways from *A*, the limits of the moveable stile will be marked out.

For any other declination, the position of the point *A* may be found in a similar manner. It will be sufficient in general to determine it for the declination of the sun when he enters each sign, and place these positions on the dial, as represented in Fig 32.

The length of the stile *A C*, or its perpendicular height *H C*, must always be of such a size that its shadow may reach the hours in the circle *F XII B*. For any declination of the sun, its length *A C* may be determined by plain trigonometry. *A XII* is always given, the inclination of the stile *D A C* is also known, the angle *A XII C* is equal to the sun's meridian altitude, and therefore the whole triangle may be easily found in the common way, or by the following trigonometrical formula:—

$$A\,C \text{ the length of the stile} = \frac{A\,XII \times \text{Sin. Merid. Alt.}}{\text{Sin. } (180° - \text{Angle of Stile} + \text{Merid. Alt.})}$$

Notwithstanding the simplicity in the construction of this dial, the motion of the stile is troublesome, and should if possible be avoided. For this purpose the idea first suggested by the celebrated La Grange will be of essential utility. He allows the stile to be fixed in the centre *A*, and describes with the radius *A B*, circles upon

Improvement upon it by La Grange.

the different points where the stile is to be placed between *A* and *D*, and on the other side of *A*, which is not marked in the figure. All these circles must be divided equally into hours like the circle *F X I I B*, and when the sun is in the summer solstice, the divisions on the circle nearest the stile are to be used; when he is in the winter solstice, the circle farthest from *A* must be employed, and the intermediate circles must be used when the sun is in the intermediate points. This advice of La Grange may be adopted also in analemmatic dials.

CHAPTER XXI.

ON THE CAUSE OF THE TIDES ON THE SIDE OF THE EARTH OPPOSITE TO THE MOON.

It has always been reckoned difficult for those unacquainted with physical astronomy, to understand why the sea ebbs and flows on the side of the globe opposite to the moon. This fact, indeed, has frequently been regarded, and sometimes adduced, by the ignorant, as an unsurmountable objection to the Newtonian theory of the tides, in which the rise of the waters is referred to the attraction of the sun and moon. From an anxiety to give a popular explanation of this subject, Mr. Ferguson has been led into an error of considerable importance, in so far as he ascribes the tides on the side of the earth opposite the moon, to the excess of the centrifugal force above the earth's attraction.[1] It cannot be questioned, indeed, that the earth revolves round the common centre of gravity of the earth and moon, at the distance of nearly 6000 miles from that centre; and that the side of the earth opposite the moon has a greater velocity, and consequently a greater centrifugal force than the side next the moon; but as the side of the earth farthest from the moon, is only 10,000 miles from the centre of gravity, it will describe an orbit of 31,415 miles in the space of 27 days 8 hours, or 656 hours, which gives only a velocity of 47 miles an hour, which is too small to create a centrifugal force, capable of raising the waters of the ocean.

The true cause of the rise of the sea may be understood

[1] See Vol. I, p. 35.

from Plate XI, Fig. 38, where *A B C* is the earth, *O* the common centre of gravity of the earth and moon, round which the earth will revolve in the same manner as if it were acted upon by another body placed in that centre. Let *A M*, *B N*, *C P*, be the directions in which the points *A*, *B*, *C*, would move, if not acted upon by the central body; and let *B b n* be the orbit into which the centre *B* of the earth is deflected from its tangential direction *B N*. Then since the waters at *A* are acted upon by a force, as much less than that which influences the centre of the earth, as the square *O B* is less than the square of *O A*, they cannot possibly be deflected as much from their tangential direction *A M*, as the centre *B* of the earth; that is, instead of describing the orbit *A m*, they will describe the orbit *e a*. In the same manner, the waters at *c* being acted upon by a force as much greater than that which influences the centre *B* of the earth, as the square of *O B* exceeds the square of *O C*, will be deflected farther from their tangential direction than the centre of the earth, and instead of describing the orbit *c p*, will describe the orbit *h c i*.

As the earth, therefore, when revolving round the centre of gravity *O*, will be acted upon by the moon in the same way as by another body placed in that centre, it will assume an oblate spheroidal form *a b c*; so that the waters at *c* will rise towards the moon, and the waters at *a* will be *left behind*, or will be *less deflected* than the other parts of the earth, by the lunar attraction, from that rectilineal direction in which all revolving bodies, if influenced only by a projectile force, would naturally move.

TABLE OF SPECIFIC GRAVITIES.

Acacia, inspissat[d]. juice of,	1.5153
Acid, nitric, - -	1.2715
nitric, highly concentrated, -	1.583
muriatic, - -	1.1940
red acetous, -	1.0251
white acetous, -	1.0135
distilled acetous,	1.0095
acetic, - -	1.007 1.0095
sulphuric, -	1.8409
highly concent[d].	2.125
fluoric, - -	1.500
phosphoric, liquid,	1.417
solid,	2.852
citric, - -	1.0345
arsenic, - -	3.391
of oranges, -	1.0176
boracic, in scales,	1.479
do. melted, -	1.803
molybdic, -	3.460
benzoic, -	0.667
formic, - -	1.11
Agalmatolite -	2.800
Agate, onyx, - -	2.6375
Mocha, -	2.5891
Alabaster of Valencia	2.638
of Malaga, pink,	2.8761
Alcohol, absolute, *Lowitz*	0.791
15 parts, water 1 part	0.8527
8 8	0.9427
1 15	0 9919
Alder-wood, *Muschenbr.*	0.8000
Allanite, *Jardine*	3.665
Aloes, hepatic, -	1.3586
socotrine, -	1.3795
Alouchi, odoriferous gum,	1.0604
Alumine, sulphate of, *Muschenbroek*	1.7140
Amber, yellow transp[t].	1.0780
Ambergris	0.7800 0.9263
Amethyst, common. See *Rock* crystal -	2.750
Amianthus, long, -	0.9088
short, -	2.3134
Ammonia, liquid, -	0.8970
muriate of, *Muschenb.*	1.4530
Analcime -	2.0 3.0
Andalusite, or hardspar, *Hauy*	3.165
Anhydrite, or Muriacite,	2.95
Anime, oriental, -	1.0284
Anthophyllite -	3.20
Antimony, glass of,	4.9464
in a metallic state fused -	6.624 6.860
Aplome - -	3.45
Apple-tree, wood of the, *Muschenbroek*	0.7930
Arctizite, or Wernerite, *Dandrada*	3.606
Argillite, or slate clay, *Kirwan*	2.600 2.680
Arnotto - -	0.5956
Arragonite, *Malus*	2.94686
Asbestinite, *Kirwan*	3.000 3.310
Asbestos, mountain cork, *Bergman*	0.6806 0.9933
Ash trunk, *Muschenbroek*	0.8450
Asphaltum, cohesive	1.450 2.060
compact -	1.070 1.165
Assafœtida -	1.3275
Aventurine, semitransp[t].	2.6667
Augite or Pyroxene *Hauy*	3.226
Automalite, Gahnite, or Fahlunite -	4.200
Axinite, or Thumerstone, *Hauy*	3.213 3.296
Azure stone, or lapis lazuli, *Brisson*	2.7675
Barytes, sulphate of, native, *Malus*	4.48141
carbonate of, native,	4.300 4.338
Basalt, from the Giant's Causeway, -	2.864
Bay tree, Spanish, *Musch.*	0.8220
Bdellium - -	1.1377
Beech-wood, *Muschenbr.*	0.8520
Bees' wax, yellow -	0.965
Benzoin, - -	1.0924

Beryl, or aquamarine *Werner* { 2.650 / 2.759
Bismuth, native, *Kirwan* 9.570
in a metallic state, fused, - { 9.756 / 9.822
Black-coal, pitch coal, *Wiedemann* 1.308
cannel do. *La Metherie* 1.270
Blende, yellow, *Gellert* { 4.044 / 4.048
Blood, human, *Jurin* 1.054
Boles, - *Kirwan* { 1.400 / 2.000
Boracite, *Westrumb* 2.566
Borax - - - 1.714
Bournonite - - 5.576
Boxwood, French, *Musch.* 0.9120
Brass, common cast, 7.824
wiredrawn, - 8.544
cast, not hammered, *Brisson* 8.395
Bronzite - - 3.20
Brick - - { 1.577 / 2.000
Butter - - 0.9423
Cachibou, gum - 1.0640
Calamine, *Brisson* 3.525
Campeachy wood, or logwood, *Muschenbroek* 0.9130
Camphor, - - 0.9887
Caoutchouc, elastic gum, or India rubber, - 0.9335
Caragna, resin of the Mexican tree caragna, 1.1244
Carnelian, stalactite, 2.5977
pale, - - 2.6301
Cat's eye, *Klaproth* { 2.600 / 2.625
Cedar tree, American, *Muschenbroek* 0.5608
Indian, ,, 1.3150
Cement, Roman, cast 1.600
Cerite, - - 4.500
Ceylanite, or Pleonaste, *Hauy* { 3.765 / 3.793
Chabasie - - 2.718
Chalcedony, bluish, 2.5867
reddish, - 2.6645
Chalk, - - { 2.315 / 2.617
Charcoal, from birch, 0.542
Charcoal, from pine, - 0.280
Cherry-tree, - 0.7150
Chrysolite of the jewellers, *Brisson* 2.782
Chrystalline lens, - 1.100
Cinnabar, dark red, from Deux-Ponts, *Kirwan* 7.786
Cinnamon-stone - 2.6
Coak, - - - - 0.744
Coal, common - 1.27
Kilkenny - 1.526
Cobalt, in a metallic state, fused, - { 7.645 / 7.811
Cocoa wood, *Muschenbr.* 1.0403
Coccolite, *Dandrada* 3.316
Columbium, *Hatchet* 5.918
transparent, - 1.0452
Copper, native, *Kirwan* { 7.600 / 7.800
arseniate of, { hexahedral 2.549 / octahedral 2.88 / trihedral 4.2
drawn into wire, 8.878
fused, - - 7.788
Cork, *Muschenbroek* 0.2400
Corundum of India, *Klaproth* 3.710
Cyanite, Sappare, or Disthene, *Saussure, jun.* 3.517
Cyder - - 1.0181
Cymophane, or Chrysoberyl, *Werner* { 3.600 / 3.720
Cypress wood, Spanish, *Muschenbroek* 0.6440
Datolite - - 2.98
Dipyre - - { 2.63 / 2.84
Diamond, oriental, colourless, - - 3.5212
orange coloured, 3.5500
Dragon's blood - 1.2045
Earth, common - { 1.520 / 1.984
loamy - - 2.016
Ebony, Indian, *Muschenb.* 1.2090
Elder tree, ,, 0.6950
Elm trunk, ,, 0.6710
Emerald, *Gahn and Berzelius* { 2.673 / 2.683
Ether, sulphuric, { from 0.716 / to 0.745

Ether, nitric, - -		0.9088
muriatic, - -		0.7296
acetic, - -		0.8664
Euclase -	*Hauy*	3.0625
Euphorbium, gum -		1.1244
Fat of beef - -		0.9232
Felspar, fresh,	*Hauy*	2.438
Filbert tree,	*Muschenbr.*	0.6000
Fir, male,	,,	0.5500
Flint - -		2.250 2.630
Gabbronite - -		2.9
Gadolinite -		4.00 4.20
Galbanum, gum -		1.2120
Gamboge - -		1.2220
Garnet, precious, of Bohemia, -	*Klaproth*	4.085
common,	*Werner*	3.576
Gas, atmospheric,* or common air, - -		1.000
phosgene, or chloro-carbonic gas,	*Davy*	3.3888
nitrous acid gas, calculated,	*Gay Lussac*	3.176
	Sir H. Davy,	2.427
vapour of sulphuret of carb.	*Gay Lussac*	2.6447
sulphuric ether	,,	2.5860
iodine, calculatd.	,,	8.6195
hydriodic ether	,,	5.4749
oil of turpentine	,,	5.0130
hydriodic acid gas	,,	4.4430
fluosilicic acid gas,	*John Davy*	3.5737
chlorine,	*Lussac and Thenard*	2.470
euchlorine,	*Sir H. D.*	2.409
	Gay Lussac	2.3144
fluoboracic gas,	*Davy*	2.3709
vapour of muriatic ether,	*Thenard*	2.219
chloro-cyanic vapour	*Gay Lussac*	2.111
Gas, sulphurous acid,	*Sir H. Davy*	2.193
	Lussac & Thenard	2.1204
vapour of alcohol,	*Dalton*	2.1
absolute alcohol,	*Gay Lussac*	1.613
cyanogen,	,,	1.806
nitrous oxide, or protoxide of azote,	*Sir H. Davy*	1.614
	Colin	1.5204
carbonic acid,	*Saussure*	1.518
	Allan and Pepys	1.524
	Biot and Arago	1.51961
muriatic acid, or hydro-chloric gas,	*Sir H. Davy*	1.278
	Biot and Arago	1.2474
sulphuretted hydrogen,	*Gay Lussac and Thenard*	1.1912
	Sir H. Davy	1.777
oxygen, mean,		1.104
	Saussure	1.114
	Kirwan and Lav.	1.103
	Biot and Arago	1.0359
	Allan and Pepys	1.127
nitrous gas, or deutoxide of azote,	*Berard*	1.0388
	Sir H Davy	1.094
olefiant gas,	*Theodore Saussure*	0.97804
azote,	*Biot & Arago*	0.96913
carbonic oxide,	*Cruickshank*	0.9569
hydrocyanic vapour,	*Gay Lussac*	0.9476
posphuretted hydroghen,	*Sir H. Davy*	0.870
steam, -	*Tralles*	0.6896
	Gay Lussac	0.62349
ammoniacal,	*Sir H. Davy*	0.590

* The specific gravities of the gases are taken from Biot's *Traité de Physique*, tom. i, p. 383; from Gay Lussac's Table in the *Annales de Chimie et de Physique*, vol. i, p. 218; and from Thomson's *Annals of Philosophy*, vol. i, p. 118. The measures for the gases, taken by MM. Biot and Arago, are calculated from Biot's formulæ. They are given in relation to atmospheric air, which is supposed to be unity. Their relation to water is easily computed.

Gas, ammoniacal	
Biot and Arago	0.59669
carburetted hydrogen, *Thomson*	0.555
Sir H. Davy	0.491
Cruickshank	0.678
Dalton	0.600
arsenical hydrogen, *Tromsdorf, Dalton*	0.529
phosphuretted ditto, *Hauy*	0.852
Sir H. Davy	0.435
hydrogen, *Thomson*	0.073
Sir H. Davy	0.074
Biot and Arago	0.072098
Gehlenite, - *Fuchs*	2.78
Girasol, - *Brisson*	4.000
Glass, crown of St. Louis, *Cauchoix, Biot*	2.487
flint of M. Dartigues, *Cauchoix, Biot*	3.20
flint used by Mr Tully for his achromatic telescopes,	3.192 3.334 3.354 3.437
Glauberite - -	2.700
Gold, native -	17.00 19.00
pure, of 24 carats, fine, fused, but not hammered, *Hauy*	19.2587
Granite, red Egyptian	2.6541
Gravel, - - -	1.749
Gum, Arabic -	1.4523
tragacanth -	1.3161
seraphic -	1.201
cherry tree -	1.4817
Bassora - -	1.4346
Acajou - -	1.4456
Monbain -	1.4206
Gutte -	1.2216
ammoniac -	1.2071
Gayac - -	1.2289
liquid, from Botany Bay, *Thomson*	1.196
lac - -	1.1390
anime, Eastern	1.0284
Western	1.0426
Gunpowder in loose heap	0.922
solid -	1.745
Gypsum, cuniform, crystallised -	2.3060
Harmotome, or crossstone	2.3333
Hazel, *Muschenbroek*	0.606
Hauyne, or Latialite,	3.20
Heliotrope, or Blood stone, *Kirwan*	2.629 2.700
Hematites. See *Ironstone*	
Hone, razor, white	2.8763
Honey - -	1.4500
Hornblende, common *Kirwan*	3.600 3.830
Hornstone, or petrosilex	2.530 2.653
Hyacinth, - *Karsten*	4.000
Jade, or Nephrite, white	2.9592
red - -	2.6612
Jenite - -	3.80 4.00
Jet, a bitumin. substance	1.2590
Indigo - -	0.7690
Iolite, or Dichroite	2.56
Iridium, ore of, discovered by Mr. Tennant, *Wollaston*	19.500
Iron, native, meteoric	6.48
chromate of, from the Uralian mountains, in Siberia, *Lauguier*	4.0579
arseniate of -	3.000
fused, but not hammer .	7.200
forged into bars	7.600 7.788
pyrites, dodecahedral *Hatchet*	4.830
magnetic	4.200 to 4.900
ore specular, *Kirwan*	4.793 5.139
Ironstone, red, ochry *Wiedemann*	2.952
Iron, native (Heleachen mass) - -	6.723
Iserine, an oxide of titanium from the Iser in Bohemia - -	4.500
Juniper tree, *Muschenbr.*	0.5560
Ivory, dry - -	1.8250
Ivy gum, from the hedera terrestris - -	1.2948
Keffekil or Meerschaum, *Klaproth*	1.6000
Labdanum, resin -	1.1862
Lapis nephriticus -	2.894
Laumonite - -	2.20

Laumonite, from Derbyshire, *Watson*	6.565 / 7.786
crystallised, *Brisson*	7.587
Lead, *Fischer, Wollaston*	11.352
arseniate of	5.00 to 6.40
carbonate of	6.00 to 7.20
muriate of -	6.00
sulphate of -	6.3
chromate of -	6.00
acetate of, *Muschenbr.*	2.3953
Lemon tree, ,,	0.7033
Lepidolite lilalite, *Klaproth*	2.816
Leucite or Amphigene *Klaproth*	2.455 / 2.490
Lignum vitæ, *Muschenbr.*	1.3330
Lime, quick - -	0.843
Limestone, compact	1.3864 / 2.7200
Linden wood, *Muschen.*	0.604
Lithomarge -	2.50
Logwood, or Campeachy wood, *Muschenbr.*	0.9130
Macle - -	2.9444
Madder root, *Muschenbr.*	0.7650
Mahogany - -	1.0630
Magnesia, native, hydrate of - -	2.336
Magnesite, or carbonate of magnesia - -	2.200
Malachite *Brisson*	3.572
Manganese *Bergman*	6.850
grey ore of, striated, *Brisson*	4.249 / 4.756
Maple wood, *Muschenbr.*	0.7550
Marble - -	2.580 / 2.840
Marl - - -	1.600 / 2.870
Mastic gum - -	1.0742
Medlar tree, *Muschenbr.*	0.9440
Meionite - - -	3.10
Melanite, or black garnet, - *Karsten*	3.691
Mellite - *Hauy*	1.586
Menachanite, *Lampadius*	4.270
Mercury at 32° of heat	13.619
at 60° - -	13.580
at 212° - -	13.375
solid state, 40° below 0 Fahr. *Biddle*	15.612
in a fluid state, 47° above 0, *Biddle*	13.545

Mercury, native, *Hauy*	13.5681
Mesotype - -	2.0833
Milk, woman's -	1.0203
ass's, - -	1.0355
cow's, - -	1.0324
Mineral pitch, elastic, or asphaltum, *Hatchet*	0.905 / 1.233
tallow, - -	0.770
Molybdena in a metallic state, saturated with water, - -	7.500
native, - *Kirwan*	4.084
Mortar - -	1.715
Mulberry tree, Spanish, *Muschenbroek*	0.8970
Myrrh - -	1.3600
Natrolite, Swedish, *Thomson*	2.779 / 2.790
Naphtha - -	0.8475
Nepheline, or Sommite *Hauy*	3.2741
Nickel in a metallic state - -	7.421 / 8.500
Nickel, ore of, called Kupfernickel of Saxony	6.648
Nigrine, or calcareo-siliceous titanic ore, *Vauquelin*	3.700
Nitre, *Muschenbroek*	1.9000
Oak, 60 years old, heart of, *Muschenbroek*	1.1700
Obsidian - -	2.348
Octohedrite - *Hauy*	3.857
Oil of filberts -	0.916
walnut - -	0.92
hemp-seed -	0.9258
poppies -	0.9238
rape-seed -	0.9193
lint-seed -	0.9403
poppy-seed -	0.929
whale - -	0.9233
ben, a tree in Arabia,	0.9119
beechmast -	0.9176
codfish -	0.9233
olives - -	0.9153
almonds, sweet	0.9170
volatile of mint, com.	0.8982
sage - -	0.9016
thyme -	0.9023
rosemary -	0.9057
calamint -	0.9116
cochlearia, -	0.9427

Oil, volatile, of wormwood	0.9073
tansy - -	0.9328
Stragan -	0.9949
Roman camomile	0.8943
sabine -	0.9294
fennel -	0.9294
fennel-seed -	1.0083
coriander-seed	0.8655
carraway-seed	0.9049
dill-seed -	0.9128
anise-seed -	0.9867
juniper-seed	0.8577
cloves -	1.0363
cinnamon -	1.0439
turpentine	0.8697
amber -	0.8865
flowers of orange	0.8798
lavender -	0.8938
hyssop -	0.8892
Olibanum gum -	1.1732
Opal, precious, *Blumenbach*	2.114
common, *Klaproth*	1.958 2.015
Opium - -	1.3365
Opoponax - -	1.6226
Orange tree, *Muschenbr.*	0.7059
Orpiment, *Kirwan*	3.048 3.435
Osmium Iridium, alloy of,	19.5
Palladium - -	11.8
Pear tree, *Muschenbroek*	0.6610
Pearls, oriental -	2.683
Peat, Edinburgh, *Thomson*	0.600
Peridot, or Olivine	3.428
Peruvian bark -	0.7849
Pewter - -	7.248
Phosphorite, or Spargel stone, whitish, from Spain, before absorbing water - - - -	2.8249
Pinite, - *Kirwan*	2.980
Pitch - - -	1.150
Pitch-stone, black *Brisson*	2.0499
Plaster, cast - -	1.286
Platina, - *Klaproth*	20.722
drawn into wire	21.0417
a wedge of, sent by Admiral Gravina to Mr. Kirwan	20.663
native -	15.601 17.200
Platina, compressed by a flatting mill -	22.069
Plum-tree, *Muschenbroek*	0.7850
Plumbago, or graphite, *Kirwan*	1.987 2.267
Pomegranate tree, *Musch.*	1.3540
Poplar wood, ,,	0 3830
Porcelain from China	2.3847
Porphyry, green -	2.6760
Potash, carbonate of	1.4594
muriate of, *Muschen.*	1.8365
sulphate of -	2.2980
Potassium at 15° centigrade, *Gay Lussac & Thenard*	0.97223
Potstone -	2.80 to 3.00
Prehnite of the Cape, *Hauy*	2.697
Pumice stone -	0.9145
Puzzolana - -	2.570 2.850
Pycnite, or shorlous beryl *Hauy*	3.5145
Pyrope, - *Klaproth*	3.718
Pyrophysalite, -	3.450
Realgar, or red orpiment *Bergman*	3.225
Rock crystal, *Malus*	2.63717
Ruby, oriental -	4.2833
spinelle -	3.7600
Rutile, - *Hauy*	4.102
Sahlite, - *Dandrada*	3.234
Sand, river - -	1.886
pit, fine -	1.523
Sapphire, oriental, white	3.991
Brazilian or occidental, *Hauy*	3.994 4.283
Sarcocolla - -	1.2684
Sardonyx, pure, *Brisson*	2.6025
Sassafras, *Muschenbroek*	0.4820
Saussurite, - -	3.260
Scapolite, or Paranthine, *Dandrada*	3.6800 3.7000
Serpentine, green, Anglesey - - -	2.683
Shale - - -	2.6
Silver, red, or ruby, *Brisson*	5.564
native, com. *Gellert*	10.000
shilling of Geo. III.	10.534
Slate - -	2.512 2.888
Soda, sulphate of, *Musch.*	2.2460
muriate of, ,,	2.1250

Sodalite - *Thomson* 2.378
Sodium, at 15° centigrade,
Guy Lussac & Thenard 0.86507
Spar, fluor, red, or false
ruby - - - 3.1911
octohedral - 3.1815
Spermaceti - - 0.9433
Spodumene, or triphane
Hauy 3.1923
Steel, *Muschenbroek* 7.767
soft - - 7.8331
hammered - 7.8404
Stilbite - - - 2.50
Stones. (See this vol. p. 250.)
Stone, sand, paving 2.4158
grinding - 2.1429
Portland - 2.496
Storax - - 1.1098
Strontian, sulphate of *Hauy* { 3.583 / 3.958
carbonate of, *Hauy* { 3.658 / 3.675
Sugar, white, *Muschenbr.* 1.6060
Sulphur, native *Hauy* 2.0332
fused - - 1.9907

Tabasheer, opaque, dry 2.059
wet 2.412
transparent, dry, wet 1.396
Talc, indurated - 2.90
Tallow - - - 0.9419
Tantalite - *Ekeberg* 7.953
Tantalium metal, *Berzelius* 5.61
in large masses,
Gahn and Berzelius 6.291
Tellurium, native { 5.700 / 6.10
yellow - - 10.6
Tile, common - 1.858
Tin, pure, from Cornwall, fused, *Watson* { 7.170 / 7.291
stone *Gellert* { 6.300 / 6.989
Titanite, Rutilite, or
Sphene - *Hauy* 4.102
Topaz, oriental - 4.0106
Brazilian - 3.5365
Tourmaline, green, *Hauy* 3.362
Tremolite - - { 2.9 / 3.2
Tungsten *Klaproth* 5.570
Turbeth mineral - 8.235
Turpentine, spirits of, 0.870
Turquoise, ivory tinged by blue calx of copper { 2.500 / 2.908
Ultramarine *Desormes and Clement* 2.360
Uran glimmer, or Uranite 2.19
Uranium - - 7.500
Urine, human - { 1.015 / 1.026
Vesuvian *Wiedemann* 3.575
Vinegar, red *Muschenbr.* 1.0251

Walnut tree of France
Muschenbroek 0.6710
Water - - - 1.000
Wavellite, or hydrargillite
Davy 2.7000
Wax, bees - - 0.9648
Willow *Muschenbroek* 0.5850
Wolfram - *Gmelin* 5.705
Woods of different kinds. (See this vol. p. 251–255.)
Yew tree, Dutch, *Muschen.* 0.7880
Spanish, ,, 0.8070
Yttrotantalite, *Ekeberg* 5.130
Yttrocerite, *Gahn and Berzelius* 3.447
Zinc, pure and compress[d]. 7.1908
sulphate of, *Muschen.* 1.9000
Zircon, or Jargon, *Hauy* { 4.3858 / 4.4161
Zoisite *Klaproth* { 3.26 / 3.31

INDEX TO VOLUME SECOND.

A

Absolute fall of water, *page* 15.
Achromatic telescope, 309. Object-glass, double, 312. Tables of their radii, 317. Triple, 313. Tables of their radii, 315. Eye-pieces, 323. Formulæ for, 324, 325.
Amontons on friction, 149.
Analemmatic dial, 381.
Anemometers, 76.
Animals, on the strength of, 2.
Axles, on the friction of, 163.

B

Backwater, how to keep it off from wheels, 63.
Balance, torsion, 234.
Ball and socket, 182.
Balloons, on the principles and construction of, 278.
Barker's water wheel, various improvements upon it, 60. Effect of, 63.
Beatson on wind-mills, 86.
Besant's undershot wheel, 44.
Bevelled wheels, on the formation of their teeth, 135.
Blowing machine, 285.
Bramah's hydrostatic press, 292.
Breast wheels, 45.
Bulfinger on friction, 149.
Burns' overshot wheel, 55.

C

Camera obscura, improvement upon it, 331.
Capstane, description of a simple and powerful one, 261.
Carriage wheels, 210. Conical rims disadvantageous in, 219. Inclination of spokes disadvantageous, 215.
Cassegrainian telescope, 300. Table for, 302.
Condenser of forces, 199.
Conical pendulum, 96.
Coulomb on wind-mills, 68. On friction, 153. On the strength of men, 2. On torsion balance, 235. On elasticity and cohesion, 241.
Course of discharge, 13. Of impulsion, *ib.*
Cranks, single and double, 181.
Crown wheels, 177.
Cycloids, on the formation of, 140.
Cylindrical wheels and pinions, 176.

D

Dial, Analemmatic, 381. Lambert's, description of, 386.
Dichroscope described, 333.
Diving bell described, 281.
Dollond's achromatic eye-piece, 327.
Double microscopes, 296.
Double corn-mills, 42.
Double overshot wheel, with a chain of buckets, 57.
Double refraction of light, 345.
Dynamical units explained, 7.

E

Elasticity, on the force of, 229.
Elements of machinery, 176.
Engine, steam, 89. Water-blowing, 285.
Epicycloids, exterior, method of forming them mechanically, 138. Geometrically, 140. Interior, 13. Spherical, 136.
Euler on wind-mills, 73. On friction, 150.
Eye-pieces, on achromatic ones, 324. Single, 328.

F

Finder, a new one for Newtonian telescopes, 304.
Float-boards, number of, 18. Size of, 20. Position of, 21.

Fluid object-glass, Dr. Blair's, 323.

Fly-wheels, on the nature and operation of, 194. How they become regulators of machinery, 195. How they become accumulators of power, 197.

Friction, how to find the momentum of, in wind-mills, 75. Nature and effects of, 148. Amonton's observations upon, 149. Bulfinger's, *ib.* Parent's, 150. Euler's, *ib.* Ferguson's, 151. Bossut's, *ib.* Vince's, 152. Coulomb's, 153. Method of diminishing the effects of, 173. Wheels, *ib.* Of axles, 163. Of ropes, 166. Of pivots, 169.

G

Gas, on the manufacture of for illumination, 271. Apparatus for oil, 273.

Glass grinding, method of, 338.

Gregorian telescopes, 300. Table for, 301.

Grinding lenses, method of, 338. Specula, 341.

Gudgeons, form and size of, 24.

H

Herschel, Mr. on achromatic object-glasses, 322. On single eye-pieces, 328.

Herschel's telescopes, 304, 307.

High pressure engines, 107.

Hooke's universal joint, 182.

Horizontal mills, 38.

Horses, way in which they draw, 224. Method of regulating their action, 201. Power of, 11.

Hydraulic ram, 288.

Hydrostatical press, Bramah's, 292.

I

Indicator of a steam engine, 101.

Iron, cast and malleable, on the strength of, 255, 256.

K

Kaleidoscope described, 336. Telescopic, 337.

L

Lagaroust, lever of, 180.

Lambert's dial, description of, 386.

Lamp, safety, for lighting mines, 275. Without flame, 278.

Lenses, method of grinding and polishing them, 338.

Locking and unlocking machinery, apparatus for, 179.

M

Machine, water-blowing, 285.

Machines, on the maximum effect of, 204. On the elements of, 176. Thrashing-machines, 267.

Materials, on the strength of, 244.

Maximum effect of machines, 204.

Mechanical agents, 1.

Men, on the strength of, 2.

Microscopes, single, 294. Tables for, 296. Compound, *ib.*

Mill-course, on the construction of it, 12.

Mills, on the performance of undershot, 28. Horizontal, 38. Double, 42. Breast, 45. Overshot, 47.

Millstone, on the formation, size, and velocity of it, 24.

Millwrights' tables, description of two on new principles, 31–35.

Montgolfier's hydraulic ram, 288.

Motion, contrivances for transferring it, 183.

N

Newtonian telescopes, 303. Table for their apertures, &c. 306. A new finder for, 304.

O

Overshot wheels, formation of their buckets, 48. Velocity of, 51. Table for, 53. Borda's observations upon, *ib.*

Overshot mills, 57. Power of, 58. Performance of, 59.

P

Parachute invented by Garnerin, 281.

Parallel motion, 100, 193.

Parent on windmills, 69.

Parent on friction, 150.

Phantasmagoria, 323.

Polarisation of light, 349. By crystals, *ib.* By reflexion, 350. By refraction, 355. By pressure, 371. By metals, 372.

Presses, on the construction of lever ones, 265.

Press, Mr. Wells' new printing one, 264. Bramah's hydrostatic one, 292.

Pivots, form and size and friction of, 169.

Prony's condenser of forces, 199.

R

Refraction, double, 345.

Ragwheels, 178.
Rackwork, method of forming its teeth, 124.
Rain wind, cause of it, 287.
Ram, hydraulic, 288.
Ramsden's achromatic eye-piece, 327.
Reflecting telescopes, 300–309. Tables for, 301, 302, 308. New one, 307.
Refracting telescopes, 298. Table for, 300.
Relative fall of water, 15.
Regulation of machinery by fly-wheels, 194.

S

Safety lamp, for lighting mines, 275.
Samuel's apparatus for regulating the action of horses, 201.
Screws, endless, 180.
Single microscopes, 294.
Smeaton's maxims on undershot mills, 30.
Smeaton on windmills, 80.
Smeaton's overshot wheel, 54.
Specific gravities, table of, 390.
Specula, method of casting, grinding, and polishing, 341.
Spur wheel, formation of, 21.
Steam-engine, 89. Watt's, 91. Steam-gage, 101. Woolff's improvements on, 104. High pressure engine, 107. On the power of, and the method of computing it, 109.
Steam boats, history of, 112. Description of, 116. Carriages described, 118.
Steel, on the strength of, 245.
Stones, on the strength of, 250.

T

Table of the strength of men, 8. Of the strength of horses, 11. Of materials, 244. Pitches of wheels, 146. Coulomb's experiments on the friction of axles, 165. Of the number of floatboards in undershot water wheels, according to Pitot, 18. For millwrights, constructed on new principles, 31–35. For breast mills, 46. Of experiments on overshot mills, 53. Of the velocity of thrashing machines, 269. Of the power of thrashing machines, 271. For refracting telescopes, 300. For Gregorian telescopes, 301. For Cassegrainian telescopes, 302. For Newtonian telescopes, 306. Of specific gravities, 390.
Tables for achromatic telescopes, 315. For single microscopes, 296.
Teeth of wheels, method of forming them, 119. Method of disposing them, 138.
Teeth of rackwork, method of forming them, 124.
Teinometer, S'Gravesende's, 230. New mechanical one, 231. Chromatic, 232.
Telescopes, achromatic, 309. Refracting, 298. Table for, 300. Gregorian, *ib.* Table for, 301. Cassegrainian, 300. Table for, 301. Newtonian, 303. Table for, 306. New reflecting one, 307. Herschel's, 304, 306. Dollond's, 310. With fluid object-glasses, 323.
Thrashing machines, 267. Driven by water, 268. Driven by horses, *ib.* On the power of, 271.
Tides, on the cause of the, 388.
Timber, on the strength of, 251.
Traction, on the line of, 222.
Trundle, formation of, 125.
Torsion, on the force of, 227, 234.

U

Undershot water wheels, construction of, 11. Table of the number of floatboards in, according to Pitot, 18.
Undershot mills, on the performance of them, 28.
Undershot wheel, Besant's, 44. At right angles to the stream, 45.
Universal joint, Hooke's, 182.

V

Valves, on the construction of, 97. Safety, 93.
Velocity of water, method of measuring it, 37. Wind, method of measuring it, 76.
Verrier's windmill, description of, 64.
Vince on friction, 152.

W

Water, how to measure its velocity, 37.
Water-blowing machine, 285.
Water wheel, undershot, 11. Number of floatboards in, 18. Position of floatboards, 21.
Wheels, on the teeth of, 119. Bevelled, on the formation of their teeth, 135. Friction, 173. Fly, 194.
Wheels of carriages, on the advantages of large ones, 212. On the position of, 221.
Wheel carriages, 210. On the formation of, 214. How to place the centre of gravity, 226.
Whitehurst's machine for raising water, 287.

Wind, method of finding its velocity, 76. By Coulomb, *ib.*
Windmill, description of Verrier's, 64. Those in Holland, 68. Sails, on the formation and position of, 69. Angle of their inclination, according to Parent, *ib.* Euler's theorem concerning, 70. Euler's observations upon, 73.
Windmills, table of their power, &c. 74. Their effect in manufacturing oil and grinding corn, 82. Horizontal, 85. Comparison between vertical and horizontal ones, 87. Smeaton's observations on, 80. Coulomb's, 68.
Wipers of stampers, &c. mode of forming them, 126.
Woolff's steam-engine, 104.

The reader is requested to correct the following

ERRATA:

VOL. I.

Page 16, *note*, dele the reference to a Supplementary Chapter on Magnetism.

VOL. II.

Page 63, line 10 from bottom, for $C\,D\,E$, read $A'\,E$.
92, line 21, for f, read f'.
93, line 17, the valve v is not seen in the Figure.
98, line 3 from bottom, the syphon e is not seen in the Figure.
101, line 11, for B read β.
107, line 6 from bottom, dele *of*.
108, line 20, for *cocks*, read *cock*.
136, line 3, for $O\,O$, read O.
136, line 9, for 4 C, read 4 $C\,c$.
161, line 4 from bottom, for *or*, read *as*.
184, line 5, for *circular*, read *circular alternating*.
191, line 20, for *Plate VII*, read *Plate V*.
191, line 6 from bottom, for *Fig.* 11, read *Fig.* 9.
189, line 3 from bottom, for $p\,D$, read $p\,D'$.
274, line 14, for *Fig.* 16, read *Fig.* 14.
288, line 1, for *Fig.* 9, read *Fig.* 10.
347, for *Fig.* 16, read *Fig.* 1, and for *Fig.* 17, read *Fig.* 2.
349, for *Fig.* 3, read *Fig.* 4.
351, lines 7 and 9 from bottom, for $B\,A\,B$, read $R\,A\,P$; for $A\,C\,O$, read $A\,D$; and for $A\,D$, read $M\,T$.
354, line 1 from bottom, for $A\,B$, read $A\,D$.

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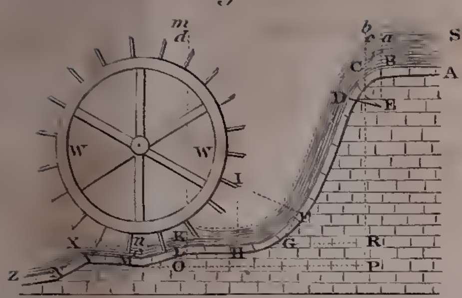

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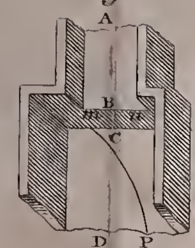

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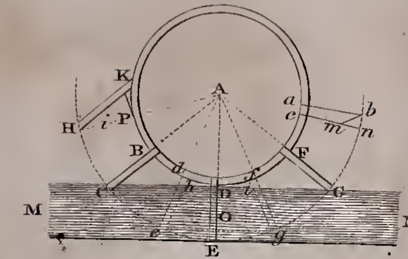

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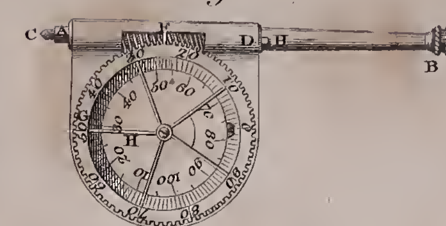

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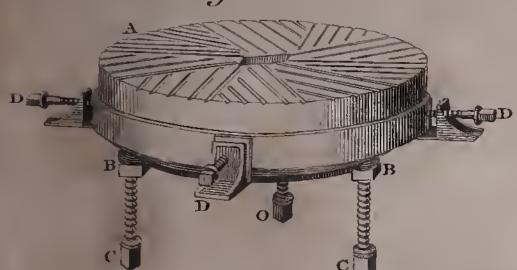

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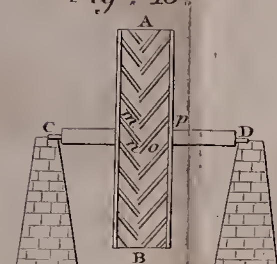

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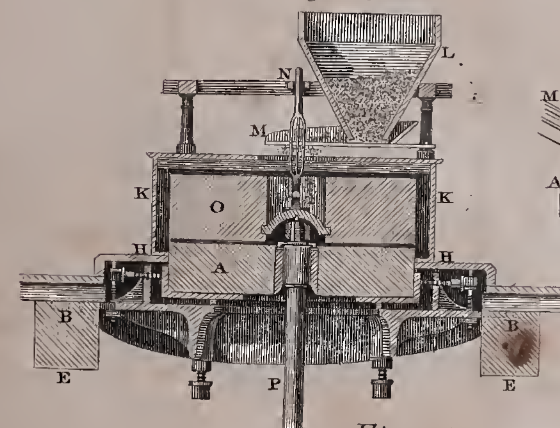

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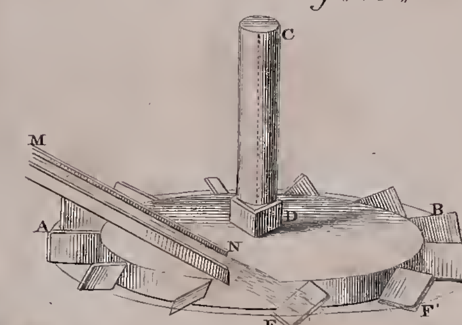

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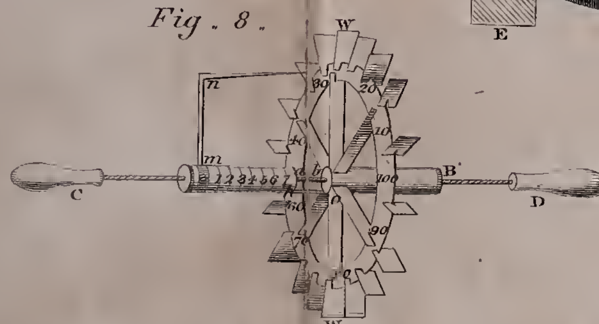

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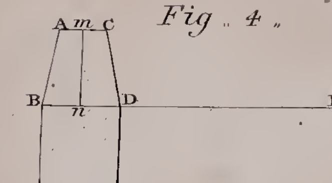

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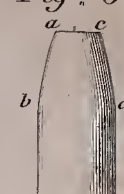

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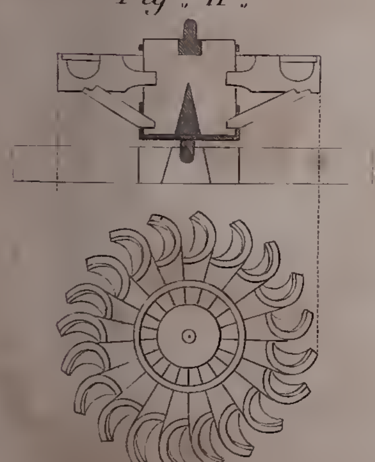

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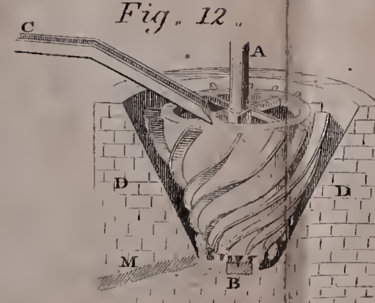

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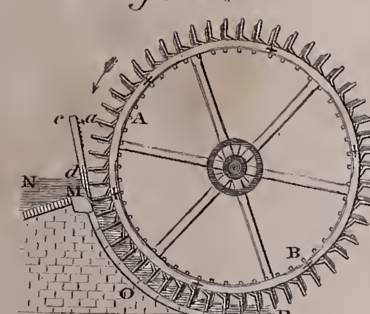

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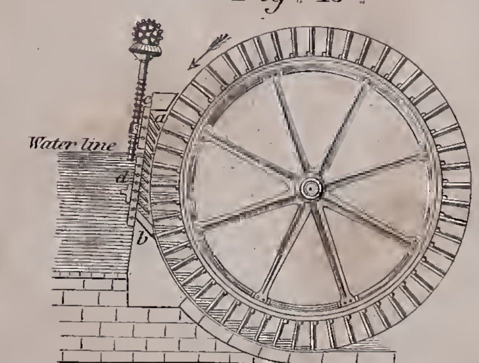

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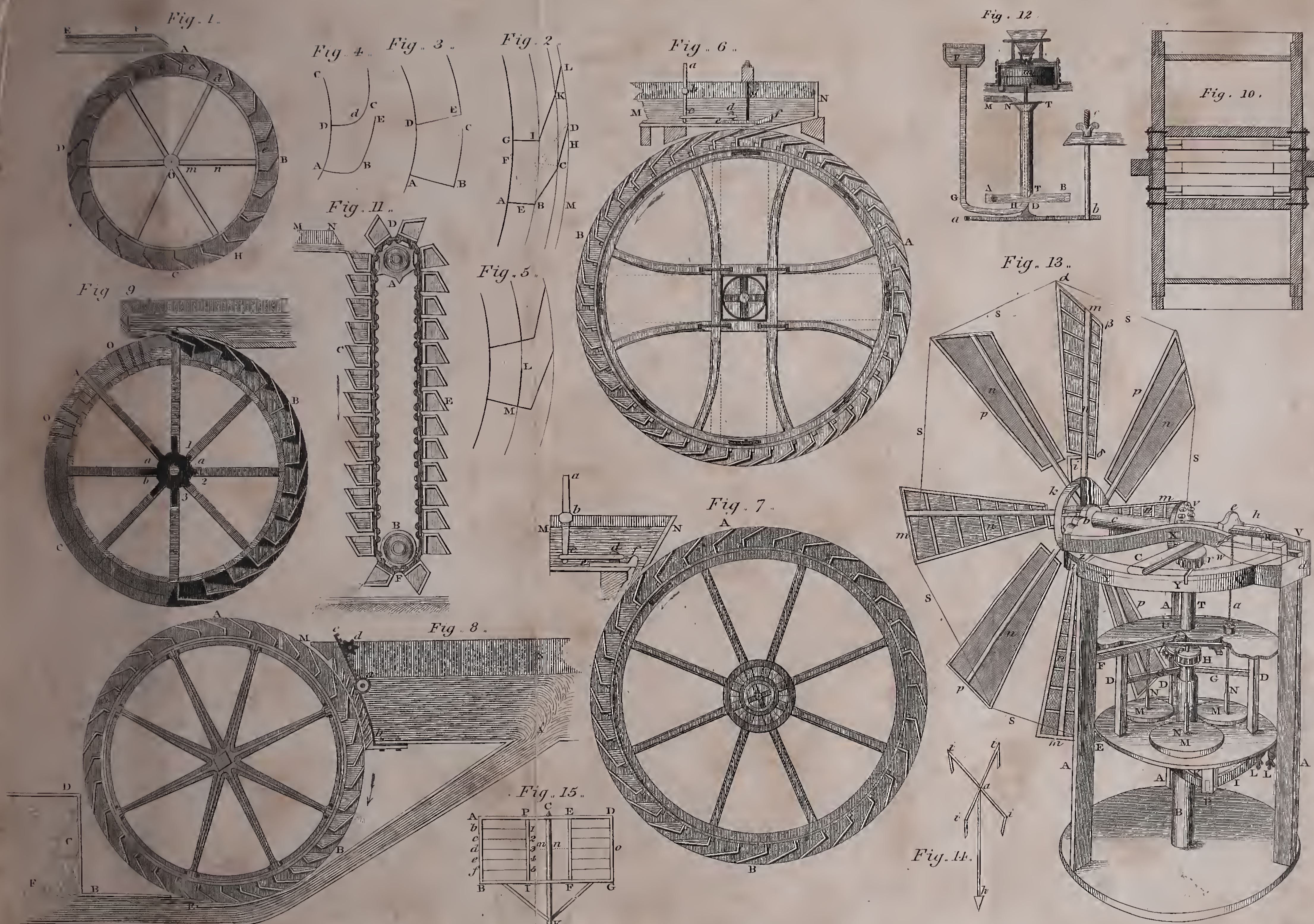

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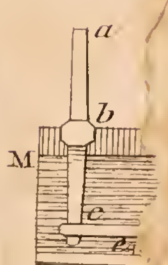

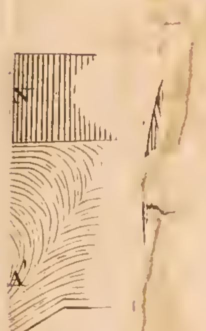

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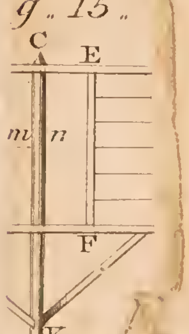

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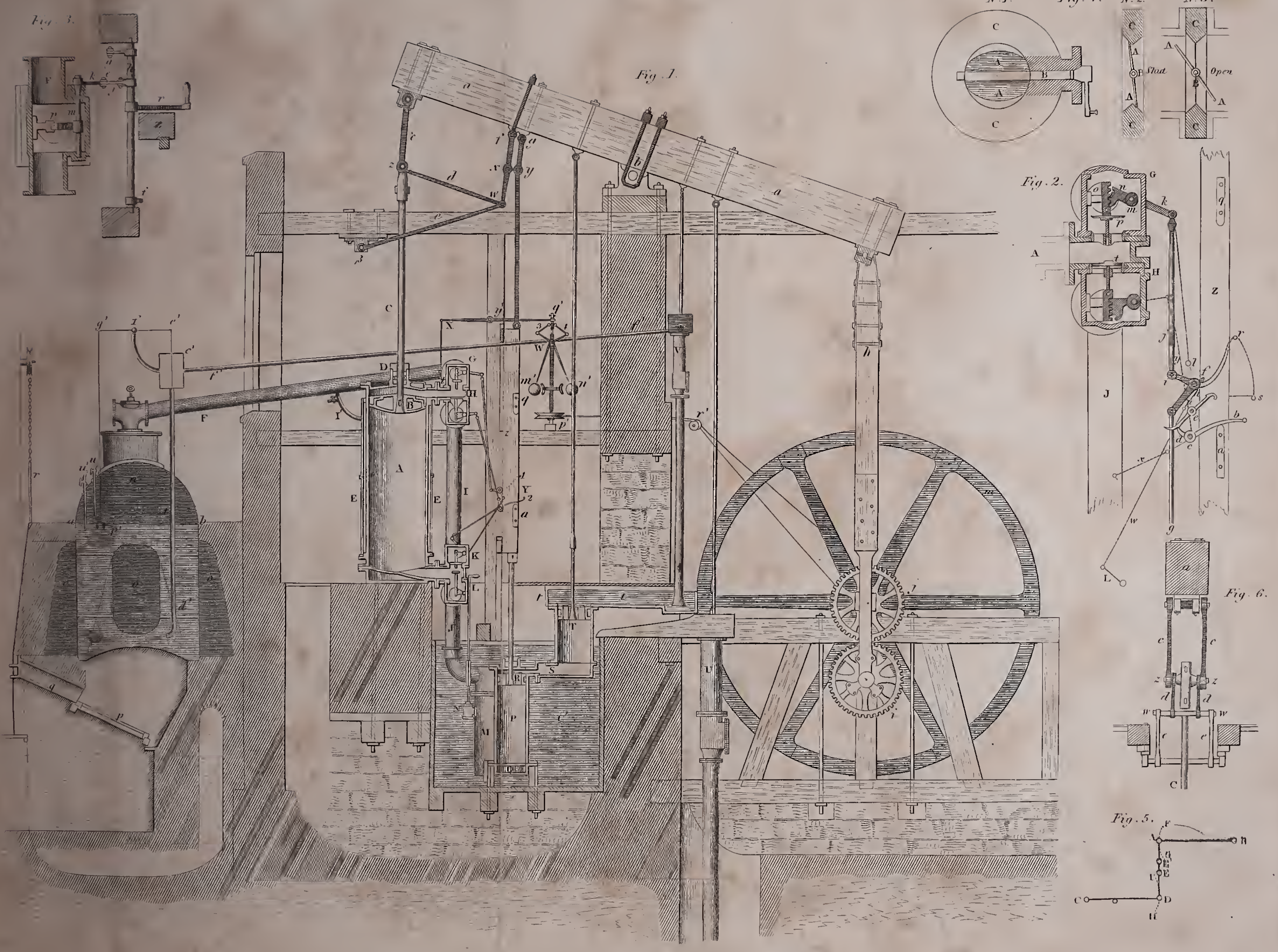

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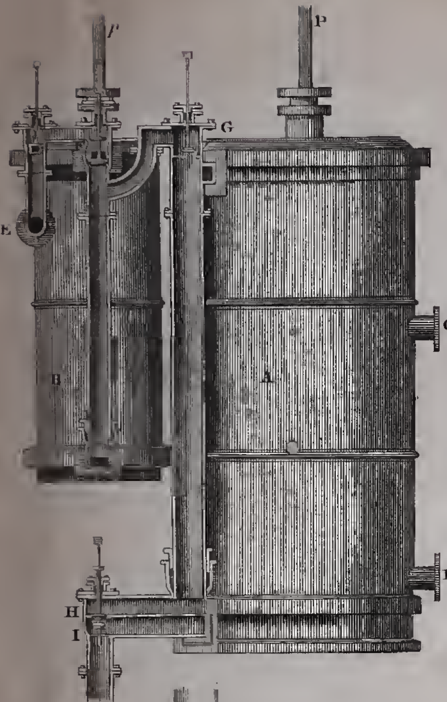

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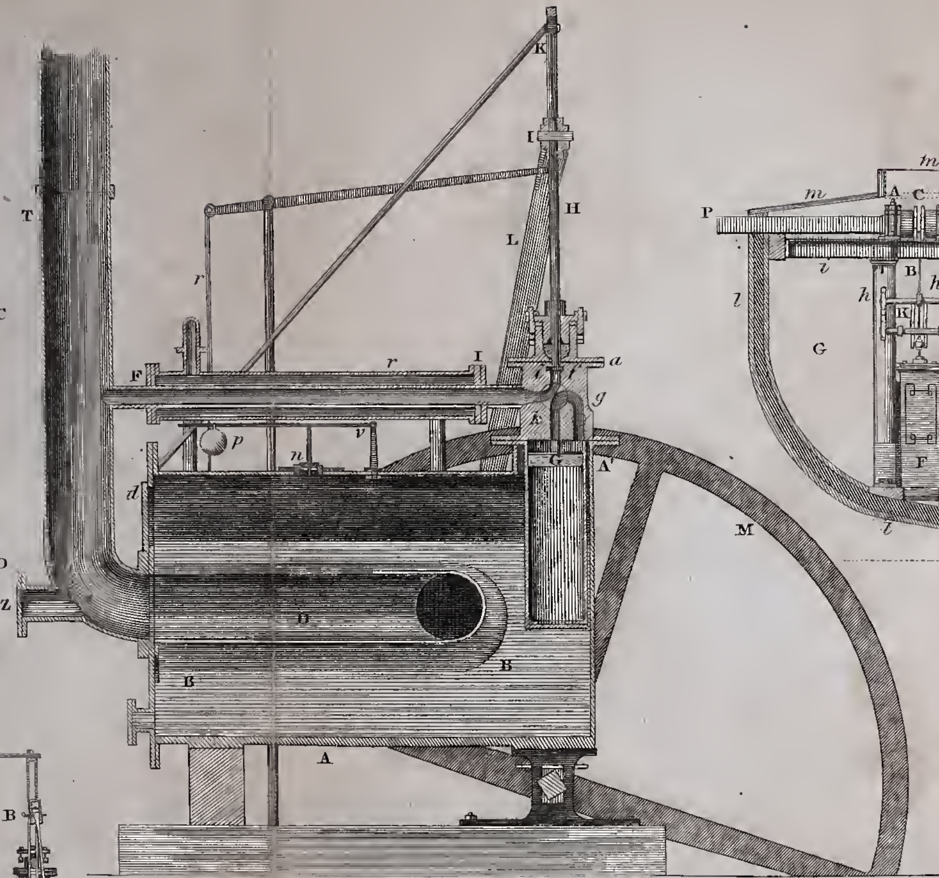

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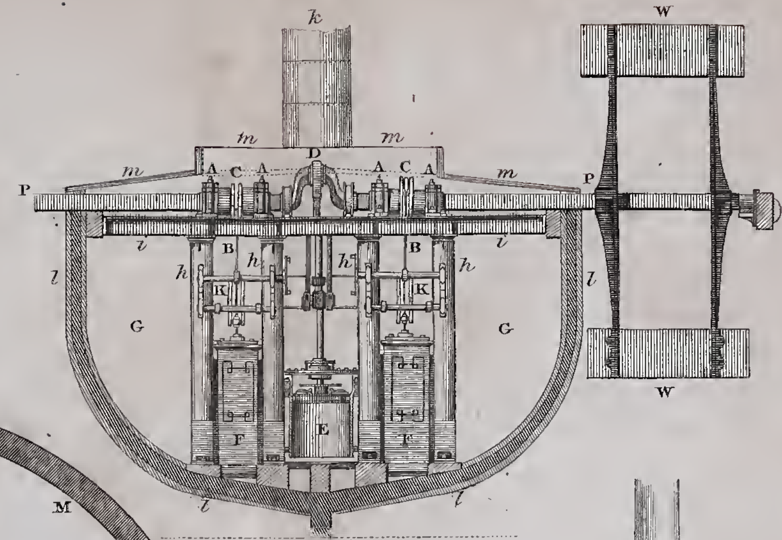

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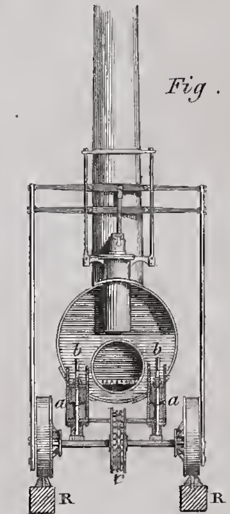

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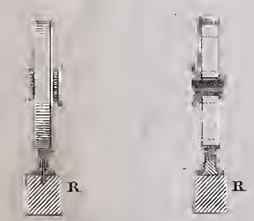

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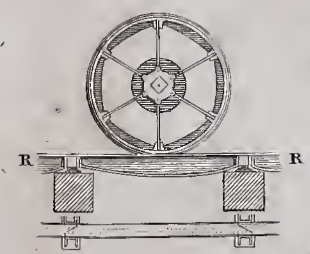

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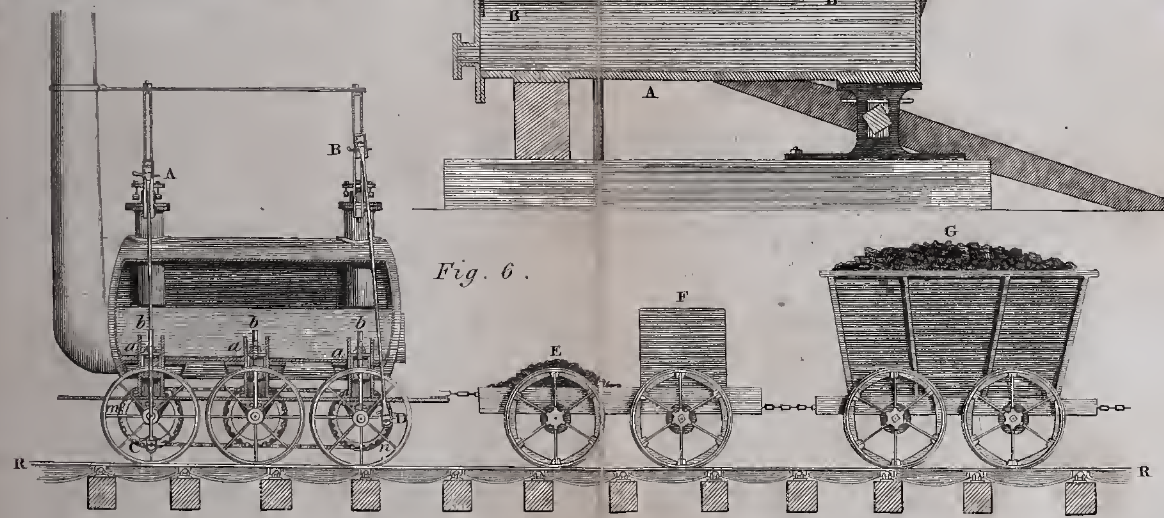

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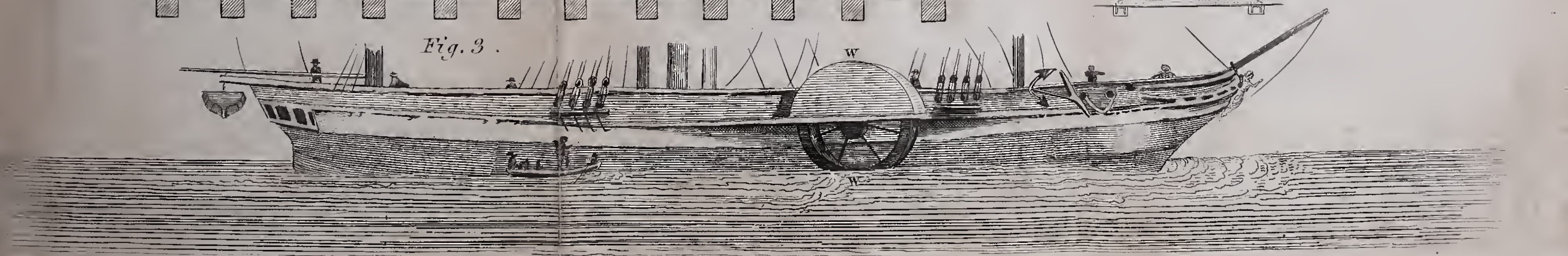

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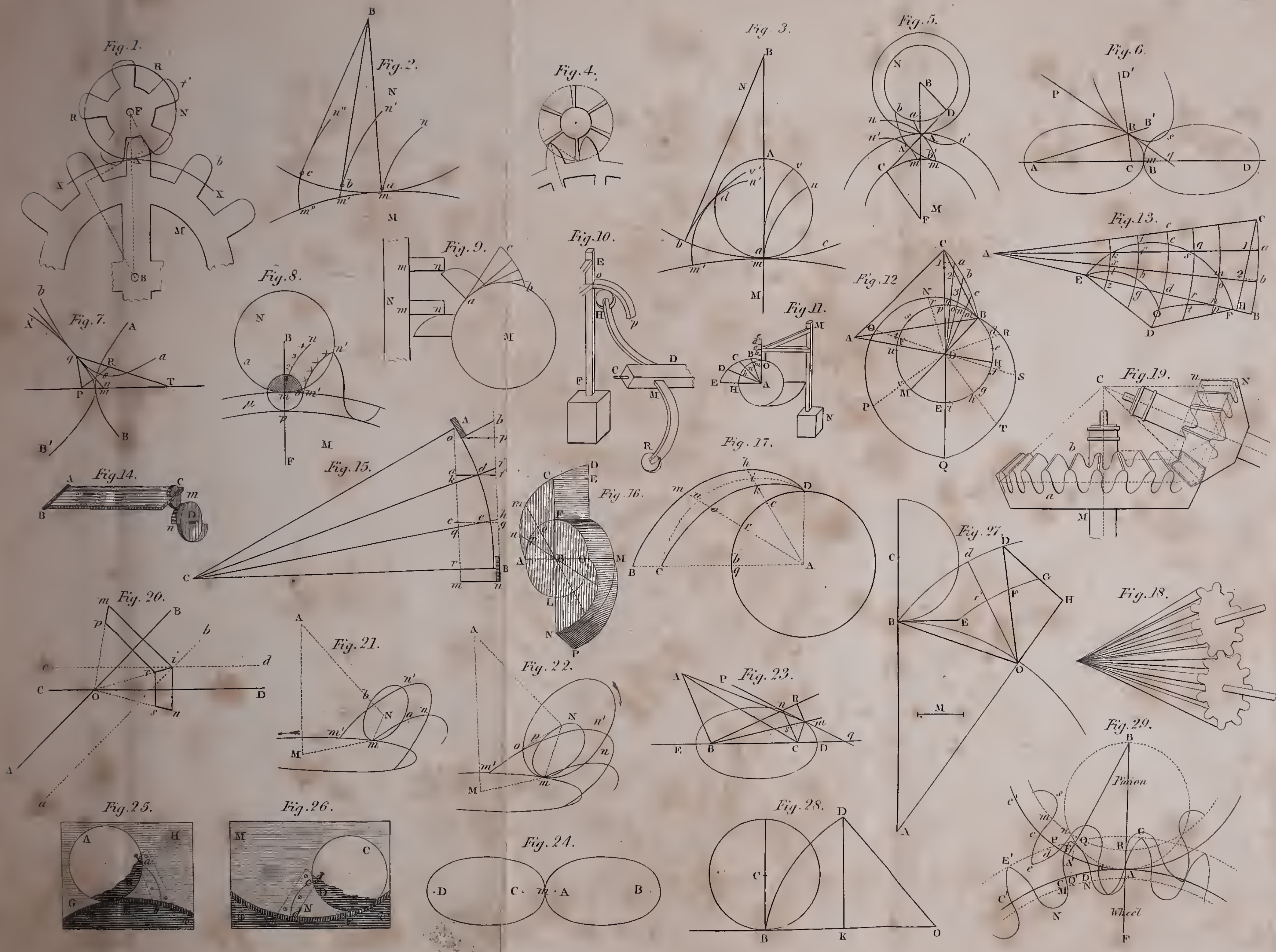

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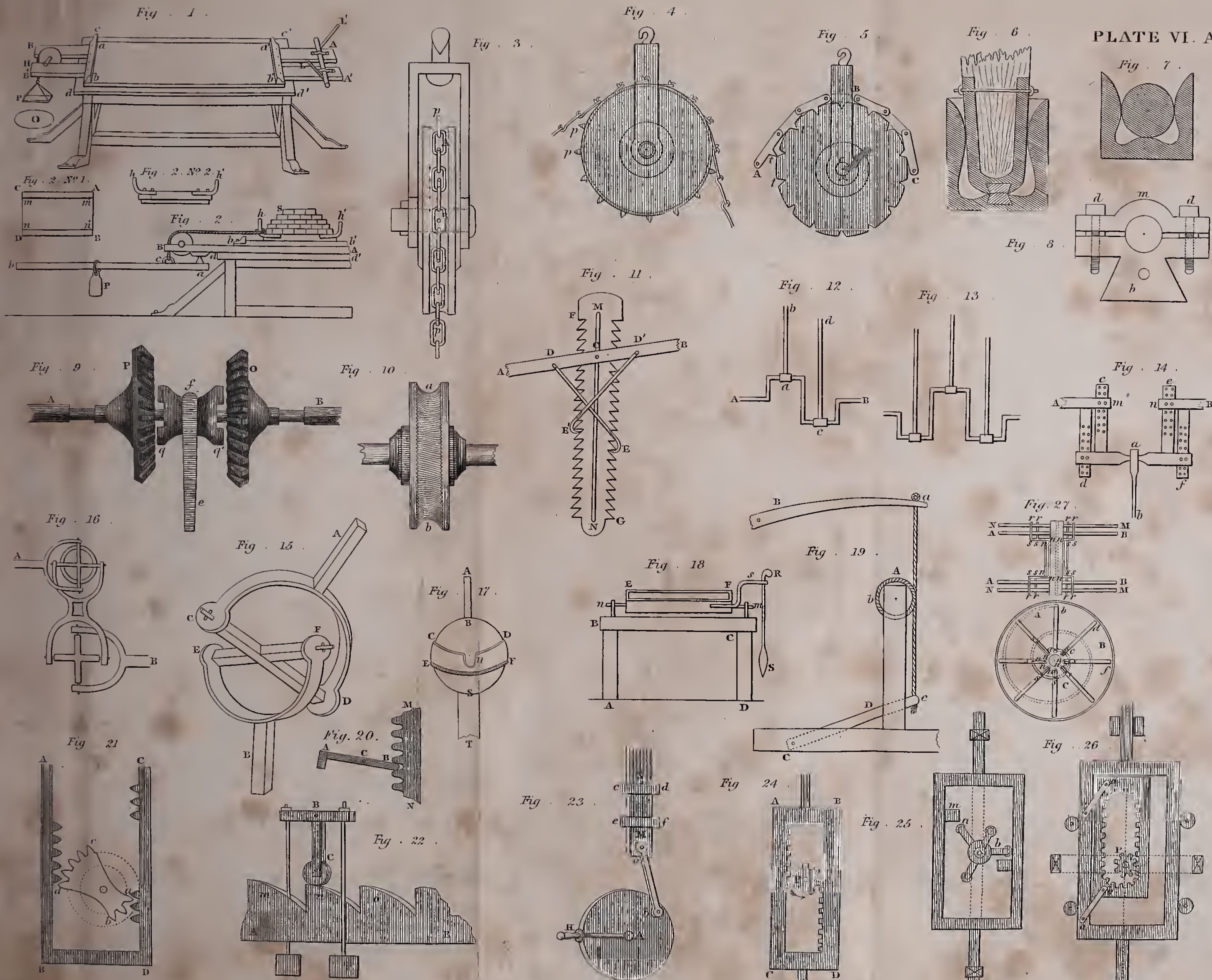

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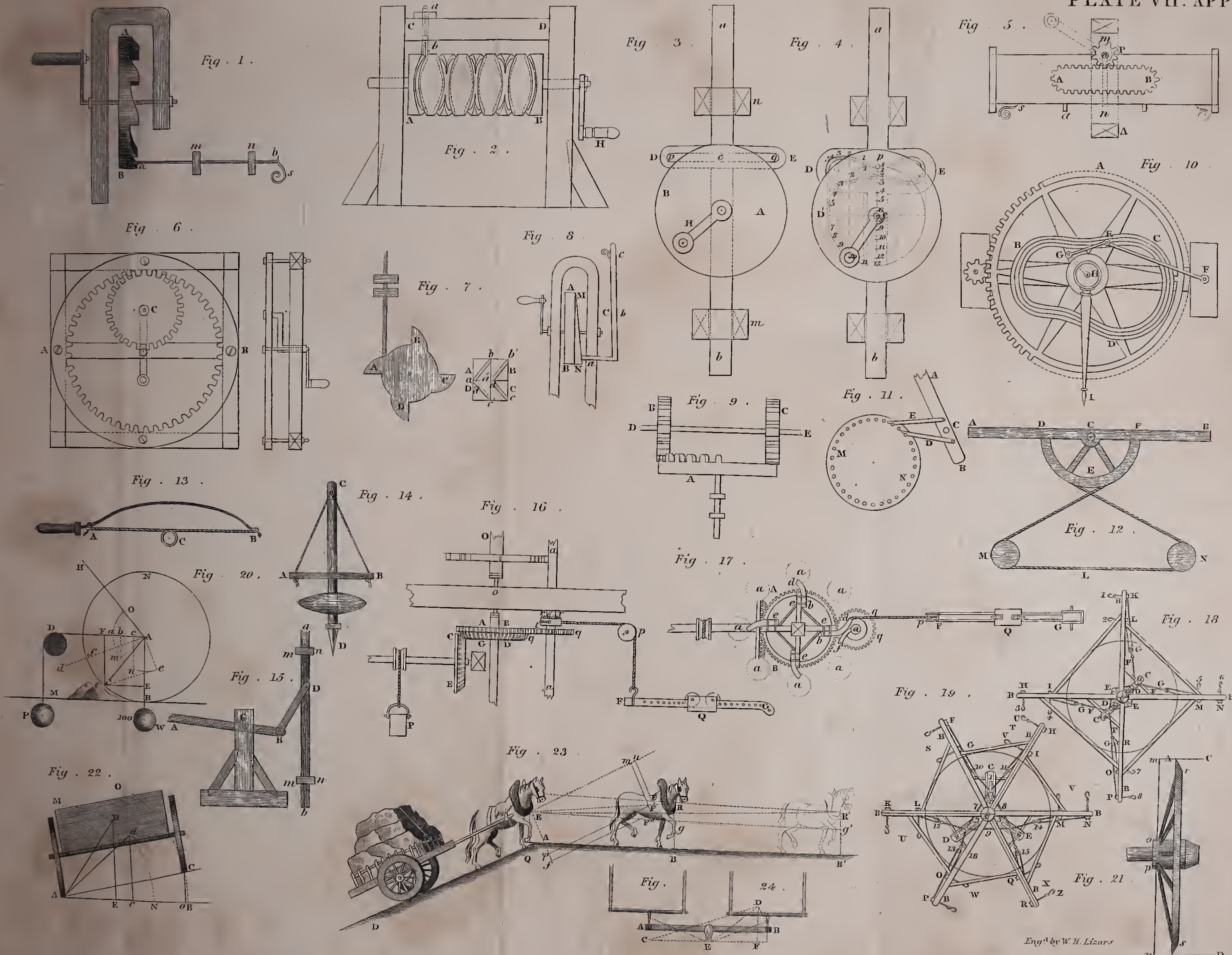

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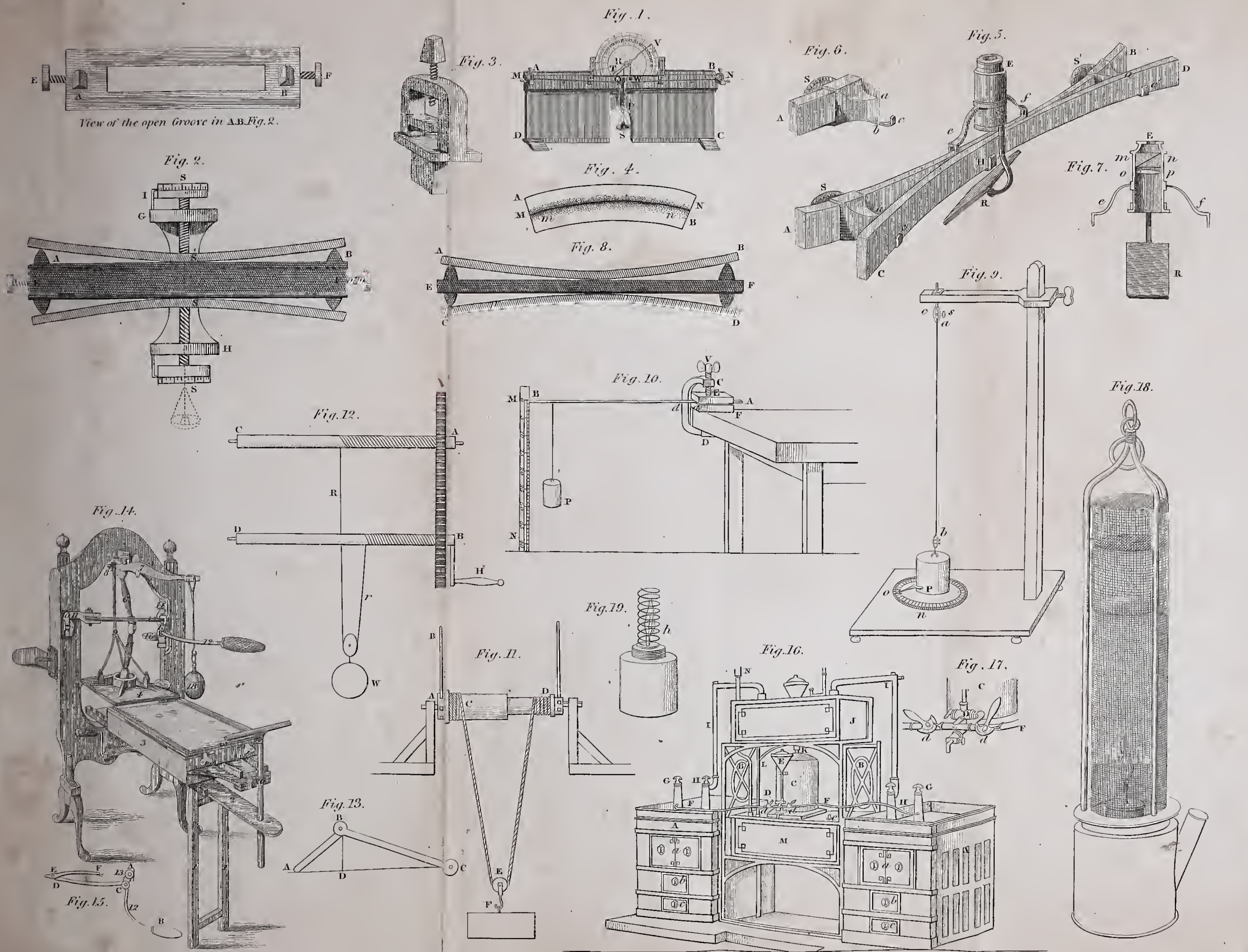

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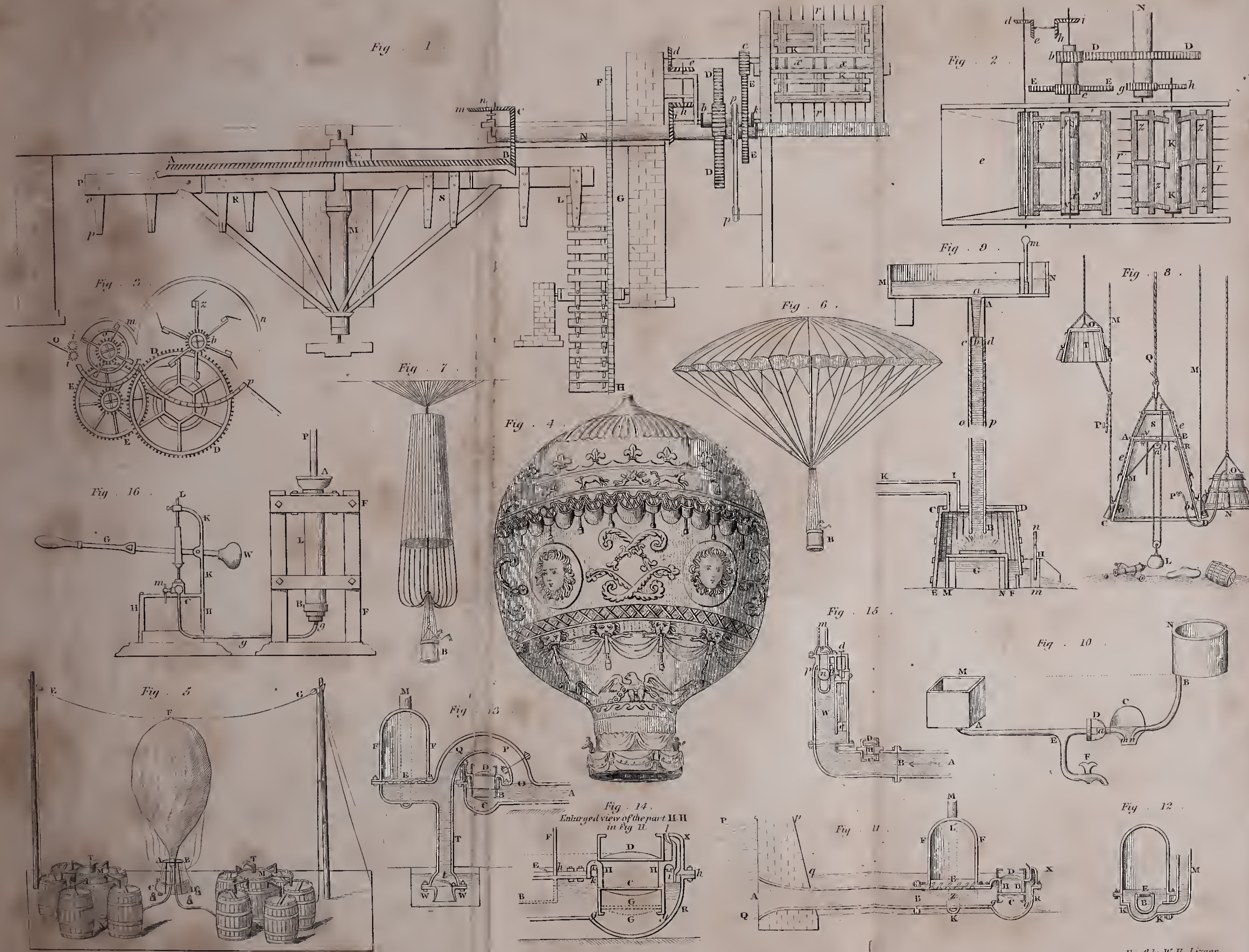

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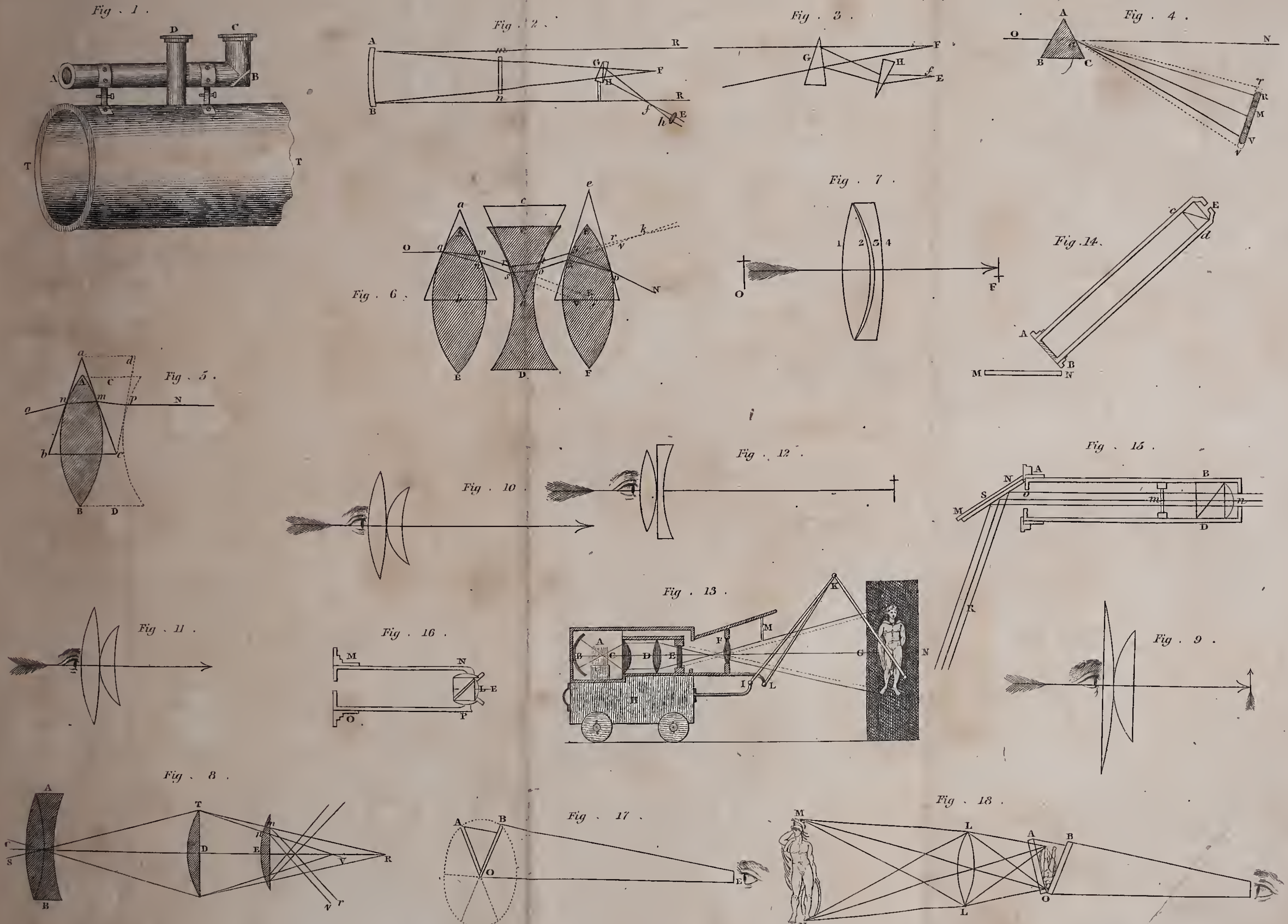

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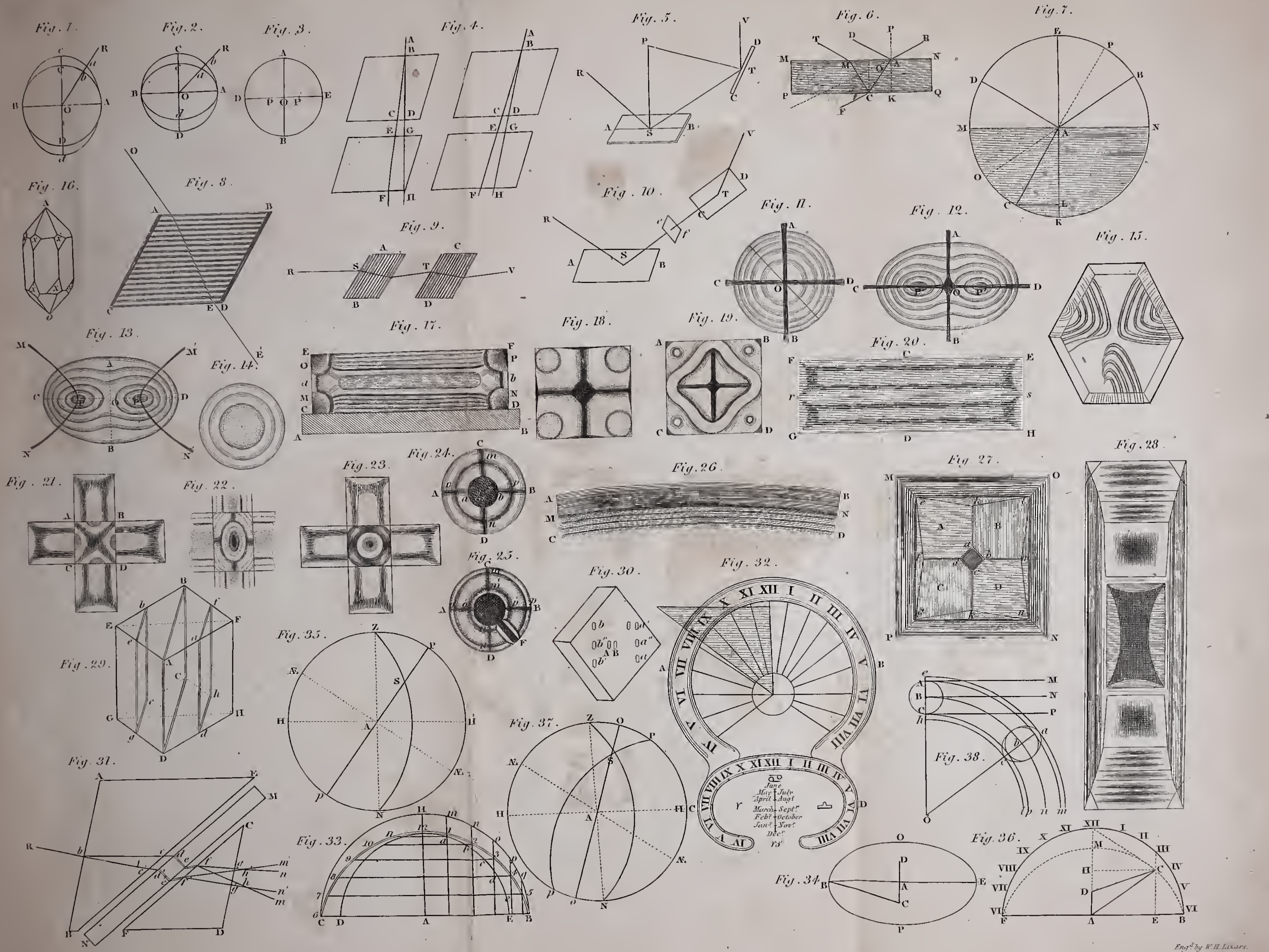

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